MATHEMATICS WORK BOOK

Class - XI

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রতন লাল নাথ
মন্ত্রী
শিক্ষা দপ্তর
ত্রিপুরা সরকার

বার্তা

শিক্ষার প্রকৃত বিকাশের জন্য, শিক্ষাকে যুগোপযোগী করে তোলার জন্য প্রয়োজন শিক্ষাসংক্রান্ত নিরন্তর গবেষণা। প্রয়োজন শিক্ষা সংশ্লিষ্ট সকলকে সময়ের সঙ্গে সঙ্গে প্রশিক্ষিত করা এবং প্রয়োজনীয় শিখন সামগ্রী, পাঠ্যক্রম ও পাঠ্যপুস্তকের বিকাশ সাধন করা। এস সি ই আর টি ত্রিপুরা রাজ্যের শিক্ষার বিকাশে এসব কাজ সুনামের সঙ্গে করে আসছে। শিক্ষার্থীর মানসিক, বৌদ্ধিক ও সামাজিক বিকাশের জন্য এস সি ই আর টি পাঠ্যক্রমকে আরো বিজ্ঞানসম্মত, নান্দনিক এবং কার্যকর করবার কাজ করে চলেছে। করা হচ্ছে সুনির্দিষ্ট পরিকল্পনার অধীনে।

এই পরিকল্পনার আওতায় পাঠ্যক্রম ও পাঠ্যপুস্তকের পাশাপাশি শিশুদের শিখন সক্ষমতা বৃদ্ধির জন্য তৈরি করা হয়েছে ওয়ার্ক বুক বা অনুশীলন পুস্তক। প্রসঙ্গত উল্লেখ্য, ছাত্র-ছাত্রীদের সমস্যার সমাধানকে সহজতর করার লক্ষ্যে এবং তাদের শিখনকে আরো সহজ ও সাবলীল করার জন্য রাজ্য সরকার একটি উদ্যোগ গ্রহণ করেছে, যার নাম ‘প্রয়াস’। এই প্রকল্পের অধীনে এস সি ই আর টি এবং জেলা শিক্ষা আধিকারিকরা বিশিষ্ট শিক্ষকদের সহায়তা গ্রহণের মাধ্যমে প্রথম থেকে দ্বাদশ শ্রেণির ছাত্র-ছাত্রীদের জন্য ওয়ার্ক বুকগুলো সুচারুভাবে তৈরি করেছেন। ষষ্ঠ থেকে অষ্টম শ্রেণি পর্যন্ত বিজ্ঞান, গণিত, ইংরেজি, বাংলা ও সমাজবিদ্যার ওয়ার্ক বুক তৈরি হয়েছে। নবম দশম শ্রেণির জন্য হয়েছে গণিত, বিজ্ঞান, সমাজবিদ্যা, ইংরেজি ও বাংলা। একাদশ দ্বাদশ শ্রেণির ছাত্র-ছাত্রীদের জন্য ইংরেজি, বাংলা, হিসাবশাস্ত্র, পদার্থবিদ্যা, রসায়নবিদ্যা, অর্থনীতি এবং গণিত ইত্যাদি বিষয়ের জন্য তৈরি হয়েছে ওয়ার্ক বুক। এইসব ওয়ার্ক বুকের সাহায্যে ছাত্র-ছাত্রীরা জ্ঞানমূলক বিভিন্ন কার্য সম্পাদন করতে পারবে এবং তাদের চিন্তা প্রক্রিয়ার যে স্বাভাবিক ছন্দ রয়েছে, তাকে ব্যবহার করে বিভিন্ন সমস্যার সমাধান করতে পারবে। বাংলা ও ইংরেজি উভয় ভাষায় লিখিত এইসব অনুশীলন পুস্তক ছাত্র-ছাত্রীদের মধ্যে বিনামূল্যে বিতরণ করা হবে।

এই উদ্যোগে সকল শিক্ষার্থী অতিশয় উপকৃত হবে। আমার বিশ্বাস, আমাদের সকলের সক্রিয় এবং নিরলস অংশগ্রহণের মাধ্যমে ত্রিপুরার শিক্ষাজগতে একটি নতুন দিগন্তের উন্মেষ ঘটবে। ব্যক্তিগত ভাবে আমি চাই যথাযথ জ্ঞানের সঙ্গে সঙ্গে শিক্ষার্থীর সামগ্রিক বিকাশ ঘটুক এবং তার আলো রাজ্যের প্রতিটি কোণে ছড়িয়ে পড়ুক।

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CONTENTS

Chapter - 1

Sets

Importnat Points and Results :

- **Set :** A set is a well defined collection of distinct objects. Each object is called an element or member of the set. A set is usually denoted by capital letters and its elements by small letters.
- **Representation of a set :** There are three ways to represent a set.

1. Tabular or Roster form : In this method, we make a list of the elements of the set and put it within braces { }.

   Ex. : (i) $\{1, 3, 5, ...\}$ (ii) $\{a, e, i, o, u\}$

2. Set builder form or Rule method : Under this method, we list the property or properties satisfied by the elements of the set.

   Ex. : (i) $\{x : x$ is a solution of $x^2+5x+6=0\}$

   (ii) $\{x : x \in R,\ x^2 = 9\}$

3. Statement or Descriptive form : In this form, we state or describe in words the elements of the set.

   Ex. : (i) Set of first five prime numbers.

   (ii) Set of all persons on the earth.

   Key points :

   (i) In a set, we do not repeat an element of a set more than once.

   (ii) A set cannot be changed if we change the order of the elements in it.

Types of set : Basically a set can be classified in two ways namely, (1) Finite set, (2) Infinite set.

(1) Finite Set :

A set is called a finite set if it contains no elements or its elements can be counted till a certain natural number.

Ex. : (i) $\{1, 4, 9, 16, 25\}$ (ii) $\{x : x$ is an integer & $|x|<5\}$

Under finite set, we have the following types.

(a) Empty set or Null set or Void set :

A set consisting of no element at all is called an empty set or a null set or a void set and it is denoted by ϕ or $\{\ \}$.

Ex. : (i) $\{x : x \in R,\ x^2 = -1\}$

(ii) A set of people who works more than 24 hours in a day.

(b) Singleton set :

A set containing only one element is called a singleton set.

Ex. : {1}, {x} etc.

(c) Equivalent sets :

Two finite sets X and Y are said to be equivalent if they have same number of elements.

Ex. : {1, 2, 3} & {a, b, c} are equivalent sets.

(d) Equal sets :

Two sets A and B are said to be equal, if every element of set A is in set B and every element of set B is in set A. It is written as A = B.

Ex. : {1, 3, 5} = {5, 3, 1}

But $\{1, 2, 3\} \neq \{a, b, c\}$

Thus, equal sets are equivalent but equivalent sets need not neccessarily be equal.

(2) Infinite set :

A set whose elements cannot be listed by the natural numbers 1,2,.............,n for any natural number is called an infinite set.

Ex.: (i) Set of all points in a plane.

(ii) $\{2,4,6,8,............\}$

(iii) $\{x : x \in R,\ 0 < x < 1\}$

We shall denote some important infinte sets by the following symbols :

N : The set of natural numbers

W : The set of whole numbers

I / Z : The set of integers

R : The set of real numbers

Q : The set of rational numbers

$\overline{Q}$: The set of irrational numbers

C : The set of complex numbers

R^+ : The set of positive real numbers

R^* : The set of non-zero real numbes

- **Power set :**

The collection of all possible subsets of a given set A is called power set of A and it is denoted by P(A) i.e., P(A)=$\{B : B \subseteq A\}$. If A is a set with n(A)=m, then $n[P(A)] = 2^m$.

- **Universal set :**

If there are some sets under consideration, then a set can be choosen which is a super set of each one of the given set. Such a set is known as the universal set and it is denoted by U or S or $\}$.

Ex. : We have, $N \subset W \subset Z \subset Q \subset R$

$\therefore$ R is the universal set.

- **Venn Diagrams :**

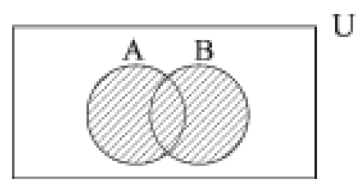
To express the relationship among sets in a perspective way, we represent them pictorially by means of diagrams, known as Venn-diagrams.

In Venn-diagram, the universal set is usually represented by a rectangular region and its subsets by a circle or ellipse inside this rectangular region.

- **Operations on sets and Venn-Diagrams :**

(a) Union of sets :

Union of two non-empty sets A and B is the set of all the elements which either belong to set A or set B or both and it is donoted by $A \cup B$ and is defined as $A \cup B = \{x : x \in A \text{ or } B \text{ or both}\}$



If $A \subset B$, then $A \cup B = B$.

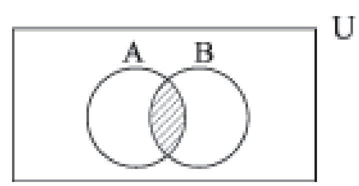
If $B \subset A$, then $A \cup B = A$.

Ex.: $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, then A .. B = {x, t} $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(b) Intersection of sets :

Intersection of two given non-empty sets A and B is the set of all the elements common to both A and B & we represent it by $A \cap B$ and its set theoretic representation is given by

$A \cap B = \{x : x \in A \text{ and } x \in B\}$



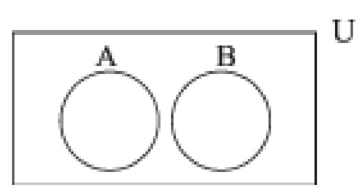
If $A \subset B$, then $A \cap B = A$.

If $B \subset A$, then $B \cap A = B$

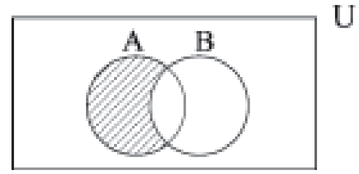
Ex. : If $A = \{x, y, z, t\}$, $B = \{x, r, s, t\}$ then $A \cap B = \{x, t\}$

(c) Disjoint sets :

Two sets A&B are said to be disjoint if they have no common elements or their intersection is a null set i.e. $A \cap B = \phi$.



(d) Difference of sets :

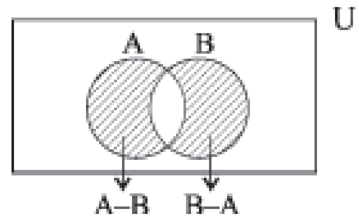


Let A and B be two given sets. Then the difference of set A and set B, donoted as A–B, is a set of all the elements which belong to set A but not to set B.

i.e., $A - B = \{x : x \in A \ \ and \ \ x \notin B\}$

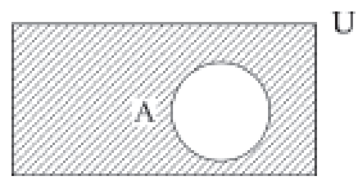
clearly, $A–B \neq B–A$ unless $A=B$.

(e) Symmetric difference of two sets :



Symmetric difference of two sets A & B is a set of elements which belongs to A or B but are not common to both the sets. It is donoted by $A\Delta B$ & is defined as $A\Delta B = (A–B)\cup(B–A)$

(f) Complement of a set :



Given a universal set U and a set A, then complement of set A is the set of all the elements which do not belong to A and it is denoted by A' or A^C or $\bar{A}$ or U–A and its set theoretic representation is given by $A' = \{x : x \in U \ \ and \ \ x \notin A\}$

Remark : (i) $A - B = A \cap B', B - A = B \cap A'$

(ii) $U' = \phi, \ \phi' = U$

(iii) $(A')' = A$

(iv) $A \cup A' = U$

(v) $A \cap A' = \phi$

- **Algebra of sets :**

(a) Idempotent laws : $A\cup A=A, \ \ A\cap A=A$

(b) Identity laws : $A\cup\phi=A, \ \ A\cap U=A$

(c) Commutative laws : $A\cup B=B\cup A, \ \ A\cap B=B\cap A$

(d) Associative laws : $(A \cup B) \cup C = A \cup (B \cup C)$
$A \cap (B \cap C) = (A \cap B) \cap C$

(e) Distributive laws : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(f) De Morgan's laws : $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

(g) Absorption laws : $A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

- **Cardinal number of a finite set :**

The numer of distinct elements contained in a finite set A is called its cardinal number and it is denoted by n(A).

If two sets A and B are equal then we write n(A) = n(B).

- **Subset :**

If A and B are two sets such that every element of A is in B, then A is a subset of set B and it is written as $A \subseteq B$.

If A is not a subset of B then we write $A \not\subseteq B$.

If $A \subseteq B$ and $B \subseteq A$, then A=B

If $A \subseteq B$ and $A \neq B$ i.e., $A \subset B$, then A is called the proper subset of B and B is called the super set of A.

Ex.: (i) $\{3\} \subset \{1,3,5\}$ (ii) $\{1,5\} \not\subset \{5,3,2\}$

Remark : (i) Every set is a subset of itself.

(ii) Empty set has no proper subset.

(iii) Empty set is the subset of every set.

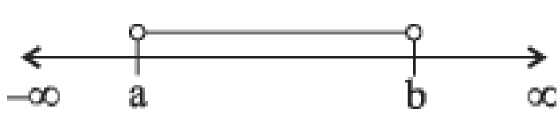
(iv) Every set (Except ϕ) has atleast two subsets.

(v) No. of subsets of a set having n elements = 2^n.

(vi) No. of proper subsets of a set having n elements = $2^n - 1$.

- **Types of Intervals :**

(a) Open interval : (a,b) = $\{x : x \in R \ and \ a < x < b\}$



(b) Closed interval : [a,b] = $\{x : x \in R \text{ and } a \le x \le b\}$

(c) Semi open or Semi closed interval :

(a,b] = $\{x : x \in R \text{ and } a < x \le b\}$

[a,b) = $\{x : x \in R \text{ and } a \le x < b\}$

Remarks : (i) [0, ∞) deonotes the set of non-negative real numbers.

(ii) (–∞,0) represents the set of negative real numbers.

(iii) (–∞,∞) denotes the set of real numbers.

(iv) b – a is called the length of any of the intervals (a,b) or [a,b] or [a,b) or (a,b].

- **Results on cardinal numbers (no. of elements in a set) of finite sets :**

If A, B and C are finite sets, and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= n(A - B) + n(B - A) + n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$, if A and B are disjoint.

(iii) $n(A - B) + n(A \cap B) = n(A)$

(iv) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= n(A) + n(B) + n(C)$, if A, B and C are mutually disjoint sets.

(vi) No. of elements in exactly one of the sets A, B, C

$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(vii) No. of elements in exactly two of the sets A,B,C

$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(viii) $n(A' \cup B') = n\left((A \cap B)'\right) = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n\left((A \cup B)'\right) = n(U) - n(A \cup B)$

Exercise – 1

Group - A

Ojective type questions : (1 or 2 marks each)

1. Multiple choice questions :

(i) Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are respectively

(a) 3, 6 (b) 3, 3 (c) 6, 3 (d) 6, 6

(ii) Number of proper subsets of a set is 63. The number of elements of the set

(a) 5 (b) 8 (c) 7 (d) 6

(iii) Let S = $\{$x|x is a positive multiple of 3 less than 100$\}$, P = $\{$x|x is a prime number less than 20$\}$. Then n(S) + n(P) is

(a) 41 (b) 31 (c) 33 (d) 30

(iv) Which one of the following is an empty set ?

(a) $A = \{x : x \in R \ \text{ and } \ x^2 = 9\}$

(b) $B = \{0\}$

(c) $C = \{x : x \in Z \ \text{ and } \ 6x^2 - 5x + 1 = 0\}$

(d) $D = \{x : x \in R \ \text{ and } \ -2 < x \leq 0\}$

(v) If $x \in A \Rightarrow x \in B$, then

(a) A=B (b) $A \subset B$ (c) $A \subseteq B$ (c) $B \subseteq A$

(vi) If $A \subseteq B$ and $B \subseteq A$, then

(a) $A = \phi$ (b) $A \cap B = \phi$ (c) A=B (d) None of these

(vii) Which of the following statements is correct ?

(a) $\{x\} \in \{x, y, z\}$ (b) $x \notin \{x, y, z\}$

(c) $x \subset \{x, y, z\}$ (d) $\{x\} \subset \{x, y, z\}$

(viii) A, B and C are finite sets such that n(A)=10, n(B)=15, n(C)=20, $n(A \cap B)=8$, $n(B \cap C)=9$, $n(C \cap A)=7$ and $n(A \cap B \cap C)=6$, then $n(A \cup B \cup C)$ is

(a) 26 (b) 27 (c) 28 (d) none of these.

(ix) Let, $A = \{x : x \in N\}$, $B = \{x : x = 2n, \ n \in N\}$, $C = \{x : x = 2n-1, \ n \in N\}$ and

$D=\{x:x \text{ is a prime}\}$. Then A∩C is

(a) B (b) C (c) D (d) $\{2\}$

(x) A= $\{1,2,3\}$, B=$\{3,4\}$, C=$\{4,5,6\}$, then A∪(B∩C) is

(a) $\{1,2,3,4,5,6\}$ (b) $\{1,2\}$ (c) $\{1,2,3,4\}$ (d) $\{1,2,4,5\}$

(xi) Let U be a universal set and A and B are two subsets of U such that n(U)=100, n(A)=40, n(B)=30 and n(A∩B)=10. The value of $(A' \cap B')$ is

(a) 20 (b) 60 (c) 30 (d) 40

(xii) n(A)=3, n(B)=6. The least value of n(A∪B) is

(a) 18 (b) 6 (c) 9 (d) none of these.

(xiii) If aN=$\{ax : x \in N\}$, then 4N∩6N equal to

(a) 6N (b) 12N (c) 24N (d) none of these.

(xiv) A=$\{x : x \in R, |x| > 1\}$, B=$\{x : x \in R, |x-1| \geq 1\}$ and A∪B=R–Y. The set Y equals to

(a) $\{x : 1 \leq x < 2\}$ (b) $\{x : 1 \leq x \leq 2\}$

(c) $\{x : 1 < x \leq 2\}$ (d) none of these.

(xv) A and B are two non-empty sets. Then, $(A \cup B^c)^c \cap (A^c \cup B)^c$ is equal to

(a) ϕ (b) U (c) A^c (d) B^c

(xvi) Which of the following sets is finite ?

(a) $\{x : x \in N \text{ and } x \text{ is prime}\}$

(b) $\{x : x^2-2x-15=0 \text{ and } x \in z^+ \text{ is prime}\}$

(c) $\{x : x \text{ is a people on earth}\}$

(d) $\{x : x \text{ is a multiple of } 3\}$

(xvii) Which of the following sets are equal ?

$A=\{x : x \in N,\ 2 \leq x \leq 3\}$, $B=\{x : x \in R,\ x^2-5x+6=0\}$

$C=\{x : x \in R,\ x^3+1=0\}$, $D=\{x : x \in R,\ 2 \leq x \leq 3\}$

(a) A, C (b) A, D (c) A, B (d) none of these.

(xviii) A = {s, o, u, r, a, v}, B = {s, a c, h, i, n} and C = {d, h, o, n, i}. Then A∩(BΔC) is

(a) {a, c, h} (b) {s, o, a} (c) {h, o, s} (d) none of these.

(xix) Among the Indians, 52% people like to drink coffee and 73% like to drink tea. If x% people like to drink both coffee and tea, then the value of x is

(a) $x \geq 25$ (b) $x \leq 52$ (c) $25 \leq x \leq 52$ (d) $x \geq 52$

(xx) A, B and C are three sets such that, $n(A)=17$, $n(B)=13$, $n(A\cap B)=9$, $n(B\cap C)=4$, $n(C\cap A)=5$, $n(A\cap B\cap C)=3$ and $n(U)=50$. Then $n(A\cap B^c\cap C^c)$ is

(a) 8 (b) 6 (c) 7 (d) none of these

(xxi) Let F_1 be the set of parallelograms, F_2 is the set of rectangles, F_3 is the set of rhombuses, F_4 is the set of squares and F_5 is the set of trapezium in a Plane. Then F_1 may be equal to

(a) $F_2\cap F_3$ (b) $F_3\cap F_4$ (c) $F_2\cap F_5$ (d) $F_2\cup F_3\cup F_4\cap F_1$

2. Very short answer type questions :

(i) Write the following sets in the roster form :

(a) $A = \{x \mid x^3 = x,\ x \in R\}$

(b) $B = \{x : x^4 - 5x^2 + 6 = 0,\ x \in R\}$

(c) $C = \left\{y \,\middle|\, \dfrac{y-2}{y+3} = 3,\ y \in R\right\}$

(d) $\{x : x$ is a prime number which is a divisor of $60\}$

(e) $\{x : x \in z$ and $|x| < 6\}$

(f) $\{x : x$ is a perfect square number and $2 < x \leq 49\}$

(ii) Write the following sets in set-builder form :

(a) A = {3, 9, 27, 81, ...}

(b) B = {........, 4,   2, 0, 2, 4, 6, ...}

(c) C = {1, 4, 9, 16, 25, ...}

(d) $\left\{1, \dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}, \ldots\ldots\ldots\ldots\right\}$

(iii) Classify the following as finite or infinite :

(a) Set of circles passing through three non-collinear points.

(b) The set of prime numbers less than 199.

(c) $\{x : x \in N$ and $(x-1)(x-2)(x-3) = 0\}$

(d) $\{x \in I : 0 < x < 2\}$

(e) The set of all letters in the word 'MATHEMATICS'.

(f) Set of concentric circles.

(iv) Classify the following pair of sets as 'equal' or 'equivalent' or ''none of these'.

(a) $A = \{\text{A, E, S, T}\}$

$B = \{x : x \text{ is a letter of the word ASSET}\}$

(b) $C = \{\oplus, \Diamond, \Leftrightarrow, \square\}$

$D = \{\alpha, \beta, \gamma, \delta\}$

(c) $E = \{x : x \text{ is a prime number less than } 14\}$

$F = \{2, 3, 5\}$

(d) $G = \{x : x^3 - 6x^2 + 11x - 6 = 0,\ x \in R\}$

$H = \{x : x^2 - 2x + 1 = 0,\ x \in R\}$

(v) Is the collection of ten most talented writers of India is a set or not ?

(vi) Write the power set of the set $X = \{x, y, z\}$.

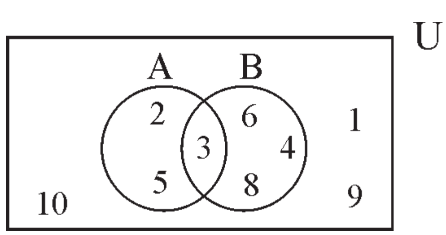
(vii) $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, 6, \ldots\ldots, 18\}$ and N is the universal set, then find $A' \cup \{A \cup B) \cap B'\}$

(viii) If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$. Find A–B.

(ix) Write all possible subsets of $\{\ 1, 0, 1\}$.

(x) What is the nature of the set $A = \{x \in z \mid x > 6\}$.

(xi) From the adjoining venn diagram, determine the following sets



(a) $A \cup B$ (b) $A \cap B$ (c) $A - B$

(d) B–A (e) $(A \cap B)'$ (f) $(A \cup B)'$

(xii) Using Venn diagram verify that,

(a) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

(b) $(A^c)^c = A$

(xiii) Applying set operations, prove that 2+3=5.

(xiv) If set $A = \{1\}$. How many elements $P\left[P\{P(A)\}\right]$ contains ?

(xv) What is the difference between ϕ and $\{\phi\}$?

(xvi) Out of 500 car owners investigated, 400 owned Maruti car and 200 owned Hyundai Car; 50 owned both cars. Is this data correct ?

(xvii) Are equal sets equivalent sets? What about the converse ?

Group - B

3. Short answer type questions : (3 marks each)

(i) If Y = { x|x is a positive factor of the number $2^{p-1}(2^p-1)$, where 2^p-1 is a prime number }. Write Y in roster form.

(ii) Given L = {1,2,3,4}, M={3,4,5,6} and N={1,3,5}. Verify that $L-(M\cup N)=(L-M)\cap(L-N)$

(iii) Let $T = \left\{ X \mid \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x} \right\}$. Is T an empty set? Justify your answer.

(iv) Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots, B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j's. Find n.

(v) For all sets A and B, show that $A-(A-B)=A\cap B$.

(vi) Applying set operations

(a) Find the H.C.F of 15, 40 and 105.

(b) Find the L.C.M of 12, 15 and 20.

(c) Show that the numbers 231 and 260 are prime to each other.

(vii) In a town of 840 persons, 450 persons read Hindi newspaper, 300 read English newspaper and 200 read both. Find the number of persons who read neither Hindi nor English newspaper.

(viii) A and B are two sets such that $n(A-B)=20+x$, $n(B-A)=3x$ and $n(A\cap B)=x+1$. If $n(A)=n(B)$, find the value of x.

(ix) In a group of 40 students, 26 take Physics, 18 take Mathematics and 8 take neither of the two subjects. How many take both Physics and Mathematics ?

(x) If set $A = \{4^n - 3n - 1,\ n \in N\}$ and set $B = \{9(n-1) : n \in N\}$. Show that A⊂B.

(xi) Let $U = \{x \in N \mid x \le 9\}$; A = { x:x is an even number 0<x<10 }, $B = \{2, 3, 5, 7\}$. Verify that $(A \cup B)' = A' \cap B'$.

(xii) $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$. Show that P=Q.

(xiii) Write all the subsets of the set $\{x : x \in z \ \& \ -1 \le x \le 2\}$.

(xiv) aN=$\{ax : x \in N\}$ and bN∩cN=dN where b, c∈N and they are coprime. Find the relation between b, c and d.

Group - C

B. Long answer type questions : (4 or 6 marks)

(i) Out of 1000 students in a college, 540 played football, 465 played cricket and 370 played volleyball; of the total 325 played both football and cricket, 260 played football and volleyball, 235 played cricket and volleyball, 125 played all the three games. How many students (a) did not play any game; (b) played only one game and (c) played just two games.

(ii) For any three seets A, B and C, prove that

(a) A∩(B–C) = (A∩B)–(A∩C)

(b) A–(B∩C) = (A–B)∪(A–C)

(iii) A market research group conducted a survey and it was found that 50% consumers like product A, 45% like product B, 40% like product C, 25% like A and B, 10% like B&C, 16% like C and A, 8% like all the three products. What percentage (a) do not like any product (b) like only product A (c) like exactly two products ?

(iv) In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows :

French=17, English=13, Sanskrit=15, French and English = 9, English and Sanskrit = 4, French and Sanskrit = 3. English, French and Sanskrit=3.

Find the number of students who study –

(a)French only, (b) English and Sanskrit but not French; (c) At least one of the three languages; (d) none of the three languages

ANSWERS

Group - A

1. (i) c (ii) d (iii) a (iv) c (v) c
(vi) c (vii) d (viii) b (ix) b (x) c
(xi) d (xii) b (xiii) b (xiv) d (xv) a
(xvi) b (xvii) c (xviii) b (xix) c (xx) b
(xxi) d

2. (i) (a) $\{0, 1, -1\}$ (b) $\{\sqrt{2}, \sqrt{3}, -\sqrt{2}, -\sqrt{3}\}$

(c) $\left\{-11/2\right\}$ (d) $\{2, 3, 5\}$

(e) $\{-5, -4, -3, -2, 0, 1, 2, 3, 4, 5\}$ (f) $\{4, 9, 16, 25, 36, 49\}$

(ii) (a) $\{x : x = 3^n, n \in N\}$ (b) $\{x : x = 2n, \text{ where } n \in Z\}$

(c) $\{x : x = n^2, \text{ where } n \in Z\}$ (d) $\left\{x : x = \frac{1}{n}, \text{ where } n \in N\right\}$

(iii) (a) Finite (b) Finite (c) Finite
(d) Infinite (e) Finite (f) Infinite.

(iv) (a) Equal (b) Equivalent (c) None of these (d) None of these

(v) Not a set as the term talented is not well defined.

(vi) $P(X) = \{\{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, X, \phi\}$

(vii) N

(viii) $\{1,3,5\}$

(ix) $\{-1\}, \{0\}, \{1\}, \{-1,0\}, \{-1,1\}, \{0,1\}, \{-1,0,1\}, \phi$

(x) Infinite set

(xi) (a) $\{2,3,4,5,6,8\}$ (b) $\{3\}$ (c) $\{2,5\}$
(d) $\{6,4,8\}$ (e) $\{1,2,4,5,6,8,9,10\}$ (f) $\{1,9,10\}$

(xiv) 16

(xv) ϕ represents an empty set & $\{\phi\}$ denotes a singleton set.

(xvi) No

(xvii) Yes. The consverse is not always true.

Group - B

3. (i) $\{1,2,2^2,.......,2^{p-1}\}$

   (iii) No.

   (iv) 45

   (vi) (a) 5 (b) 60

   (vii) 290

   (viii) 10

   (ix) 12

   (xiii) {–1}, {0}, {1}, {2}, {–1,0}, {–1,1}, {–1,2}, {0,1}, {0,2}, {1,2}, {–1,0,1}, {0,1,2}, {1,2,–1}, {–1,0,1,2}, ϕ, {2,–1,0}

Group - C

4. (i) (a) 320 (b) 110 (c) 445

   (iii) (a) 8% (b) 17% (c) 27%

   (iv) (a) 6 (b) 1 (c) 30 (d) 20

Chapter - 2

Relations and Functions

Important points and Results :

- **Orderded Pair :**

  An ordered pair consists of two elements written within parenthesis in a fixed order.

  In Cartesian plane, the first element of an ordered pair is called x coordinate (abscissa) and the second element is called y-coordinate (ordinate). It helps to locate a point on a plane.

  e.g. (2,3), (1,0), (x,y) etc.

  Two ordered pairs (a,b) and (c,d) are said to be equal if their first entries and second entries are equal i.e. $(a,b)=(c,d) \Leftrightarrow a=c$ and $b=d$.

- **Cartesian Product :**

  Let A and B be two given non-empty sets. Then their Cartesian product, denoted by A×B is the set of all ordered pairs (a,b) such that $a \in A$ and $b \in B$.

  In symbolic form, it is written as,

  $A \times B = \{(a,b) : a \in A \ and \ b \in B\}$

  e.g. If A = $\{x,y,z\}$ and B=$\{a,b\}$, then

  $A \times B = \{(x,a),(x,b),(y,a),(y,b),(z,a),(z,b)\}$

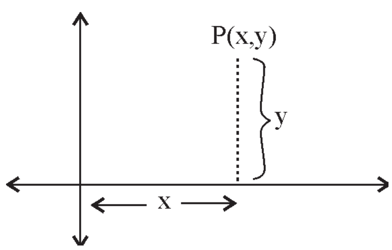
  $B \times A = \{(a,x),(a,y),(a,z),(b,x),(b,y),(b,z)\}$

  Clearly, $A \times B \neq B \times A$, unless A=B.

  For three given sets A, B and C

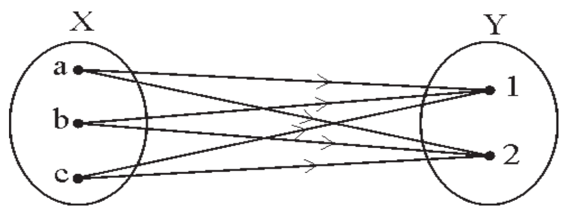
  $A \times B \times C = \{(a,b,c) : a \in A,\ b \in B \ and \ c \in C\}$ where (a,b,c) is called ordered triplet.
  We extend this notion for more than three sets also. We use tree diagram to find the elements of the Cartesian product of three or more sets.

- Ordered pair (x,y) can be plotted on a plane by drawing two perpendicular lines taken as x-axis and y-axis.



- The diagrammatic representation of Cartesian product of two sets can be shown by using arrow diagram.

  Ex. If $X=\{a,b,c\}$, $Y=\{1,2\}$, then the following figure gives the arrow diagram of $X \times Y$.



- If A and B are two finite sets, then $n(A\times B) = n(A)\times n(B)$.

  If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$.

  Remark :

  i) If either A or B is an infinite set, then $A\times B$ is an infinite set.

  ii) If A, B, C are finite sets, then, $n(A\times B\times C) = n(A)\times n(B)\times n(C)$

  iii) $A_1\times A_2\times ....\times A_n=\{(a_1,a_2..........,a_n) : a_1 \in A_1,\ a_2 \in A_2,..........,a_n \in A_n\}$

- **Relation :**

  If A and B are two non-empty sets then a relation R from the set A to the set B is defined as a subset of $A\times B$ and it is written as $R\subseteq A\times B$. Also, $(x,y)\in R \Rightarrow (x,y)\in A\times B$. If $(x,y)\in R$, we say x is in relation R to y and write, xRy. If x is not in relation R to y then we write $x \not R y$.

  Ex. : A relation R is defined from set $A=\{2,3,4,5\}$ to a set $B=\{3,6,7,10\}$ as follows : $(x,y)\in R$ $\Leftrightarrow$ x divides y. Thus, $R=\{(2,6), (2,10), (3,3), (3,6), (5,10)\}$

  Here, the set of all first elements of ordered pair in a relation R is called the domain and set of all the second elements of ordered pairs in a relation R is called the range. The set B is called the co-domain of relation R.

  Range $\subseteq$ Co-domain.

  Thus, domain of R = $\{a : (a,b)\in R\}$

  range of R = $\{b : (a,b)\in R\}$

- If n(A)=m and n(B)=n then the total number of relations from A to B is 2^{mn}.

- **Representation of a Relation**

  i) Roster form :

  Let $A = \{-2,-1,0,1,2\}$ and $B = \{0,1,4,9\}$.

If R is a relation from set A to B such that aRb $\Leftrightarrow$ a^2=b

Then, R can be described in roster form as R = {(0,0), (–1,1), (–2,4), (1,1), (2,4)}

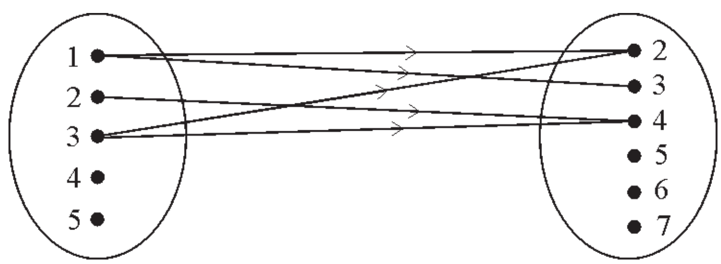
ii) Set-builder form

R = $\left\{(a,b): a \in A, b \in B \text{ and } b = \frac{1}{a}\right\}$

iii) By arrow diagram

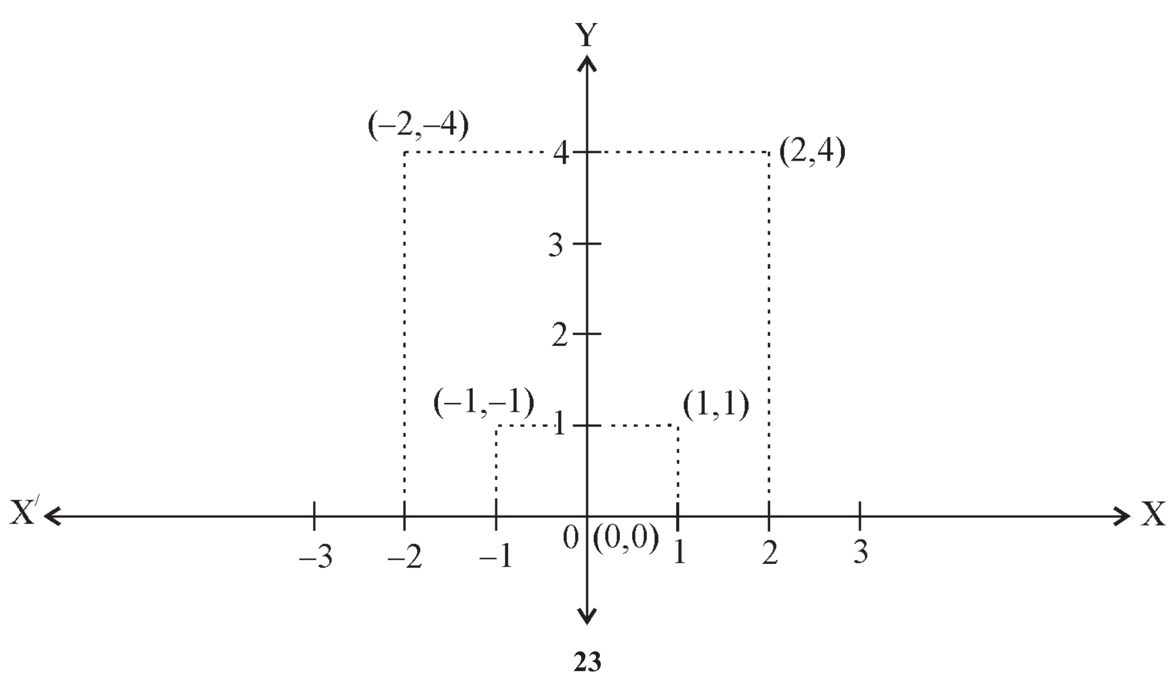
Let R = $\{$(1,2), (2,4), (3,2), (1,3), (3,4)$\}$ be a relation from set A = $\{$1,2,3,4,5$\}$ to B = $\{$2,3,4,5,6,7$\}$.

Then, it can be represented by the following arrow diagram.



iv) By Lattice

If R =$\{$ (–2,4), (–1,1), (0,0), (1,1), (2,4)$\}$ is a relation from set A=$\{$–3,–2,–1,0,1,2,3$\}$ to set B=$\{$0,1,2,3,4,5,6,7,8,9$\}$, then R can be represented by the following lattice.



- **Types of Relations :**

  i) Void or empty Relation : For any set A, $\phi \subset A \times A$

  ii) Universal Relation : For any set A, $A \times A \subseteq A \times A$

  iii) Identity Relation : $I_A = \{(a,a) : a \in A\}$

  iv) Reflexive Relation : aRa, $\forall\, a \in A$.

  v) Symmetric Relation : $aRb \Rightarrow bRa$, $\forall\, a, b \in A$

  vi) Anti-symmetric Relation : aRb and bRa $\Rightarrow a=b$, $\forall\, a, b \in A$

  vii) Transitive Relation : $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$, $\forall\, a,b,c \in A$.

  viii) Equivalence Relation : If a relation R is reflexive, symmetric and transitive on a set A.

  ix) Partial order Relation : If R is reflexive, symmetric and anti-symmetric on a set A.

  x) Total order Relation : If R is a partial order relation on A.

- **Some useful Results :**

  For any four sets A, B, C & D

  i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ and

  ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

  iii) $A \times (B - C) = (A \times B) - (A \times C)$

  iv) $A \times B = B \times A \Leftrightarrow A = B$

  v) If $A \subseteq B$ then $A \times A \subseteq (A \times B) \cap (B \times A)$

  vi) If $A \subseteq B$, then $A \times C \subseteq B \times C$

  vii) If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

  viii) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

  ix) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

  x) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

  xi) If A and B having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.

  xii) For a non-empty set A, $A \times B = A \times C \Rightarrow B = C$.

  xiii) $A \times \phi = \phi$ and $\phi \times A = \phi$

  xiv) If $A \neq B$, then $A \times B \neq B \times A$

- Let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by

  $R^{-1} = \{(b,a) : (a,b) \in R\}$

Also, Dom(R) = Range(R^{-1}) and Range(R) = Dom(R^{-1})

- **Function / Mapping :**

  A relation R from set A to set B is known as a function if its first entry is not repeated. We also define function f from set A to set B as a rule which assigns to each element of set A a unique element of set B, and we write as f : A→B or $A \xrightarrow{f} B$.

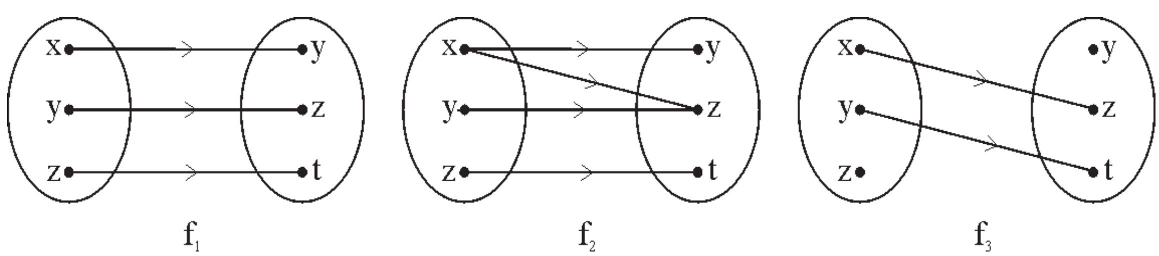
  Ex. : Let A = {x,y,z}, B={y,z,t} and f_1, f_2 and f_3 be three subsets of A×B as given by :

  f_1= {(x,y), (y,z), (z,t)}

  f_2 = {(x,y), (x,z), (y,z), (z,t)}

  f_3 = {(x,z), (y,t)}

  Here, f_1 is a function from A to B but f_2 and f_3 are not functions. We also represent f_1, f_2 and f_3 are not functions. We also represent f_1, f_2 and f_3 by arrow diagram.



- If f : A→B then set A is known as domain of the function and set B is known as its co-domain. Set of all the second entries of ordered pairs in a function is called range of the function. Range is a subset of co-domain.

  Ex. : Let A={2,3,5,7,11} and B={1,2,3,.....,200} be two sets and f : A→B such that f = {(2,4), (3,9), (5,25), (7,49), (11,121)}

  Then, Domain of f (D_f) = A, Co-domain of f = B and Range of f (D_f) (R_f) = {4, 9, 25, 49, 121}.

- If f : A→B and element x of set A under rule 'f' takes the value 'y' from set B, then 'x' is known as independent variable and 'y' is known as dependent variable. We also call y as image of x or x as pre-image of y.

- Two functions f and g are said to be equal if f –

  a) domain of f = domain of g

  b) co-domain of f = co-domain of g, and

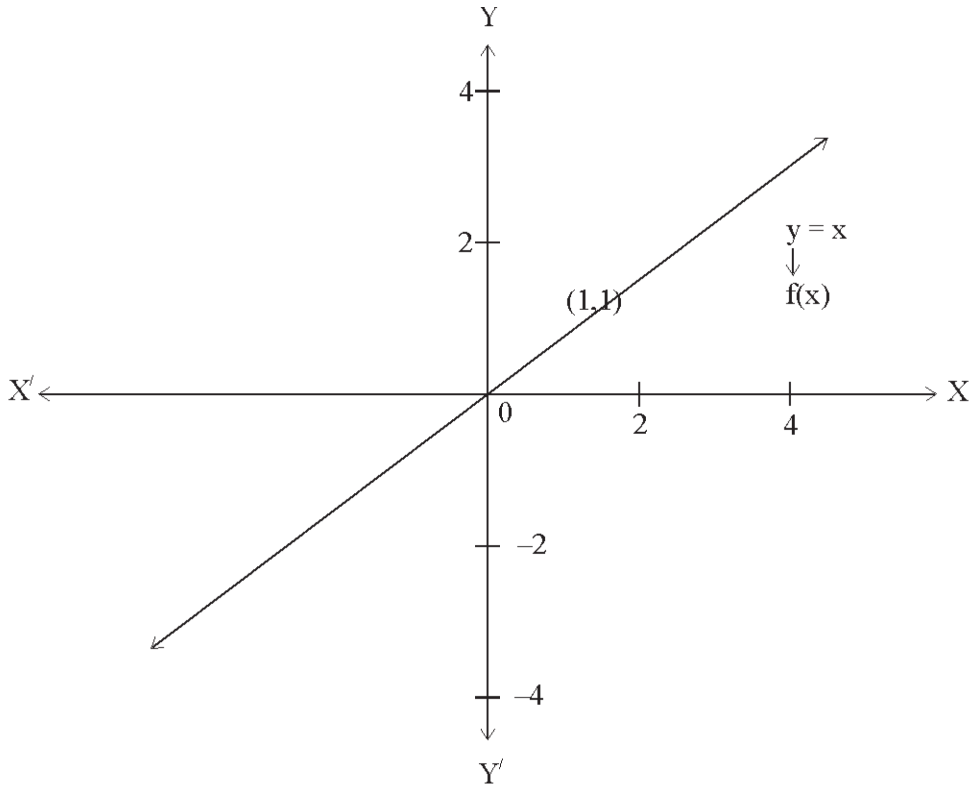
  c) f(x) = g(x) for every x belonging to their common domain and we write f = g.

- A function whose domain and co-domain are the sets of real numbers is known as a real valued function i.e., f : R→R.

- **Some standard real functions and their graphs.**

a) Identity function :

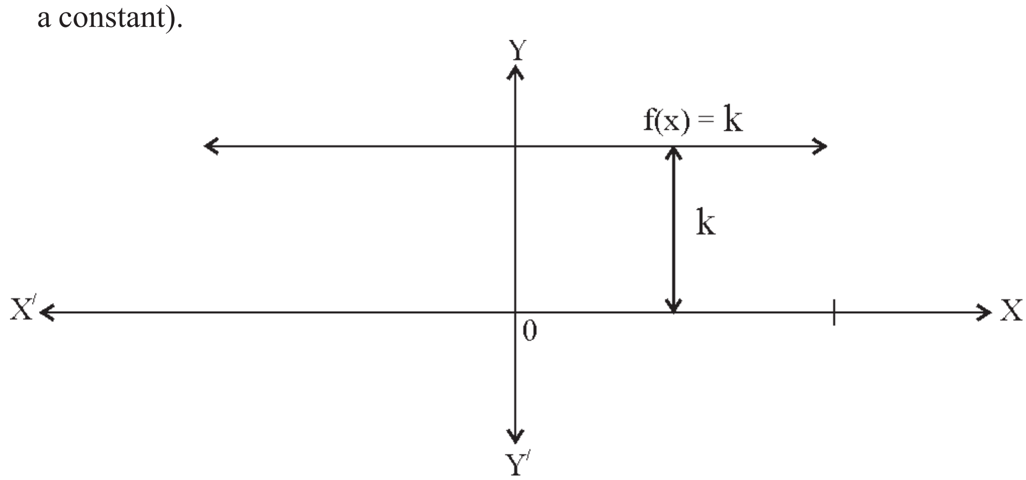
The function $f : R \rightarrow R$ defined by $f(x) = x$, for all $x \in R$ is called the identity function.



$D_f = R$, $R_f = R$

b) Constant function :

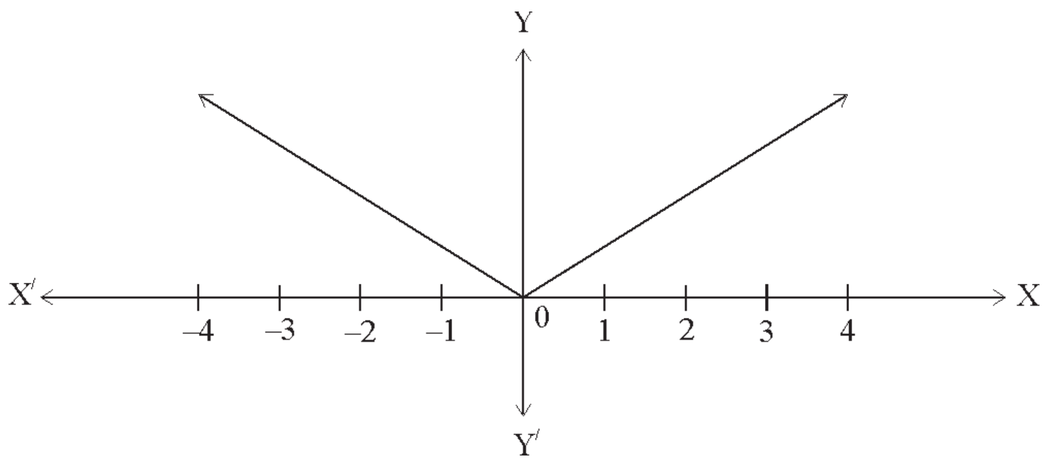
The function $f : R \rightarrow R$ defined by $f(x) = k \; \forall \; x \in R$ is called constant function (where K is a constant).



$D_f = R$, $R_f = \{k\}$

c) Modulus function :

f : R→R defined by $f(x)=|x|=\begin{cases} x, & when\ x \geq 0 \\ -x, & when\ x < 0 \end{cases}$ is called the modulus function or absolute value function.



D_f=R, R_f=$R^{+} \cup \{0\}$

Properties :

i) For any real x, $\sqrt{x^2}=|x|$

ii) $|x| \leq a \Leftrightarrow -a \leq x \leq a$

iii) $|x| \geq a \Leftrightarrow x \leq -a\ or,\ x \geq a$

iv) $a \leq |x| \leq b \Leftrightarrow x \in [-b,-a] \cup [a,b]$

v) $a < |x| < b \Leftrightarrow x \in (-b,-a) \cup (a,b)$

vi) For real nos x and y, we have

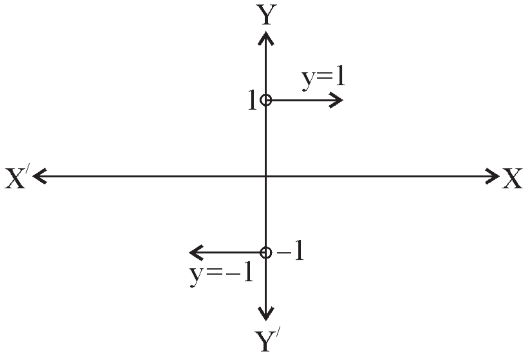
$|x+y| \leq |x|+|y|$

$|x-y| \geq \big||x|-|y|\big|$

d) Signum function :

The function f : R→R defined by

$$f(x)=\frac{|x|}{x}=\begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases}$$



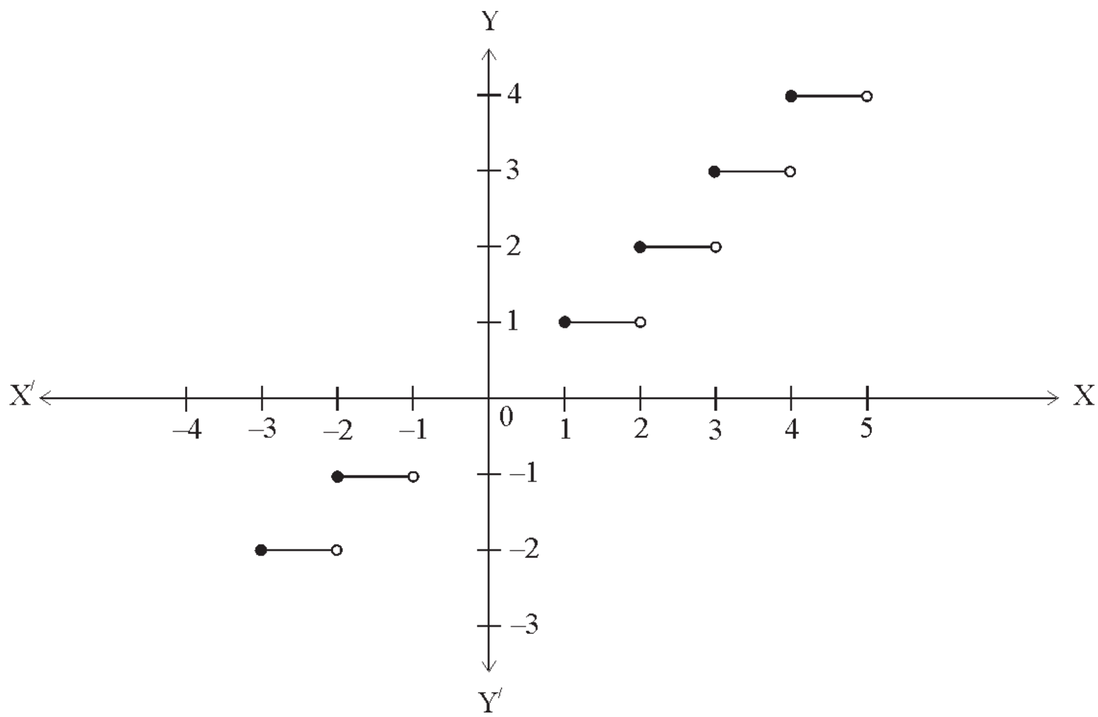
is known as signum function.

$D_f = R$, $R_f = \{-1, 0, 1\}$

e) Greatest Integer function (Floor function)

The function f : R→R defined by $f(x)=[x]$ or $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.

e.g. $[2.35] = 2$, $[-1.75] = -2$



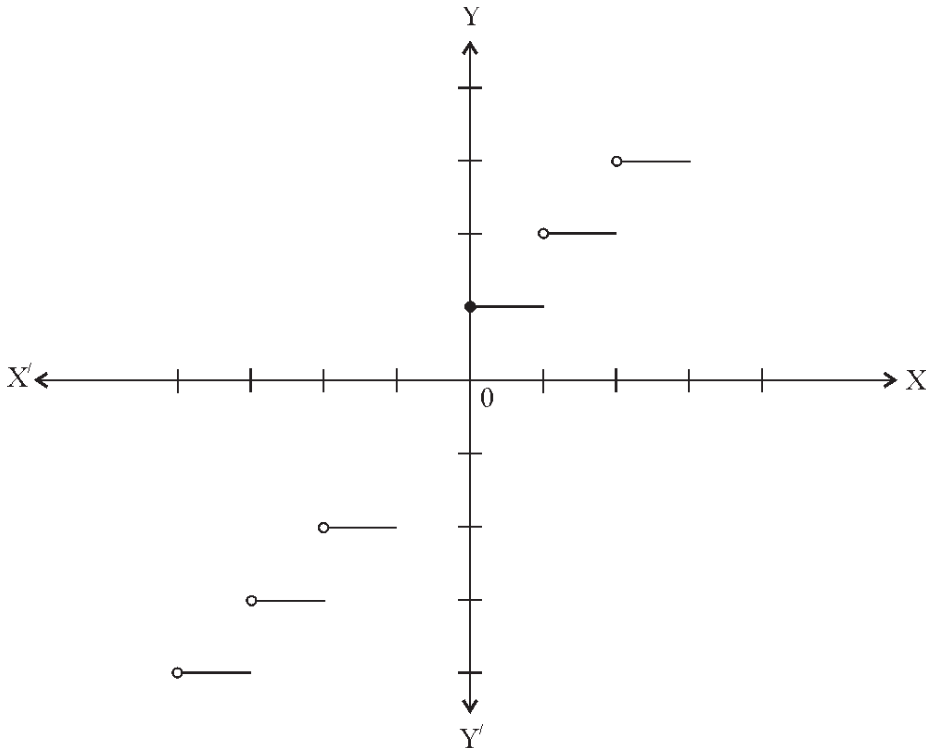
$$[x] = -1, \text{ for } -1 \le x < 0$$
$$= 0, \text{ for } 0 \le x < 1$$
$$= 1, \text{ for } 1 \le x < 2 \text{ etc.}$$

$D_f = R$, $R_f = Z$

f) Smallest integer function (ceiling function)

The function f : R→R defined by $f(x) = \lceil x \rceil$ denote the smallest integer greater than or equal to x.

e.g. $\lceil 4.7 \rceil = 5$, $\lceil -7.2 \rceil = -7$



$D_f = R,\ R_f = Z$

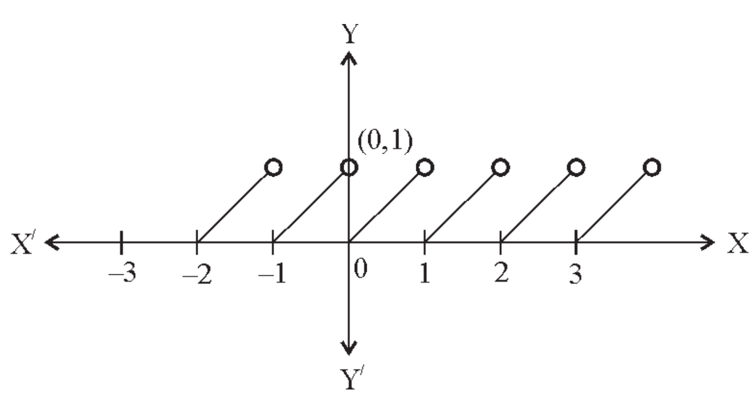
g) Rational function :

A function $f(x) = \dfrac{p(x)}{q(x)}$, where p(x) and q(x) are polynomial functions and $q(x)\neq 0$ is known as a rational function $D_f = R-\{\text{zeroes of } q(x)\}$

h) Fractional Part function :

The function $f : R\to R$ such that $f(x)=\{x\}$ is called the fractional part function.

It can also be written as $f(x) = \{x\}=x-[x]$, for all $x\in R$



$D_f = R,\ R_f = [0,1)$

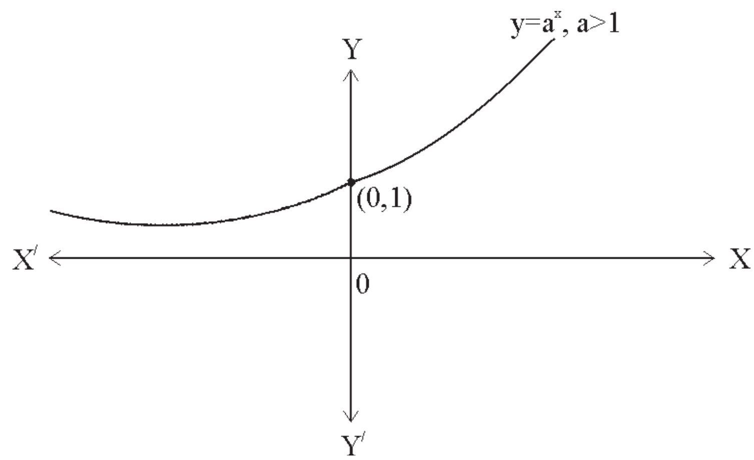
i) Exponential function :

A function $f : R \to R$ defined by $f(x)=a^x$, where $a>0$ and $a\neq 1$ is called the exponential function.

$D_f = R$, $R_f = (0,\infty)$

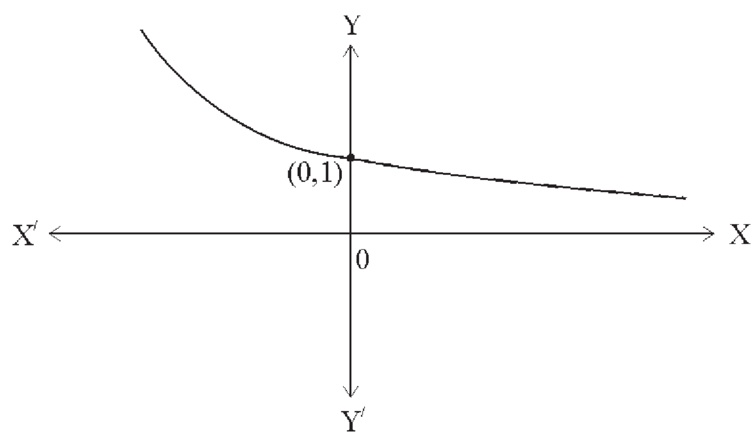
Case-I : When $a>1$

$$f(x) = a^x \begin{cases} <1, & for\ x<0 \\ =1, & for\ x=0 \\ >1, & for\ x>0 \end{cases}$$



Case-2 : When $0<a<1$

$$f(x) = a^x \begin{cases} >1, & for\ x<0 \\ =1, & for\ x=0 \\ <1, & for\ x>0 \end{cases}$$



Remark : We have $2<e<3$. Therefore, graph of $f(x)=e^x$ is identical to that of $f(x)=a^x$ for $a>1$ and $f(x)=e^{-x}$ is identical to that of $f(x)=a^x$ for $0<a<1$.

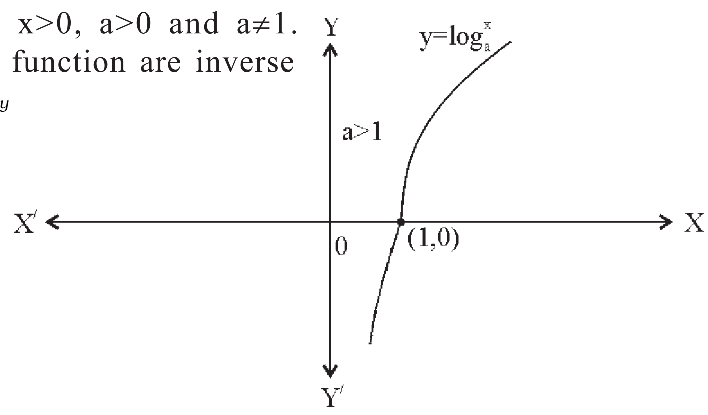
j) Logarithmic function :

The function $f : R \to R$ defined by $f(x)=\log_a x$ is called the logarithmic function where $x>0$, $a>0$ and $a\neq 1$. Logarithmic and exponential function are inverse functions i.e. $\log_a x = y \Leftrightarrow x = a^y$

$D_f = (0,\infty)$, $R_f = R$

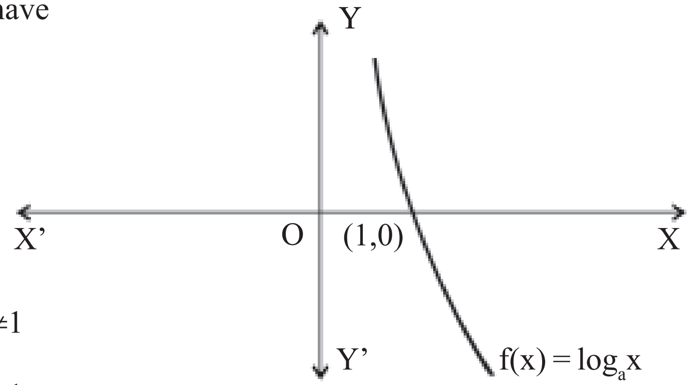
Case-I : When $a>1$, we have

$$y = \log_a x \begin{cases} <0 & for\ 0<x<1 \\ =0 & for\ x=1 \\ >0 & for\ x>1 \end{cases}$$



Case-II : When 0<a<1, we have

$$y = \log_a x \begin{cases} > 0 & for\ 0 < x < 1 \\ = 0 & for\ x = 1 \\ < 0 & for\ x > 1 \end{cases}$$



Properties :

i) $\log_a 1 = 0$, where a>0, a≠1

ii) $\log_a a = 1$, where a>0, a≠1

iii) $\log_a (xy) = \log_a |x| + \log_a |y|$, where a>0, a≠1 and xy>0

iv) $\log_a \left(\frac{x}{y}\right) = \log_a |x| - \log_a |y|$, where a>0, a≠1 and $\frac{x}{y}$>0

v) $\log_a (x)^n = n \log |x|$, where a>0, a≠1 and x^n>0

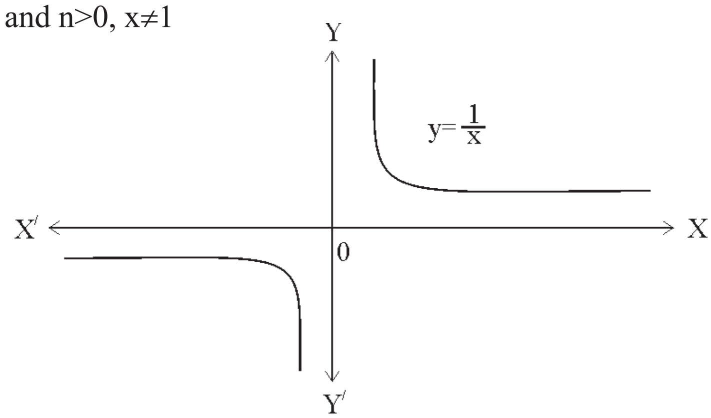
vi) $a^{\log_a^x} = x$

vii) $\log_a^x = \frac{1}{\log_x^a}$ for a>0, a≠1 and n>0, x≠1

viii) $\log_a^x = \log_b^x \times \log_a^b$

k) Reciprocal function :

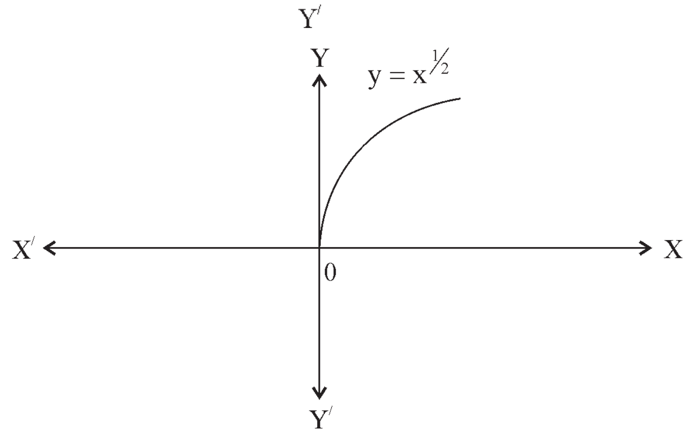
The function f : R–{0}→R defined by f(x) = $\frac{1}{x}$ is called the reciprocal function.



D_f = R–{0}, R_f = R–{0}

***l*) Square root function :**

The function f : R^+→R defined by f(x) = $+\sqrt{x}$ is called the square root function.

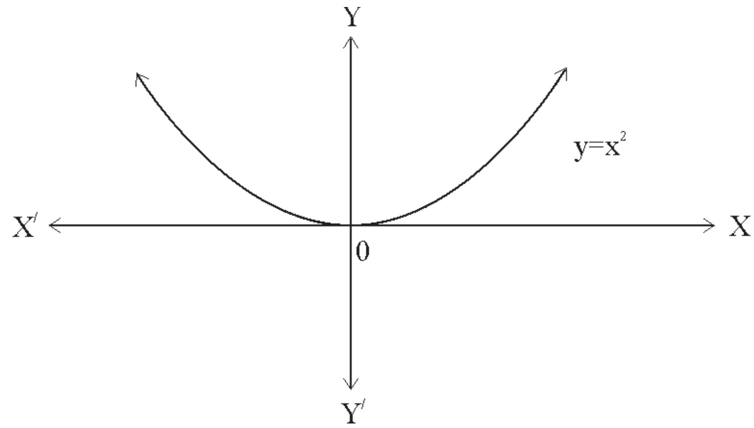


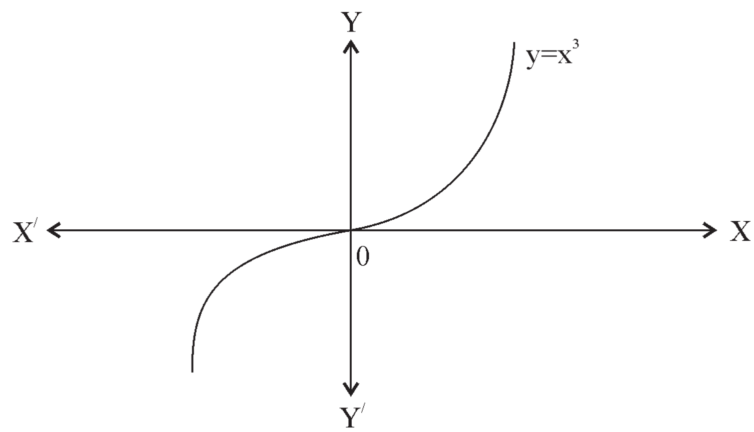
D_f = [0,∞), R = [0,∞)

m) **Square function :**

The function $f : R \to R$ defined by $f(x) = x^2$ is called the square function

$D_f = R$, $R_f = [0, \infty)$





n) **Cube function :**

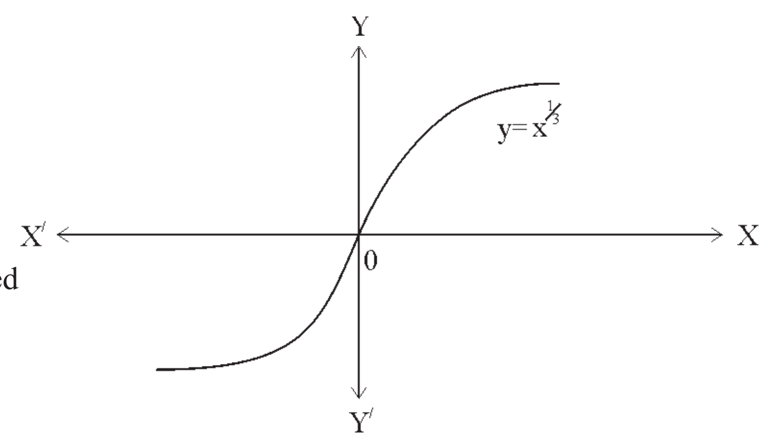
The function $f : R \to R$ defined by $f(x) = x^3$ is called the cube function.

$D_f = R_f = R$

o) **Cube root function :**

The function $f : R \to R$ such that $f(x) = x^{1/3}$ is called the cube root function

$D_f = R_f = R$



p) **Even function :**

A function 'f' is said to be an even function, if $f(-x) = f(x)$.

Ex. : x^2, cosx etc.

q) **Odd function :**

A function 'f' is said to be an odd function if $f(-x) = -f(x)$.

Ex. : x^3, $\sin^7 x$ etc.

- **Algebra of functions :**

If 'f' and 'g' are two given functions with domains D_1 and D_2, then

i) $(f \pm g)(x) = f(x) \pm g(x)$

ii) $(fg)(x) = f(x).g(x)$

iii) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$,

iv) $(cf)(x) = cf(x)$ where, c is scalar.

Domain of $f \pm g$, fg is $D_1 \cap D_2$ and Domain of $\frac{f}{g}$ is $D_1 \cap D_2 - \{\text{zeroes of } g(x)\}$

Exercise – 2

Group - A

Objective Type questions : [1 or 2 marks]

1. **Multiple Choice Questions :**

i). If R be a relation from a set P to a set Q, then

a) $R = P \cup Q$ b) $R = P \cap Q$ c) $R \subseteq P \times Q$ d) $R \subseteq Q \times P$

ii) If the set X has x elements, Y has y elements, then the number of elements in $X \times Y$ is

a) x+y b) x+y–1 c) x^2 d) xy

iii) If $R = \{(x, y) : x, y \in z,\ x^2 + y^2 \leq 4\}$ is a relation on Z, then domain of R is

a) {0, 1, 2} b) {0, –1, –2} c) {–2, –1, 0, 1, 2} d) none of these

iv) If R is a relation on a finite set having n elements, then the number of relations on A is

a) 2^n b) 2^{n^2} c) n^2 d) n^n

v) If A={1,2,4}, B={2,4,5}, C={2,5}, then (A–B)×(B–C) is

a) {(1,4), (2,3)} b) {(1,4)} c) {(1,2), (1,5), (2,5)} d) none of these

vi) If R is a relation on the set A = {1,2,3,4,5,6,7,8,9} given by $xRy \Leftrightarrow y=3x$, then R =

a) {(3,1), (6,2), (8,2), (9,3)} b) {(3,1), (6,2), (9,3)}

c) {(1,3), (2,6), (3,9)} d) none of these

vii) Let A={1,2,3}, B={1,3,5}. If relation R from A to B is given by R ={(1,5), (2,3), (3,3), (2,5)}. Then R^{-1} =

a) {(3,3), (3,1), (5,2)} b) {(5,1), (3,2), (3,3), (5,2)}

c) {(1,3), (5,2), (5,1)} d) none of these

viii) If A and B are two sets having 3 elements in common. If n(A) = 5, n(B) = 4, then $n\left[(A\times B)\cap(B\times A)\right]$

a) 8 b) 12 c) 9 d) none of these.

ix) Let R be set of points inside a rectangle of sides a and b (a,b>1) with two sides along the positive direction of x-axis and y-axis. Then –

a) $R=\{(x,y): 0\le x\le a,\ 0\le y\le b\}$

b) $R=\{(x,y): 0\le x< a,\ 0\le y\le b\}$

c) $R=\{(x,y): 0< x< a,\ 0< y< b\}$

d) $R=\{(x,y): 0\le x\le a,\ 0< y< b\}$

x) If sets A and B are defined as A=$\left\{(x,y)\,\middle|\,y=\frac{1}{x}, x\ne 0, x\in R\right\}$, B=$\left\{(x,y)\,\middle|\,y=-x, x\in R\right\}$ then

a) A∩B = A b) A∩B = ϕ c) A∩B = B d) A∪B = A

xi) If (x–2, y+5) = $\left(2, \frac{1}{2}\right)$ then the values of x and y are

a) $x=-4,\ y=\frac{14}{3}$ b) $x=-4,\ y=-\frac{14}{3}$ c) $x=4,\ y=\frac{14}{3}$ d) $x=4,\ y=-\frac{14}{3}$

xii) If $\left\{(x,|x|)\,\middle|\,x\in R\right\}$ is a relation, then the range of R is

a) (0, ∞) b) (–∞, 0) c) [0, ∞) d) (–∞, ∞)

xii) Range of f(x)=$\frac{1}{1-2\cos x}$ is

a) $\left[\frac{1}{3},1\right]$ b) $\left[-1, \frac{1}{3}\right]$ c) $(-\infty,-1]\cup\left[\frac{1}{3},\infty\right)$ d) $\left[-\frac{1}{3}, 1\right]$

xiv) If f(x) = $\sqrt{1+x^2}$, then

a) f(xy) = f(x).f(y) b) f(xy) ≥ f(x).f(y) c) f(xy) = f(x)+f(y) d) f(xy) ≤ f(x).f(y)

xv) Domain of $\sqrt{c^2 - x^2}$ (c>0) is

a) (–c, c) b) [–c, c] c) [0, c] d) (–c, 0]

xvi) If f(x) = px+q, where p and q are integers, f(–1) = –5 and f(3) = 3, then p and q are equal to

a) p = –3, q = –1 b) p = 2, q = –3 c) p = 0, q = 2 d) p = 2, q = 3

xvii) The domain for which the function defined by f(x) = $3x^2$–1 and g(x) =3+x are equal is

a) $\left\{-1, \frac{4}{3}\right\}$ b) $\left[-1, \frac{4}{3}\right]$ c) $\left(-1, \frac{4}{3}\right)$ d) $\left[-1, \frac{4}{3}\right)$

xviii) The domain of the function $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$ is

a) R–{3,–2} b) R–{–3,–2} c) R–[3,–2] d) R–{–3,–2}

xix) The domain and range of the function f given by f(x) = 2–|x–5| is

a) Domain = R^+, Range = (–∞, 1] b) Domain=R, Range = (–∞, 2)

c) Domain = R, Rannge = (–∞, 2) d) Domain = R^+, Range = (–∞, 2]

xx) The domain of $f(x) = \sqrt{4 - x} + \dfrac{1}{\sqrt{x^2 - 1}}$ is equal to

a) $(-\infty, -1) \cup (1, 4]$ b) $(-\infty, -1] \cup (1, 4]$ c) $(-\infty, -1) \cup [1, 4]$ d) $(-\infty, -1] \cup [1, 4)$

xxi) The range of the function $f(x) = \dfrac{|x - 5|}{5 - x}$ is

a) {5, –5} b) {2, –2} c) {1, –1} d) none of these

xxii) Which one of the following is not a function ?

a) $\{(x, y) : x, y \in R,\ x^2 = y\}$ b) $\{(x, y) : x, y \in R,\ y^2 = x\}$

c) $\{(x, y) : x, y \in R,\ x = y^3\}$ d) $\{(x, y) : x, y \in R,\ y = x^3\}$

xxiii) The range of f(x) = cos[x], for $-\pi/2 < x < \pi/2$ is

a) {–1, 1, 0} b) {cos1, cos2, 1} c) {cos1, –cos1, 1} d) [–1,1]

xxiv) If x≠1 and $f(x) = \dfrac{x + 1}{x - 1}$ is a real function then $f(f(f(2)))$ is

a) 1 b) 2 c) 3 d) 4

xxv) If $3f(x)+5f\left(\frac{1}{x}\right)=\frac{1}{x}-3$ for all non-zero x, then f(x)=

a) $\frac{1}{14}\left(\frac{3}{x}+5x-6\right)$ b) $\frac{1}{14}\left(-\frac{3}{x}+5x-6\right)$ c) $\frac{1}{14}\left(-\frac{3}{x}+5x+6\right)$ d) none of these

2. Very short answer type questions :

i) Is the given relation a function ? Give reasons for your answer.

a) $\{(4,6), (3,9), (-11,6), (3,11\}$

b) $\{(x,x)$: x is a real number$\}$

c) $\{(n,\frac{1}{n})$: n is a positive integer$\}$

d) $\{(x,3)$: x is a real number$\}$

ii) If f and g are two real valued functions defined as f(x) = 2x+1, g(x) = x^2+1, then find

a) f + g b) f – g c) fg d) $\frac{f}{g}$

e) $f\left(\frac{1}{2}\right)\times g(14)$ f) $\frac{f(t)-f(5)}{t-5}, t\neq 5$ g) f(3) + g(–5)

iii) Find the domain of the following functions :

a) $\frac{1}{\sqrt{9-x^2}}$ b) $\frac{1}{3x-2}$ c) 4sinx–3cosx d) $\frac{1}{\sqrt{1-\cos x}}$

e) x|x| f) $\frac{1}{\sqrt{x+|x|}}$ g) $\frac{x^3-x+3}{x^2-1}$ h) [x] + x

iv) Find the range of the following functions given by :

a) 1+3cos2x b) $\sqrt{16-x^2}$ c) 1–|x–2| d) |x–3|

e) $\frac{x}{1+x^2}$ f) $\frac{1}{\sqrt{9-x^2}}$

v) If A = $\{-2,2\}$, find A×A×A

vi) If $A\times B=\{(l,p),(l,q),(m,p),(m,q),(n,p),(n,q)\}$, find A and B.

vii) If $f(x)=\frac{1+x}{1-x}$, show that $\frac{f(x).f(x^2)}{1+[f(x)]^2}=\frac{1}{2}$

viii) Let set A has 3 elements and set B has 2 elements. How many relations are possible from set A to set B?

ix) If $f(x)=\dfrac{2x}{1-x^2}$, evaluate f(tanθ)

x) Find domain and range of the relation R, given by

$$R=\left\{(x,y):y=x+\frac{6}{x},\ x,y\in N,\ x<6\right\}$$

xi) Show that f(x) = x^2+cosx, is an even function.

xii) Show that $f(x)=\log\left(x+\sqrt{x^2+1}\right)$ is an odd function.

xiii) Find the range of the function f(x) = 4sinx–3cosx.

xiv) If $P=\{x:x<3,\ x\in N\}$, $Q=\{x:x\le 2,\ x\in W\}$ Find (P∪Q)×(P∩Q).

xv) Let f and g be real functions defined by f(x) = 2x+1 and g(x) = 4x–7. For what real numbers x, f(x) < g(x) ?

xvi) Let n(A) = m, n(B) = n. Then find the total number of non-empty relations that can be defined from A to B.

xvii) Find the solution set of $[x]^2-5[x]+6=0$.

Group - B

3. Short answer type questions : [3 marks each]

i) Lef f : R→R such that f(x) = x^2+3. Find

a) {x : f(x)=28}

b) The pre-images of 39 under f.

ii) Let A={1,2,3,4,6} and R be a relation on A defined by $\{(a,b):a,b\in A,\ a \text{ divides } b\}$.

a) Write R in roster form

b) Find the range of R

iii) Let R be a relation on N, defined as R=$\{(x,y)\in N\times N : x+2y=39\}$. Find the domain and the range of R.

iv) Is g={(1,1), (2,3), (3,5), (4,7)} a function ? If this is described by the formula, g(x) = αx+β, then what values should be assigned to α and β ?

v) Given function f : R–{1}→R defined as $f(x)=\dfrac{x^2-1}{x-1}$ and the function g : R→R defined as g(x) = x+1. Find whether function f is equal to function g or not.

vi) Let A={1,2,3}, B={3,4} and C={4,5,6}. Find $A\times(B\cap C)$ and $(A\times B)\cup(A\times C)$.

vii) For a non zero x, $pf(x)+qf\left(\frac{1}{x}\right)=\frac{1}{x}-5$, where $p\neq q$. Find f(x).

viii) If $f(x)=\frac{5x+3}{4x-5},\left(x\neq\frac{5}{4}\right)$, show that $f\{f(x)\}$ is an identity function.

ix) If $f(x)=\frac{x-1}{x+1}$, then show that

a) $f\left(\frac{1}{x}\right)=-f(x)$ b) $f\left(-\frac{1}{x}\right)=-\frac{1}{f(x)}$

x) If $A\times B\subseteq C\times D$ and $A\times B\neq\phi$, show that $A\subseteq C$ and $B\subseteq D$

xi) Let f : R→R be such that $f(x)=2^x$. Determine

a) $\{x : f(x)=1\}$

b) Whether f(x+y) = f(x).f(y) holds.

xii) Let f : R→R defined as $f(x) = x^2+2$. Find a) $f^{-1}(-6)$ b) $f^{-1}(27)$

xiii) If $f(x)=y=\frac{ax-b}{cx-a}$, then prove that $f(y) = x$.

Group - C

Long answer type questions : [4 or 6 marks each]

i) Redefine the function $f(x) = |x-2|+|2+x|$, $-3\leq x\leq 3$

ii) Find the domain and range of function $f(x)=\frac{1}{\sqrt{x-5}}$

iii) If $f(x)=\sqrt{x}$, g(x)=x be two functions defined in the domain $R^+\cup\{0\}$. Find

a) (f+g)(x) b) (f–g)(x) c) (fg)(x) d) $\left(\frac{f}{g}\right)(x)$

iv) A={1,2,3,4,5}, S={(x,y) : $x\in A$, $y\in A$}, then find the ordered pair which satisfy the conditons given below :

a) x+y=5 b) x+y<5 c) x+y>8

v) If A={x : $x\in$ w, x<2}, b={x : $x\in$ N, 1<x<5} and C={3,5}, then find

a) $A\times(B\cap C)$ b) $A\times(B\cup C)$

vi) Find domain and range of the real-valued function f(x), defined by $f(x)=\begin{cases}1-x, & x<0\\ 1, & x=0\\ x-1, & x>0\end{cases}$

and draw its graph.

vii) If f : R→R is defined by f(x)=[x], the greatest integer function (g.i.f), find its domain, range and draw its graph.

viii) Let X= {1, 2, 3, 4, 5}, Y={1,2,3,4,.........., 26}. Find the domain and range of each, f : X→Y such that

a) f(x) = 3x+7 b) f(x) = x^2+1

ix) Define the function f(x) = | x –1 | + | 1 + x |, –2 ≤ x ≤ 2 and draw its graph.

x) a) Given set A = {honest, violence}, set B = {peace, prosperity, destruction, hatred}

Write the set A×B, choose one element of A×B which you would like to have your values in life

b) Hardwork and success of a student are interelated i.e., one is a function of the other. A survey in particular school class XI is conducted and 80% of the students are found to be hard working and 70% successful. What advice you would like to give to the students who are hard working ?

ANSWERS

Group - A

1. i) (c) ii) (d) iii) (c) iv) (b) v) (b) vi) (c) vii) (b) viii) (c) ix) (c)

x) (b) xi) (d) xii) (c) xiii) (c) xiv) (d) xv) (b) xvi) (b) xvii) (a) xviii) (a) xix) (b) xx) (a) xxi) (c) xxii) (b) xxiii) (b) xxiv) (c) xxv) (b)

2. i) (a) No. (b) Yes (c) Yes (d) Yes

ii) (a) x^2+2x+2 (b) x(2–x) (c) $2x^3+x^2+2x+1$ (d) $\frac{2x+1}{x^2+1}$ (e) $\frac{1363}{4}$ (f) t+5 (g) 6

iii) (a) (–3,3) (b) $R-\{\frac{2}{3}\}$ (c) real numbers (d) R–{2nπ, n∈I} (e) R (f) R^+

(g) R–{1,–1} (h) R

iv) (a) [–2, 4]   (b) [–4,4]   (c)]–∞, 1]   (d) $R^{+}\cup\{0\}$ (e) $\left[-\frac{1}{2},\frac{1}{2}\right]-\{0\}$ (f) $\left[\frac{1}{3},\infty\right)$

v) $\{$(–2,–2,–2), (–2,–2, 2), (–2, 2,–2), (2,–2,–2), (–2, 2, 2), (2,–2, 2), (2, 2,–2), (2, 2, 2)$\}$

vi) A=$\{l$,m,n$\}$, B=$\{$p,q$\}$ viii) 64   ix) tan2θ   (x) Domain =$\{1,2,3\}$, Range = $\{7,5\}$

xiii) [–5, 5]   (xiv) $\{$(0,1), (0,2), (1,1), (1,2), (2,1), (2,2)$\}$ xv) x>4   xvi) $2^{mn}-1$ xvii) $x\in[2,4)$

Group - B

3. i) (a) {–5,5}   (b) ±6

ii) (a) R=$\{$(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)$\}$

(b) Range = $\{1,2,3,4,6\}$

iii) $D_R = \{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37\}$ $R_R = \{1,2,....,19\}$

iv) Yes, $\alpha = 2$, $\beta = -1$

v) Not equal

vi) $\{$(1,4), (2,4), (34)$\}$

$\{$(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6)$\}$

vii) $\dfrac{2p-10x(p-q)-2qx^2}{x\left(p^2-q^2\right)}$

xi) (a) $\{0\}$ (b) Yes

xii) (a) Not possible   (b) x = ±5

Group - C

i) $f(x)\begin{cases}-2x, & -3\le x<-2\\ 4, & -2\le x<2\\ 2x, & 2\le x\le 3\end{cases}$

ii) $D_f = (5, \infty)$, $R_f = R^{+}$

iii) (a) $\sqrt{x}+x$ (b) $\sqrt{x}-x$ (c) $x^{3/2}$ (d) $\dfrac{1}{\sqrt{x}}$

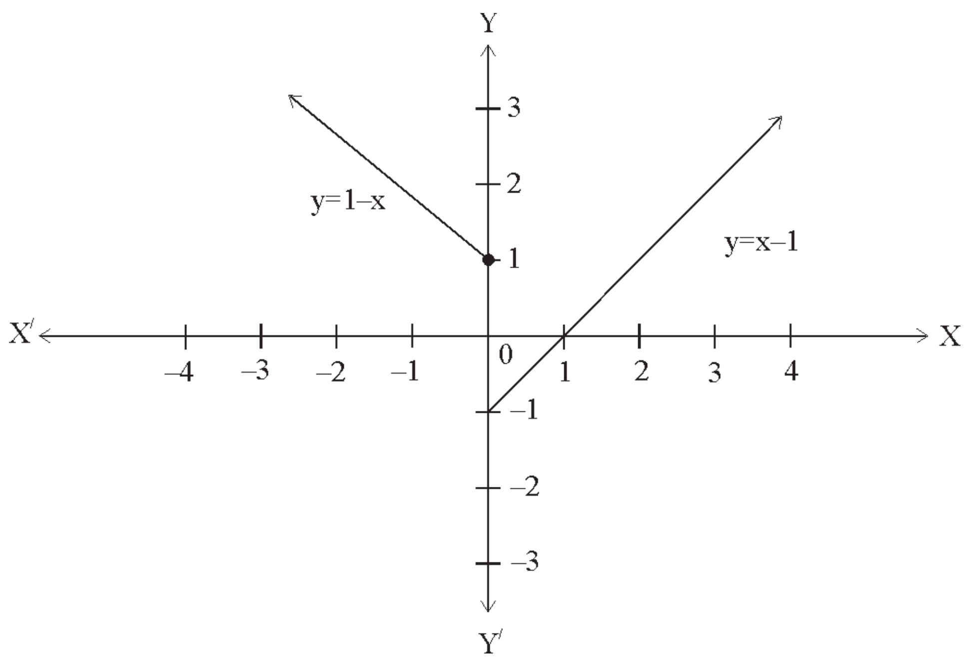
iv) (a) $\{$(1,4), (2,3), (3,2), (4,1)$\}$

(b) $\{$(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)$\}$

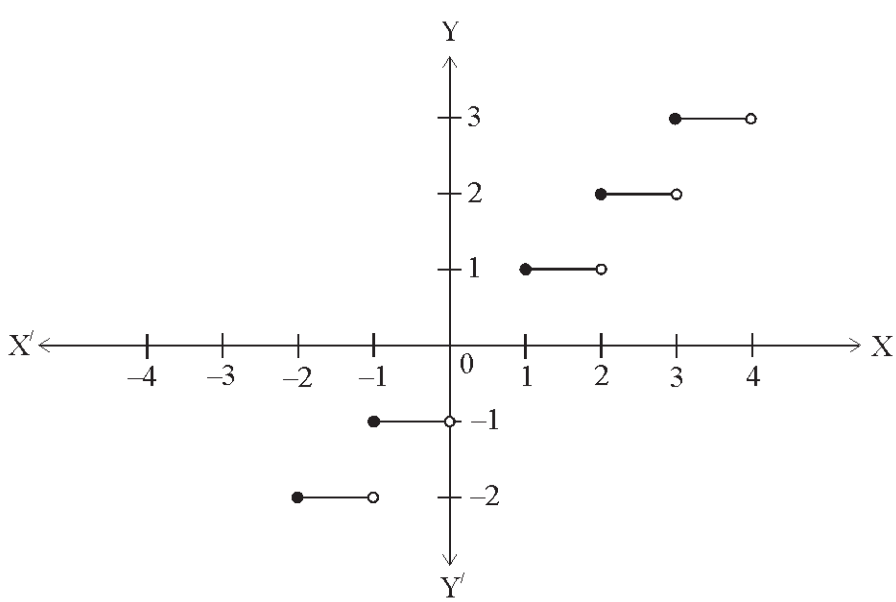
(c) $\{(4,5), (5,4), (5,5)\}$

v) (a) $\{(0,3), (1,3)\}$ (b) $\{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$

vi) Domain = R, Range = $\{x \in R : x > -1\}$



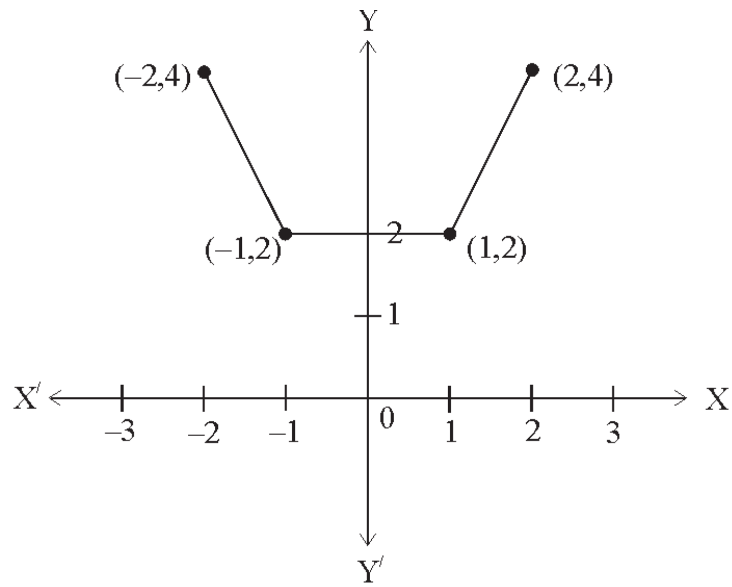
vii) Domain = R, Range = Integers



viii) (a) $\{(1,10), (2,13), (3,16), (4,19), (5,22)\}$, Domain = X, Range = $\{10,13,16,19,22\}$

(b) $\{(1,2), (2,5), (3,10), (4,17), (5,26)\}$, Domain = X, Range = $\{2,5,10,17,26\}$

ix) $f(x) = \begin{cases} -2x, -2 \le x < -1 \\ 2, -1 \le x < 1 \\ 2x, 1 \le x \le 2 \end{cases}$



x) (a) A×B = $\{$(honest, peace), (honest, prosperity), (honest, destruction), (honest, hatred), (violence, peace), (violence, prosperity), (violence, destruction), (violence, hatred), (honest, peace)$\}$

(b) Pay attention to the quality working not quantity working.

Chapter - 3
Trigonometric Functions

Important Points and results :

- **System of measuring Angles :**

  There are three different systems of units are used in the measurement of trigonometrical angles –

  i) Sexagesimal System

  ii) Centesimal System

  iii) Circular System

  i) Sexagesimal System :

  1 right angle = 90 degrees (or 90^0)

  1 degree (1^0) = 60 minutes ($60^/$)

  1 minute ($1^/$) = 60 second ($60^{//}$)

  ii) Centesimal System :

  1 right angle = 100 grades (100g)

  1 grade (1^g) = 100 minutes ($100^/$)

  1 minute ($1^/$) = 100 seconds ($100^{//}$)

  iii) Circular System :

  1 radian = $\frac{2}{\pi}$ right angle = $57^0 17^/ (44.8)^{//}$ (approx)

  $\therefore$ π radian = 2 right angles.

- **Relation among the three sytems :**

  If the sexagesimal, centesimal and circular measures of a given angle be D^0, G^g and R^c respectively then, $\frac{D}{180} = \frac{G}{200} = \frac{R}{\pi}$.

- If an are of lenght l of a circle of radius r subtends an angle θ radian at its centre, then $\theta = \frac{l}{r} \Rightarrow l = r\theta$.

- **Relation between Degree and radian :**

  Redian measure = $\frac{\pi}{180} \times$ Degree measure

Degree measure = $\frac{180}{\pi} \times$ Radian measure

Domain, Range and Period of trigonometric functions :

Trigonometric Ratio	**Domain**	**Range**	**Period**
sinθ	R	[–1, 1]	2π
cosθ	R	[–1, 1]	2π
tanθ	$R-\left\{(2n+1)\frac{\pi}{2}; n \in I\right\}$	R i.e. (–∝, ∝)	π
cosecθ	$R-\{n\pi; n \in I\}$	R – (–1, 1)	2π
secθ	$R-\left\{(2n+1)\frac{\pi}{2}; n \in I\right\}$	R – (–1, 1)	2π
cotθ	$R-\{n\pi; n \in I\}$	R i.e. (–∝, ∝)	π

- **Trigonometrical Identities :**

i) $\sin\theta = \frac{1}{\cos ec\theta}$

ii) $\cos\theta = \frac{1}{\sec\theta}$

iii) $\tan\theta = \frac{1}{\cot\theta}$

iv) $\cos ec\theta = \frac{1}{\sin\theta}$

v) $\sec\theta = \frac{1}{\cos\theta}$

vi) $\cot\theta = \frac{1}{\tan\theta}$

vii) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

viii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

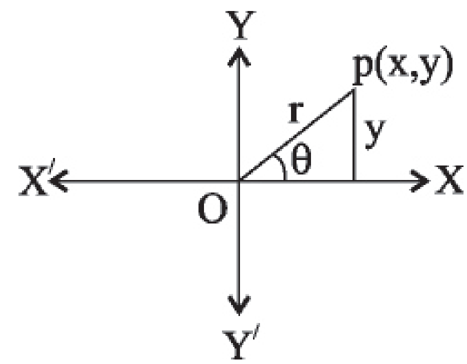
ix) $\sin^2\theta+\cos^2\theta=1 \Rightarrow \sin^2\theta = 1-\cos^2\theta \Rightarrow \cos^2\theta = 1-\sin^2\theta$

x) $1+\tan^2\theta = \sec^2\theta$

xi) $1+\cot^2\theta = \text{cosec}^2\theta$

- **Signs of Trigonometrical Ratios :**

(i) In first quadrant : We have, x>0, y>0



$\sin\theta = \frac{y}{r} > 0$ $\quad$ $\cos ec\theta = \frac{r}{y} > 0$

$\cos\theta = \frac{x}{r} > 0$ $\quad$ $\sec\theta = \frac{r}{x} > 0$

$\tan\theta = \frac{y}{x} > 0$ and $\cot\theta = \frac{x}{y} > 0$

Thus in the first quadrant all trigonmetric functions are positive.

(ii) In second quadrant : We have, x<0, y>0

$\sin\theta = \frac{y}{r} > 0$ $\quad$ $\cos ec\theta = \frac{r}{y} > 0$

$\cos\theta = \frac{x}{r} < 0$ $\quad$ $\sec\theta = \frac{r}{x} < 0$

$\tan\theta = \frac{y}{x} < 0$ and $\cot\theta = \frac{x}{y} < 0$

Thus in the second quadrant sine and cosecant functions are positive and all others are negative.

(iii) In third quadrant : We have x<0, y<0

$\sin\theta = \frac{y}{r} < 0$ $\quad$ $\cos ec\theta = \frac{r}{y} < 0$

$\cos\theta = \frac{x}{r} < 0$ $\quad$ $\sec\theta = \frac{r}{x} < 0$

$\tan\theta = \frac{y}{x} > 0$ and $\cot\theta = \frac{x}{y} > 0$

Thus in the third quadrant all trigonometric functions are negative except tangent and cotangent.

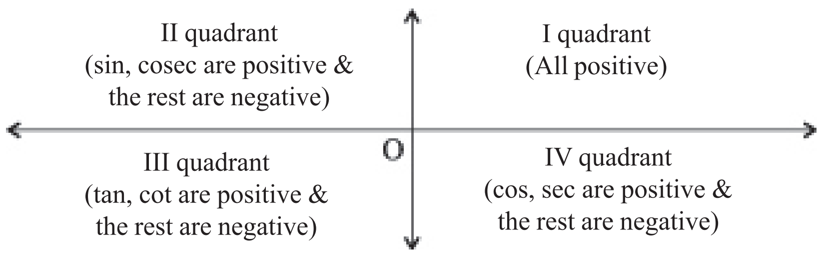
(iv) In fourth quadrant : We have x>0, y<0

$\sin\theta = \frac{y}{r} < 0$ $\quad$ $\cos ec\theta = \frac{r}{y} < 0$

$\cos\theta = \frac{x}{r} > 0$ $\quad$ $\sec\theta = \frac{r}{x} > 0$

$\tan\theta = \frac{y}{x} < 0$ and $\cot\theta = \frac{x}{y} < 0$

Thus in the fourth quadrant all trigonometric functions are negative except cosine and secant



- **Trigonmetrical ratios of Compound angles : (Sum and difference of two angles)**

i) sin(A+B) = sinAcosB+cosAsinB

ii) cos(A+B) = cosAcosB–sinAsinB

iii) sin(A–B) = sinAcosB–cosAsinB

iv) cos(A–B) = cosAcosB+sinAsinB

v) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, where A, B and (A+B) are not odd multiple of $\frac{\pi}{2}$.

vi) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$, where A, B and (A–B) are not odd multiple of $\frac{\pi}{2}$.

vii) $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$, where A, B and (A+B) are not multiple of π.

vii) $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$, where A, B and (A–B) are not multiple of π.

- **Some Important Results :**

i) $\sin(A+B)\sin(A-B) = \sin^2A - \sin^2B = \cos^2B - \cos^2A$

ii) $\cos(A+B)\cos(A-B) = \cos^2A - \sin^2B = \cos^2B - \sin^2A$

iii) sin(A+B+C) = sinAcosBcosC + cosAsinBcosC + cosAcosBsinC – sinAsinBsinC

iv) cos(A+B+C) = cosAcosBcosC – sinAsinBcosC – sinAcosBsinC – cosAsinBsinC

v) $\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan c - \tan C \tan A}$

- **Transformation of Product into Sum or Difference :**

i) 2sinAcosB = sin(A+B)+sin(A–B)

ii) 2cosAsinB=sin(A+B)–sin(A–B)

iii) 2cosAcosB = cos(A+B)+cos(A–B)

iv) 2sinAsinB = Cos(A–B)–cos(A+B)

- **Transformation of sum or difference into product :**

i) sinC+sinD $= 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

ii) sinC–sinD $= 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}$

iii) cosC+cosD $= 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

iv) cosC – cosD $= -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

$= 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

- **Trigonmetric functions of multiple of angles :**

For all values of angle A

a) $\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$

b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$

$\Rightarrow$ 1+cos2A = $2\cos^2 A$ and 1–cos2A = $2\sin^2 A$

c) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

d) $\cot 2A = \frac{2\cot A}{\cot^2 A - 1}$

e) sin3A = 3sinA – $4\sin^3 A$

f) cos3A = $4\cos^3 A$ – 3cosA

g) $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

h) $\cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$

- **Trigonmetric functions of submultiples of angles :**

  For all angles of angle A

i) $\cos A = \cos^2\left(\frac{A}{2}\right) - \sin^2\left(\frac{A}{2}\right) = 2\cos^2\left(\frac{A}{2}\right) - 1 = 1 - 2\sin^2\left(\frac{A}{2}\right) = \frac{1-\tan^2\left(\frac{A}{2}\right)}{1+\tan^2\left(\frac{A}{2}\right)}$

ii) $\sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right) = \frac{2\tan\left(\frac{A}{2}\right)}{1+\tan^2\left(\frac{A}{2}\right)}$

iii) $\tan A = \frac{2\tan\left(\frac{A}{2}\right)}{1-\tan^2\left(\frac{A}{2}\right)}$

iv) $\cot A = \frac{2\cot\left(\frac{A}{2}\right)}{\cot^2\left(\frac{A}{2}\right)-1}$

v) $\sin A = 3\sin\left(\frac{A}{3}\right) - 4\sin^3\left(\frac{A}{3}\right)$

vi) $\cos A = 4\cos^3\left(\frac{A}{3}\right) - 3\cos\left(\frac{A}{3}\right)$

vii) $\tan A = \frac{3\tan\left(\frac{A}{3}\right) - \tan^3\left(\frac{A}{3}\right)}{1-3\tan^2\left(\frac{A}{3}\right)}$

viii) $\cot A = \frac{\cot^3\left(\frac{A}{3}\right) - 3\cot\left(\frac{A}{3}\right)}{3\cot^2\left(\frac{A}{3}\right)-1}$

Trigonmetrical ratios of some useful angles :

i) Value of $\sin 18^0 = \frac{\sqrt{5}-1}{4}$

ii) Value of $\cos 36^0 = \dfrac{\sqrt{5}+1}{4}$

iii) Value of $\sin 36^0 = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$

iv) Value of $\cos 18^0 = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

- **Solution of Trigonometric equations :**

Solution of trigonometric equations is of two types. i.e i) Principal Solution, ii) General Solution

Standard form	General solution
1. i) $\sin\theta=0$	$\theta=n\pi$, $n\in Z$, where Z is the set of integers.
ii) $\sin\theta=\sin\alpha\left(-\frac{\pi}{2}\le\alpha\le\frac{\pi}{2}\right)$	$\theta=n\pi+(-1)^n\alpha$, $n\in Z$
iii) $\sin\theta=1$	$\theta=(4n+1)\frac{\pi}{2}$, $n\in Z$
iv) $\sin\theta=-1$	$\theta=(4n-1)\frac{\pi}{2}$, $n\in Z$
2. i) $\cos\theta=0$	$\theta=(2n+1)\frac{\pi}{2}$, $n\in Z$
ii) $\cos\theta=\cos\alpha(0\le\alpha\le\pi)$	$\theta=2n\pi\pm\alpha$, $n\in Z$
iii) $\cos\theta=1$	$\theta=2n\pi$, $n\in Z$
iv) $\cos\theta=-1$	$\theta=(2n+1)\pi$, $n\in Z$
3. i) $\tan\theta=0$	$\theta=n\pi$, $n\in Z$
ii) $\tan\theta=\tan\alpha$ $\left(-\frac{\pi}{2}<\alpha<\frac{\pi}{2}\right)$	$\theta=n\pi+\alpha$, $n\in Z$
4. $\left.\begin{array}{l}\sin^2\theta=\sin^2\alpha\\ \cos^2\theta=\cos^2\alpha\\ \tan^2\theta=\tan^2\alpha\end{array}\right\}$	$\theta=n\pi\pm\alpha$, $n\in Z$

Maximum and Minimum value of $a\sin\theta+b\cos\theta$

1. Maximum value of $a\sin\theta+b\cos\theta=\sqrt{a^2+b^2}$

2. Minimum value of asinθ+bcosθ = $-\sqrt{a^2+b^2}$

Exercise – 3

Group – A

Objective Type Questions : [1 or 2 mark each]

I. Multiple choice type questions :

1. The radian measure of $48^0 37^{/} 30^{//}$ is

a) $\frac{389\pi}{1440}$ radian b) $\frac{752\pi}{1440}$ radian c) $\frac{389\pi}{1440}$ radian d) $\frac{752\pi}{1440}$ radian

2) The value of $\tan 1^0 \tan 2^0 \tan 3^0 .... \tan 89^0$ is

a) 0 b) 1 c) $\frac{1}{2}$ d) Not defined

3) If $\sin\theta + \operatorname{cosec}\theta = 2$ then $\sin^2\theta + \operatorname{cosec}^2\theta$ is equal to

a) 1 b) 4 c) 2 d) None of these

4) If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then the value of $\theta+\phi$ is

a) $\frac{\pi}{6}$ b) π c) 0 d) $\frac{\pi}{4}$

5) The value of $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $-\frac{1}{4}$ d) 1

6) If $\tan\theta = 3$ and θ lies in third quadrant, then the value of sinθ is

a) $\frac{1}{\sqrt{10}}$ b) $-\frac{1}{\sqrt{10}}$ c) $\frac{-3}{\sqrt{10}}$ d) $\frac{3}{\sqrt{10}}$

7) The value of $\tan 75^0 - \cot 75^0$ is

a) $2\sqrt{3}$ b) $2+\sqrt{3}$ c) $2-\sqrt{3}$ d) 1

8) The value of $\sin(45^0+\theta) - \cos(45^0-\theta)$ is

a) 2cosθ b) 2sinθ c) 1 d) 0

9) If $\sin\theta = -\frac{4}{5}$ and θ lies in third quadrant then the value of $\cos\frac{\theta}{2}$ is

a) $\frac{1}{5}$ b) $-\frac{1}{\sqrt{10}}$ c) $-\frac{1}{\sqrt{5}}$ d) $\frac{1}{\sqrt{10}}$

10) The value of $\cos^2 48^0 - \sin^2 12^0$ is

a) $\frac{\sqrt{5}+1}{8}$ b) $\frac{\sqrt{5}-1}{8}$ c) $\frac{\sqrt{3}+1}{5}$ d) $\frac{\sqrt{5}+1}{2\sqrt{2}}$

Very Short Answer Type Question :

1) Find the radian measure corresponding to the degree measure $125^0 30'$

2) Find the value of $\frac{\sin 50^0}{\sin 130^0}$.

3) If $\tan A = \frac{1-\cos B}{\sin B}$, then find the value of tan2A.

4) Find the value of $\frac{1-\tan^2 15^0}{1+\tan^2 15^0}$.

5) Evalute : $\sin 50^0 - \sin 70^0 + \sin 10^0$

6) If $\alpha + \beta = \frac{\pi}{4}$, then find the value of $(1+\tan\alpha)(1+\tan\beta)$

7) Evalute : $\sin\frac{\pi}{18} + \sin\frac{\pi}{9} + \sin\frac{2\pi}{9} + \sin\frac{5\pi}{18}$

8) If $\tan\alpha = \frac{1}{7}$ and $\tan\beta = \frac{1}{3}$, then find the value of $\cos 2\alpha$.

9) Find the principal solution of the trigonometric equation $\sin x = -\frac{\sqrt{3}}{2}$

10) Find the general solution of the trigonmetric equation $\sec 2\theta = -2$.

Group - B

Short answer type question : [3 marks each]

1) If $\cos\theta = -\frac{3}{5}$ and θ lies in the third quadrant then find the value of $\sin\theta$ and $\tan\theta$.

2) The minute hand of a watch is 1.8cm long. How far does its tip move in 30 min?

3) In any quadrilateral ABCD, prove that cos(A+B) = cos(C+D).

4) If $\cos ec\theta + \cot\theta = \frac{11}{2}$, then find the value of tanθ.

5) Prove that $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$

6) Find the value of $\tan 22^0 30'$.

7) Prove that $\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{5\pi}{8} + \cos\frac{7\pi}{8} = 0$

8) If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

9) Prove that $8\sin^2\frac{\pi}{6} + \cos ec^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

10) If θ is the positive acute angle, then solve the equation $3\cos^2\theta - 4\sin\theta = 1$

Group - C

Long answer type questions : [4/6 marks each]

1) If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$, then prove that $\cos(\alpha+\beta) - 4ab\cos(\alpha-\beta) = 1 - 2a^2 - 2b^2$.

2) If $\frac{\sin^4\alpha}{a} + \frac{\cos^4\alpha}{b} = \frac{1}{a+b}$, then show that $\frac{\sin^8\alpha}{a^3} + \frac{\cos^8\alpha}{b^3} = \frac{1}{(a+b)^3}$

3) If $\cos(\theta+\phi) = m\cos(\theta-\phi)$, then prove that $\tan\theta = \frac{1-m}{1+m}\cot\phi$

4) Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right) + \sin^4(3\pi+\alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right) + \sin^6(5\pi-\alpha)\right]$$

5) If $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = -\frac{3}{2}$, then show that $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\cos(\alpha-\beta) = -\frac{1}{2}$.

6) Show that $\cot 7\frac{1}{2}^0 = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

7) If $x = \sec\phi - \tan\phi$ and $y = \operatorname{cosec}\phi + \cot\phi$ then show that $xy + x - y + 1 = 0$.

8) If θ lies in the first quadrant and $\cos\theta = \frac{8}{17}$, then the value of

$\cos(30^0+\theta)+\cos(45^0-\theta)+\cos(120^0-\theta)$.

9) Find the value of $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}$

10) Eliminate θ from $x\sin\theta-y\cos\theta=\sqrt{x^2+y^2}$ and $\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}=\frac{1}{x^2+y^2}$.

11) If $\tan\beta=\frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$, show that $\tan(\alpha-\beta)=(1-n)\tan\alpha$.

12) If $\sin\theta+\sin\phi=a$, $\cos\theta+\cos\phi=b$, prove that $\tan\frac{\theta-\phi}{2}=\pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$

13) If $\sin\alpha=\frac{4}{5}$, $\sin\beta=\frac{3}{5}$ and α, β are positive acute angles, then find the value of $\cos\frac{\alpha-\beta}{2}$ and $\sin\frac{\alpha-\beta}{2}$.

14) If $\tan(\alpha-\beta)=\frac{\sin 2\beta}{5-\cos 2\beta}$, find the value of $\tan\alpha : \tan\beta$.

15) Prove that, $\cos\frac{\pi}{32}=\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$. Hence, find $\sin\frac{\pi}{32}$.

16) Find the general solution of the equation $5\cos^2\theta+7\sin^2\theta-6=0$

17) Find the general solution of the equation, $\sin x-3\sin 2x+\sin 3x=\cos x-3\cos 2x+\cos 3x$.

18) Find the general solution of the equation $(\sqrt{3}-1)\cos\theta+(\sqrt{3}+1)\sin\theta=2$

19) Solve : $2+2\cos 2x\cos 5x=\sin^2 2x$.

20) If $\cos x+\sin x=\cos\alpha-\sin\alpha$, prove that $x-\frac{\pi}{4}=2n\pi\pm\left(\alpha+\frac{\pi}{4}\right)$.

ANSWERS

Group - A

I. 1. (c) $\frac{389\pi}{1440}$ radian 2.(b) 1 3.(c) 2 4.(d) $\theta+\phi=\frac{\pi}{4}$

5 .(c) $-\frac{1}{4}$ 6.(c) $-\frac{3}{\sqrt{10}}$ 7.(a) $2\sqrt{3}$ 8.(d) 0

9.(c) $-\frac{1}{\sqrt{5}}$ 10.(a) $\frac{\sqrt{5}+1}{8}$

II. (1) $\frac{251\pi}{360}$ (2) 1 (3) tanB (4) $\frac{\sqrt{3}}{2}$

(5) 0 (6) 2 (7) $\sin 70^0+\sin 80^0$ (8) $\sin 4\beta$

(9) (10) $\theta=n\pi\pm\frac{\pi}{3}, n\in Z$

Group - B

(1) $\sin\theta=\frac{-4}{5}$ *and* $\tan\theta=\frac{3}{4}$ (2) 5.66cm (4) $\frac{44}{117}$

(6) $\frac{1}{\sqrt{2}+1}$ (10) $\theta=30$

Group - C

(4) 1 (8) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)$ (9) $\frac{3}{2}$

(10) $\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$ (14) 3:2 (15) $\sin\frac{\pi}{32}=\frac{\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2}$

(16) $\theta=n\pi\pm\frac{\pi}{4}$ (17) $x=\frac{n\pi}{2}+\frac{\pi}{8}$ (18) $\theta=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{12}$

(19) x=(2m+1)π, where m = any integer.

Chapter - 4

Principle of Mathematical Induction

Important Points and results :

- One key basis for mathematical thinking is deductive reasoning. In contrast to deduction, inductive reasoning depends on working with different cases and developing a conjecture by observing incidences till we have observed each and every case. Thus, in simple language we can say the word 'Induction' means the generalisation from particular cases or facts.

  The principle of mathematical induction is one such tool which can be used to prove a wide variety of mathematical statements. Each such statement is assumed as P(n) associated with positive integer n, for which the correctness for the case n=1 examined. Then assuming the truth of P(K) for some positive integers K, the truth of P(K+1) is established.

  Statement involving mathematical relations are known as the mathematical statements.

- **First Principle of Mathematical Induction :**

  Let P(n) be a statement involving the natural number n such that

  (i) P(1) is true i.e. P(n) is true for n=1 and

  (ii) P(m+1) is true, whenever P(m) is true i.e. P(m) is true $\Rightarrow$ P(m+1) is true, (for any positive integer m) Then, P(n) is true for all natural numbers n.

- **Some Mathematical Statements :**

  (i) Sum of first n-natural numbers = $\frac{1}{2}n(n+1)$

  (ii) If $n\geq1$ is a natural number then, $n<2^n$.

  (iii) (n^2+n+41) is a prime number; where $n\geq1$ is a natural number.

  (iv) $n\geq2$ is a natural number then n(n+1) always divisible by 3.

  (v) Sum of the square of first 'n' natural numbers = $\frac{1}{6}n(n+1)(2n+1)$

  (vi) Sum of the cube of 'n' natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

  (vii) n^2+n is even natural number $\forall\, n\in N$.

  (viii) n(n+1)(2n+1) is divisible by 6 for all $n\in N$.

  (ix) Sum of the cube of three consecutive positive integer is always divisible by 9.

  (x) n is a positive even integers, then a^n-b^n always divisible by a+b.

  (xi) $1+3+5+\cdots\cdots\cdots+(2n-1)=n^2,\ n\geq1$.

- **A proof by induction consists of two steps –**
  - i) In step one we will prove that P(1) is true. The step is also called basic step.
  - ii) In 2nd step, the inductive step,we assume P(k) is true for n=k, a positive integer and we prove P(k+1) is true for n=k+1. Then the statement is true for all $n \in N$.

Exercise

Group - A

Objective Type Questions : (1 or 2 marks each)

1. Multiple Choice Questions :

(i) Sum of 'n' natural numbers –

(a) n (b) $\frac{n(n+1)}{2}$ (c) $\frac{1}{6}n(n+1)(2n+1)$ (d) $\left[\frac{n(n+1)}{2}\right]^2$

(ii) Sum of the square of first n natural numbers –

(a) n^2 (b) $\frac{n(n+1)}{2}$ (c) $\frac{1}{6}n(n+1)(2n+1)$ (d) $\left[\frac{n(n+1)}{2}\right]^2$

(iii) If $10^n+3.4^{n+2}+K$ is divisible by 9, for all $n \in N$, then the least positive integral value of K is –

(a) 5 (b) 3 (c) 7 (d) 1

(iv) For all $n \in N$, $3.5^{2n+1}+2^{3n+1}$ is divisible by –

(a) 19 (b) 17 (c) 21 (d) 25

(v) If $x^n–1$ is divisible by x–K, then the least positive integral value of K is –

(a) 1 (b) 2 (c) 3 (d) 4

(vi) Sum of the cube of first n natural numbers is –

(a) n^3 (b) $\frac{n(n+1)}{2}$ (c) $\frac{1}{6}n(n+1)(2n+1)$ (d) $\left[\frac{n(n+1)}{2}\right]^2$

(vii) If $I_n=5^n+3$, then the last digit of I_n is –

(a) 5 (b) 8 (c) 3 (d) None of these.

(viii) If $n \in N$ and $n < \left(\sqrt{2}+1\right)^6$, then the maximum value of n is –

(a) 199 (b) 197 (c) 195 (d) 198

(ix) If $3^{n+1}<4^n$ is true, then the minimum positive integer of n is –

(a) For 1 (b) For 2 (c) For 3 (d) For 4

(x) $3^{4n+2}+5^{2n+1}$, $n\in N$ is –

(a) Divisible by 8 (b) Divisible by 14 (c) Divisible by 4 (d) None of these.

(xi) $n!>2^{n-1}$ is true, when

(a) For all $n>1$, $n\in N$ (b) For all $n>2$, $n\in N$

(c) For all $n\in N$ (d) None of these.

(xii) If n is positive integer, then 4^n-3n-1 is –

(a) Divisible by 5 (b) Divisible by 9 (c) Divisible by 8 (d) Divisible by 27

(xiii) For $2^n>n^2$ is true, for all integers $n\geq K$, then K =

(a) 5 (b) 3 (c) 2 (d) None of these.

(xiv) n be any natural number then $a^{(n+1)}+(a+1)^{2n-1}$ always divisible by –

(a) a^2+a+1 (b) $(a+1)^2$ (c) a^2-a+1 (d) a^2+1

2. Very short answer type question : (each question caries 1 or 2 marks)

(i) If P(n) is the statement 'n(n+1) is even', then what is P(3) ?

(ii) If P(n) is the statement '$2^n\geq 3n$' and if P(r+1) is true, prove that P(n+1) is true.

(iii) Give an example of a statement P(n) such that it is true for al $n\in N$.

(iv) Prove that $n^2<2^n$ for all natural number $n\geq 5$.

(v) If x^n-1 is divisible by x–K, then find the least positive integral value of K.

(vi) Prove that $2n<(n+2)!$, for all natural number n.

(vii) If P(n) is the statement "n(n+1)(n+2) is divisible by 12" then prove that P(3) and P(4) are true, but P(5) is not true.

(viii) If x and y are any two distinct integers and P(n) is the statement "x^n-y^n is divisible by x–y" for all n, If P(r) is true, then prove that P(r+1) is true.

(ix) For all $n\in N$, P(n) is the statement "3^{2n} when divided by 8, the remainder is always 1". If P(r) is true, then prove that P(r+1) is true.

(x) For all $n\in N$, P(n) is the statement "$10^n+3.4^{n+2}+5$ is divisible by 9". If P(r) is true, then prove that P(r+1) is also true.

(xi) For all $n\in N$ and P(n) is the statment $(ab)^n=a^n.b^n$. If P(r) is true, then prove that P(r+1) is also true.

(xii) For all $n\in N$ and P(n) is the statement '$x^{2n}-y^{2n}$' is divisible by x + y. If P(r) is true, then prove that P(r+1) is also true.

(xiii) For all $n\in N$, P(n) is the statement $\sqrt{2+\sqrt{2+\sqrt{2+......n\,terms}}}<2$. If P(r) is true, then prove that P(r+1) is also true.

(xiv) For all n, P(n) is the statment that "$n^3+(n+1)^3+(n+2)^3$ is divisible by 9". If P(r) is true, then prove that P(r+1) is also true.

(xv) For all $n \in N$, is P(n) is the statement "$10^{2n-1}+1$ is divisible by 11". If P(r) is true, then prove that P(r+1) is true.

Group - B

3. Short answer type questions : (each question carries 3 marks)

(i) For all $n \in N$. If $2n+7<(n+3)^2$, shwo that $2(n+1)+7<(n+4)^2$

(ii) If $1.1!+2.2!+3.3!+............+n.n! = (n+1)!-1$, then show that $1.1!+2.2!+3.3!+............+n.n!+(n+1)(n+1)! = (n+2)!-1$.

(iii) If $n \in N$, $x>-1$ and $(1+x)^n \geq 1+nx$, then prove that $(1+x)^{n+1} \geq 1+(n+1)x$.

(iv) $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$, for all natural numbers $n \geq 2$.

(v) A sequence a_1, a_2, a_3............. is defined by letting $a_1=3$ and $a_k=7a_{k-1}$ for all natural numbers $k \geq 2$. Show that $a_n=3.7^{n-1}$ for all natural numbers.

(vi) A sequence b_0, b_1, b_2 .... is defined by letting $b_0=5$ and $b_k=4+b_{k-1}$, for all natural number k show that $b_n = (n-1)\, 4+9$ for all $n \in N$.

(vii) If $n \geq 3$ and $n \in N$ then fove $2^n<n^2$

(viii) If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove by Mathematical Induction that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for all $n \in N$.

(ix) Prove by the Mathematical Induction, $3^n < n!$, where $n \geq 7$ and n is integer.

(x) Prove by the Mathematical Induction, $1.2+2.3+3.4+ .... +n(n+1) = \frac{1}{3}n(n+1)(2n+1)$

(xi) Prove by the Mathematical Induction, $1^2+2^2+3^2+.............n^2 > \frac{n^3}{3}$

(xii) Prove by the Mathematical Induction, $2+3.2+4.2^2+.............(n+1).2^{n-1} = n.2^{2n}$

Group - C

Long Answer Type Questions : [4/6 Marks Each]

Prove by the principle of Mathematical Induction that for all $n \in N$.

(i) $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + ... + \frac{1}{1+2+3+........+n} = \frac{2n}{n+1}$

(ii) $1.2+2.2^2+3.2^3+ .... +n.2^n = (n-1).2^{n+1}+2$

(iii) $\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+\cdots\cdots\cdots+\cos[\alpha+(n-1)\beta]=\dfrac{\cos\left[\alpha+\frac{n-1}{2}.\beta\right].\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$

(iv) $\cos\theta.\cos2\theta.\cos2^2\theta..........\cos2^{n-1}\theta=\dfrac{\sin 2^n\theta}{2^n\sin\theta}$

(v) Prove $(5^{2n+2}-24n-25)$ is divisible by 576 for all $n\in N$.

(vi) Prove $(\cos\theta+i\sin\theta)^n=(\cos n\theta+i\sin n\theta)$

(vii) $\left(1-\dfrac{1}{2^2}\right)\left(1-\dfrac{1}{3^2}\right)\left(1-\dfrac{1}{4^2}\right)...........\left(1-\dfrac{1}{(n-1)^2}\right)=\dfrac{n+2}{2n+2}$

(viii) ${}^nC_0+{}^nC_1+{}^nC_2+...........+{}^nC_n=2^n$

(ix) Prove (x^n-y^n) is always divisible by $(x-y)$.

(x) Prove by the principle of Mathematical Induction.

$3^{2n+2}-8n-9$ always divisible by 64.

(xi) Prove by the principle of Mathematical Induction $|\sin nx|\le n|\sin x|$, for all positive integers of n.

(xii) Prove by the principle of Mathematical Induction, $P^{n+1}+(P+1)^{2n-1}$ is always divisible by (P^2+P+1), where n, P is positive integers.

ANSWERS

Gropu - A

1. i) b) ii) c iii) a iv) b

   v) a vi) d vii) b viii) b

   ix) d x) b xi) b xii) b

   xiii) a xiv) a

2. i) p(3) : 3 (3+1) in even

   v) k = 1

Chapter - 5

Complex Numbers and Quadratic Equations

Important Points and results :

- **Definition of Complex numbers :**

If an ordered pair (x,y) of two real number x and y is represented by the symbol x+iy, where $i=\sqrt{-1}$ then the ordered pair (x,y) is called a complex number. If Z=x+iy, then x is called the real part $R_e(Z)$ of the complex number Z, and y is called its imaginary part Im(Z).

A complex number Z = x+iy is called purely real if y=0 i.e. Im(Z)=0 and is called purely imaginary if x=0 i.e Re(Z)=0.

If x, y are real and $i=\sqrt{-1}$ then the complex numbers x+iy and x–iy are said to be conjugate to each other. Conjugate of a complex number Z is denoted by $\bar{Z}$.

Properties of Conjugate of Complex numbers :

If Z = x+iy then by definition $\bar{Z}$.

i) $\bar{\bar{Z}}=Z$, where $\bar{Z}$ is the conjugate of complex number Z and $\bar{\bar{Z}}$ is the conjugate of complex number $\bar{Z}$.

ii) $Z+\bar{Z}=2x=2\,\mathrm{Re}(Z)$

iii) $Z-\bar{Z}=2iy=2i\ Im(Z)$

iv) $Z\bar{Z}=x^2+y^2$

v) $\overline{Z_1+Z_2}=\bar{Z}_1+\bar{Z}_2$

vi) $\overline{Z_1-Z_2}=\bar{Z}_1-\bar{Z}_2$

vii) $\overline{Z_1.Z_2}=\bar{Z}_1.\bar{Z}_2$

viii) $\overline{\left(\frac{Z_1}{Z_2}\right)}=\frac{\bar{Z}_1}{\bar{Z}_2},\ provided\ Z_2,\bar{Z}_2\neq 0$

Two complex numbers $Z_1=x_1+y_1$ and $Z_2=x_2+iy_2$ are said to be equal if $x_1=x_2$ and $y_1=y_2$.

- **Algebra of Complex numbers :**

Let, two complex numbers are $Z_1=x_1+iy_1$ and $Z_2=x_2+iy_2$, then their

i) Addition is defined as

$$Z_1+Z_2 = (x_1+x_2) + i(y_1+y_2)$$

ii) Subtraction is defined as

$$Z_1-Z_2 = (x_1-x_2) + i(y_1-y_2)$$

iii) Multiplication is defined as

$$Z_1Z_2 = (x_1+iy_1)(x_2+iy_2) = (x_1x_2-y_1y_2) + i(x_1y_2+y_1x_2)$$

iv) Division of defined as

$$\frac{Z_1}{Z_2} = Z_1Z_2^{-1} = \left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}\right) + i\left(\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}\right)$$

If Z=x+iy then the positive square root of (x^2+y^2) is called the modulus or absolute value of Z and is denoted by |Z| or mod Z. Thus if Z=x+iy then $|Z| = \sqrt{x^2+y^2}$.

If Z=x+iy then the unique value of θ satisfying $x=|Z|\cos\theta$, $y=|Z|\sin\theta$ and $-\pi<\theta\leq\pi$ is called the principal value of argument (or amplitude) of Z and is denoted by arg Z or amp Z.

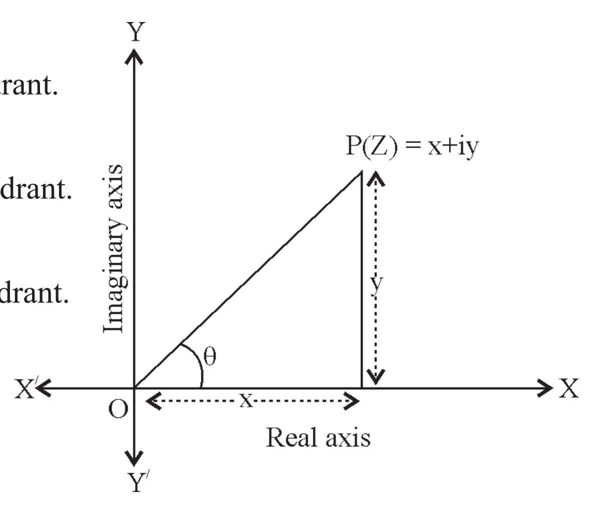
If the point P(Z) in the argand diagram represents the complex number Z=(x,y)=x+iy, and arg Z=θ then.

i) $0 < \theta < \frac{\pi}{2}$, when P lies in the first quadrant.

ii) $\frac{\pi}{2} < \theta < \pi$, when P lies in the second quadrant.

iii) $-\pi < \theta < -\frac{\pi}{2}$, when P lies in the third quadrant.

iv) $-\frac{\pi}{2} < \theta < 0$, when p lies in the fourth quadrant.



- The polar form of a complex number Z is $Z=r(\cos\theta+i\sin\theta)$, where $r=|Z|$ and $\theta=\arg Z$ and (r,θ) is polar coordinate, $-\pi<\theta\leq\pi$ is called the modulus - amplitude form or polar form of the complex number Z.
- If $x+iy=0$ then $x=0$ and $y=0$
- $i=\sqrt{-1}$, $i^2=-1$, $i^3=-i$, $i^4=1$
- Any integral power of i is i or $(-i)$ or (-1) or 1.
- $|Z_1+Z_2|\leq|Z_1|+|Z_2|$
- $|Z_1Z_2|=|Z_1||Z_2|$
- $\left|\dfrac{Z_1}{Z_2}\right|=\dfrac{|Z_1|}{|Z_2|}$
- $\arg(Z_1Z_2)=\arg Z_1+\arg Z_2+m$
- $\arg\left(\dfrac{Z_1}{Z_2}\right)=\arg Z_1-\arg Z_2+m$

  where $m=0$, 2π or (-2π)

  The unique value of θ such that $-\pi<\theta\leq\pi$ is called the principal value of the argument (or amplitude) or principal argument.

  Cube root of 1 are 1, ω, ω^2

  Where $\omega=\dfrac{-1+\sqrt{3}i}{2}$ or $\omega^2=\dfrac{-1-\sqrt{3}i}{2}$

  Here ω and ω^2 are called the imaginary cube roots of 1.

  If ω be an imaginary cube root of unity then $\omega^3=1$ and $1+\omega+\omega^2=0$

- **Quadratic equation :**

  An equation of the form $ax^2+bx+c=0$, $(a\neq0)$ is called quadratic equation in variable x.

  - A quadratic equation has two and only two roots.
  - If a, b and c are real numbers, then equation is called quadratic equation and its roots are given by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

If α and β be the roots of the equation $ax^2+bx+c=0$, $(a\neq 0)$ then $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$, where (b^2-4ac) is called the discriminant of equation $ax^2+bx+c=0$.

- **Nature of the roots :**

i) If discriminant is positive (i.e. $b^2-4ac>0$) then the roots α and β of equation $ax^2+bx+c=0$, are real and distinct.

ii) If discriminant is zero (i.e. $b^2-4ac=0$) then the roots of equation $ax^2+bx+c=0$ are real and equal.

iii) If discriminant is negative (i.e. $b^2-4ac<0$) then the roots of equation $ax^2+bx+c=0$ are unequal and imaginary.

iv) If discriminant is positive and a perfect square then the roots of equation $ax^2+bx+c=0$ are real, rational and unequal.

Again if discriminant is positive but not a perfect square then the roots of equation $ax^2+bx+c=0$ are real, irrational and unequal.

v) If b^2-4ac (i.e. the discriminant) is a perfect square but any one of a or b is irrational then the roots of equation $ax^2+bx+c=0$ are irrational.

Exercise – 5

Group - A

Objective Type Questions : [1/2 Marks Each]

I. Multiple choice type question :

1) If $Z=-2-\sqrt{-5}$ then $\bar{Z}$ is

a) $-2+\sqrt{-5}$ b) $2-\sqrt{-5}$ c) $2+\sqrt{-5}$ d) $-\sqrt{5}+2i$

2) If $\bar{Z}=-3+5i$ then Z is

a) –3–5i b) 3+5i c) 5+3i d) 5–3i

3) For any complex number Z, the minimum value of $|Z|+|Z-1|$ is

a) 0 b) 2 c) –1 d) 1

4) Which one is true ?

a) 2+3i>1+4i b) 3+3i>6+2i c) 5+9i>5+6i d) None of these

5) The modulus of the complex number $\left(1+\sqrt{-3}\right)$ is

a) 2 b) –2 c) 3 d) –3

6) The amplitude of the complex number Z = –2 is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

7) The argument of the complex number Z = 3i is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

8) If x, y are real and x+iy = i then

a) x=0, y=1 b) x=1, y=0 c) x=1, y=1 d) x=0, y=0

9) The value of $(Z+3)(\bar{Z}+3)$ is equivalent to

a) $|Z+3|^2$ b) $|Z-3|$ c) Z^2+3 d) None of these

10) If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

a) x=2n+1 b) x=4n c) x=2n d) x=4n+1

where $n \in N$

11) If $x, y \in R$, then x+iy is a non-real complex number if

a) x=0 b) y=0 c) $x \neq 0$ d) $y \neq 0$

12) If a+ib = c+id, then

a) $a^2+b^2=0$ b) $b^2+c^2=0$ c) $a^2+b^2=c^2+d^2$ d) $b^2+d^2=0$

13) Roots of $x^2+2 = 0$ are

a) $\pm\sqrt{2}i$ b) 2 c) 2i d) None of these

14) If $x^2+x+1 = 0$ then which of the following is correct

a) $x = \frac{-1 \pm \sqrt{3}i}{2}$ b) $x = \frac{-i-\sqrt{3}}{2}$ c) $x = \frac{1 \pm \sqrt{3}i}{2}$ d) $x = \frac{1-\sqrt{3}i}{2}$

15) If difference between the roots of the equation $x^2 - px + 8 = 0$ is 2, then p is equal to

a) ± 2 b) ± 6 d) ± 1 d) ± 5

II. Very short answer type question : [1/2 mark each]

1) Simplify : $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

2) Express $(7+5i)(7-5i)$ in the form of a+ib.

3) Express $\left(-\sqrt{3}+\sqrt{-2}\right)\left(2\sqrt{3}-i\right)$ in the form of a+ib.

4) Find the value of $\sqrt{-25} \times \sqrt{-9}$.

5) Evaluate : $\dfrac{(1-i)^3}{1-i^3}$

6) Find the values of x and y if $x+i(3x-y) = 3-6i$

7) Find the principal argument of the complex number $-1-i\sqrt{3}$.

8) If $|1-i|^n = 2^n$, then find the value of n.

9) If $(1+i)Z = (1-i)\bar{Z}$, then show that $Z = -i\bar{Z}$.

10) If the roots of the equation $5x^2 - 7x - K = 0$ are reciprocal to one another, then find the value of K.

11) If α and β are the roots of the equation $x(x-3) = 4$, find the value of $\alpha^2+\beta^2$.

12) If $2+\sqrt{3}i$ is a root of the equation $x^2+px+q = 0$, where p and q are real find p and q.

Group - B

Short answer type questions : [3 marks each]

1) If $\dfrac{(1+i)^2}{2-i} = x + iy$, then find the value of x+y.

2) Express $\left[\left(\sqrt{5}+\dfrac{1}{2}\right)\left(\sqrt{5}-2i\right)\right] \div (6+5i)$ in the form a+ib.

3) Find the real values of x and y for which $(1+i)y^2+(6+i) = (2+i)x$.

4) Find x and y if $(3x-2iy)(2+i)^2=10(1+i)$.

5) If $|Z| = 4$ and $\arg(Z) = \frac{5\pi}{6}$, then find the value of Z.

6) What is the Conjugate of $\frac{2-i}{(1-2i)^2}$?

7) Find the principal argument of $\left(1+i\sqrt{3}\right)^2$.

8) Find the conjugate and modulus of the complex number $\frac{2+3i}{3+2i}$.

9) If $Z_1 = \sqrt{2} - 3i$ and $Z_2 = 5 - i\sqrt{2}$, then find the quadrant in which $\frac{Z_1}{Z_2}$ lies.

10) Find the modulus and principal argument of $\left(-\sqrt{3}+i\right)$.

11) Discuss the nature of the roots of the equation $3x^2-7x+3 = 0$

12) If α and β be the roots of the equation $ax^2+bx+c = 0$, find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$.

13) Discuss the nature of the roots of the equation $2x^2-3x+5 = 0$.

14) If a, b, c, are in G.P, prove that the roots of the equation $ax^2+2bx+c = 0$ are equal.

15) Find m, given that the difference of the roots of the equation $2x^2-12x+m+2 = 0$ is 2.

Group - C

Long answer type questions : [4/6 marks each]

1) If $x = -2 - \sqrt{3}i$, then find the value of $2x^4+5x^3+7x^2-x+41$.

2) Evaluate : $2x^3+2x^2-7x+72$, when $x = \frac{3-5i}{2}$.

3) Find the values of x and y if $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$

4) Find real values of x and y for which the complex numbers $-3+ix^2y$ and x^2+y+4i are conjugate of each other.

5) If $|Z+1| = Z+2(1+i)$, then find the complex number Z.

6) Show that $\left|\frac{Z-2}{Z-3}\right| = 2$ represents a circle. Find its centre and radius.

7) Z_1 and Z_2 are two complex numbers such that $|Z_1| = |Z_2|$ and $\arg(Z_1)+\arg(Z_2) = \pi$ then show that $Z_1 = -\bar{Z}_2$.

8) If Z_1, Z_2 and Z_3, Z_4 are two pairs of conjugate complex numbers, then find $\arg\left(\frac{Z_1}{Z_4}\right)+\arg\left(\frac{Z_2}{Z_3}\right)$.

9) If for the complex numbers Z_1 and Z_2, $\arg(Z_1) - \arg(Z_2) = 0$, then show that $|Z_1-Z_2| = |Z_1| \sim |Z_2|$

10) Find the complex number satisfying the equation $Z+\sqrt{2}\,|(Z+1)|+i=0$

11) Find $\left|(1+i)\frac{(2+i)}{(3+i)}\right|$

12) If $f(Z) = \frac{7-Z}{1-Z^2}$, where $Z=1+2i$, then find $|f(Z)|$.

13) If $\frac{Z-1}{Z+1}$ is a purely imaginary number ($Z\neq-1$), then find the value of $|Z|$.

14) Write the complex number $Z = \frac{1-i}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ in polar form.

15) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\left(\frac{\pi}{4}\right)$ show z lies on a circle whose centre is at (0, 1) and radius $\sqrt{2}$.

16) Convert the complex number $-\sqrt{3}-i$ into polar form and determine the modulus and the principal value of the argument of given complex number.

17) Find th square root of $a^2+\frac{1}{a^2}-4\left(a+\frac{1}{a}\right)i-2$

18) Solve the equation $25x^2-30x+11 = 0$ by using the general expression for the roots of a quadratic equation and show that the roots are complex conjugate.

19) If $z = x + iy$ and $\frac{z-i}{z-i} = ib$ then prove $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{2}$

20) Solve the following equation in the complex plane C.

$6x^2 - (18+5i)x + 18 + i = 0$

ANSWERS

Group - A

I. 1(a) 2(a) 3(d) 4(d) 5(a) 6(a) 7(b) 8(a) 9(a) 10(b) 11(d) 12(c) 13(a) 14(a) 15(b)

II. 1) 1+4i 2) 74+0i 3) $\left(-6+\sqrt{2}\right)+i\left(\sqrt{3}+2\sqrt{6}\right)$ 4) –15 5) –2 6) x=3, y=15 7) $-\frac{2\pi}{3}$

8) n=0, 10) –5, 11) 17 12) p=–4 and q=13.

Group - B

1) $\frac{2}{5}$ 2) $\frac{72-15\sqrt{5}}{122} - i\left(\frac{30+9\sqrt{5}}{61}\right)$ 3) (x=5, y=2) or (x=5, y=–2) 4) $x = \frac{14}{15}, y = \frac{1}{5}$

5) $-2\sqrt{3}+2i$ 6) $-\frac{2}{25}-\frac{11}{25}i$ 7) $\frac{2\pi}{3}$ 8) $\bar{Z} = \frac{12}{13}-\frac{5}{13}i$ and |Z|=1 9) 4th quadrant 10) 2, $\frac{5\pi}{6}$

11) real, irrational and unequal 12) $\frac{3abc-b^3}{c^3}$ 13) imaginary and unequal 15) m=14

Group - C

1) 6 2) 4 3) x=3 and y=–1 4) (x=1, y=–4) or (x=–1, y=–4) 5) $Z = \frac{1}{2} - 2i$

6) centre=$\left(\frac{10}{3}, 0\right)$, radius=$\frac{2}{3}$ 8) 0, 10) Z=–2–i 11) 1 12) $\frac{\sqrt{5}}{2}$ 13) 1

14) $\sqrt{2}\left[\cos\left(\frac{-5\pi}{12}\right)+i\sin\left(\frac{-5\pi}{12}\right)\right]$ 16) |Z|=2 and $\arg(Z) = -\frac{5\pi}{6}$, 17) $\pm\left(a+\frac{1}{a}-2i\right)$

18) $\alpha = \bar{\beta}$ and $\beta = \bar{\alpha}$ i.e. roots are complex conjugate of each other 20) $\left(2+\frac{3i}{2}\right)$ or $\left(1-\frac{2i}{3}\right)$

Chapter - 6

Linear Inequality

Important points and Results :

1] Two real numbers or two algebraic expression related by the symbol '<', '>' '≤' or '≥' form an inequality.

i) $3<5$; $7>5$ are the examples of numerical inequalities.

ii) $x<5$; $y>2$: $x\geq3$; $y\leq4$ are some linear inequalities.

iii) $2<y\leq4$; $3\leq x<5$ are double inequalities.

iv) $ax+b<0$; $ax+b>0$; $ax+by<c$; $ax+by>c$; $ax^2+bx+c>0$; $ax^2+bx+c<0$; $ax^3+bx^2+cx+d>0$; $ax^3+bx^2+cx+d<0$ are some strict inequalities.

v) $ax+b\leq0$; $ax+b\geq0$; $ax+by\geq c$; $ax+by\leq c$; $ax^2+bx+c\geq0$; $ax^3+bx^2+cx+d\geq0$ are some slack inequalities.

2] An inequation may contain one or more variables. Also, it may be linear or quadratic or cubic etc.

i) Linear inequation in one variable :

Let a be a non-zero real number and x be a variable. Then inequations of the form $ax+b<0$; $ax+b\leq0$; $ax+b>0$ and $ax+b\geq0$ known as linear inequations in one variable x.

ii) Linear inequaltion in two variables :

Let b be non-zero real number and x, y be variables. $ax+by>c$; and $ax+by\geq c$ are known as linear inequation with variables x and y.

iii) Quadratic inequations :

Let a be a non-zero real numbers.Then an inequation of the form $ax^2+bx+c<0$ or $ax^2+bx+c\leq0$ or $ax^2+bx+c>0$ or $ax^2+bx+c\geq0$ is known as quadratic inequation.

3] Algebraic solutions of linear ineuations in one variable and their Graphical representation :

i) A solution of an equation is the value(s) of the variable(s) that makes it a true statement : Example : $\frac{3-2x}{5}<\frac{x}{3}-4$

ii) Solution set : The set of all possible solutions of an inequation is known as its solution set.

Example : The solution set of the inequation $x^2+1\geq0$ is the set R of all real numbers whereas the solution set of the inequation $x^2+1<0$ is the null set ϕ.

4] **Solving linear inequations in one variable :**

Rule 1 : Same number may be added to (or substrated from) both sides of an inequation without changing the sign of inequality.

Rule 2 : Both sides of an inequations can be multiplied (or divided) by the same positive real number without changing the sign of inequality. However, the sign of inequality is reversed when both sides of an inequaltion are multiplied or divided by a negative number.

Rule 3 : Any term of an inequation may be taken to the other side with its sign changed without affecting the sign of inequality.

5] **Some important characteristics of inequations :**

i) If $x>y$ then $-x<-y$ and if $x<y$ then $-x>-y$.

ii) If $x>y$ then $x+z>y+z$.

iii) $x>y$ then $x-z>y-z$.

iv) $x>y$ and $z>0$ then $x.z>y.z$.

v) $x>y$ and $z>0$ then $\frac{x}{z}>\frac{y}{z}$

vi) $x>y$ and $y>z$ then $x>z$.

vii) $x>y$ and $z<0$ then $xz<yz$.

viii) $x<y$ and $z<0$ then $xz>yz$.

6] **Inequations related to modulus function and Determination of solution set :**

i) If a is a positive real number then

a) $|x|<a \Leftrightarrow -a<x<a$ i.e. $x\in(-a,a)$

b) $|x|\le a \Leftrightarrow -a\le x\le a$ i.e. $x\in[-a,a]$

ii) If a is a positive real number, then

a) $|x|\ge a \Leftrightarrow x\le -a$ or $x\ge a$

b) $|x|>a \Leftrightarrow x<-a$ or $x>a$

iii) Let r be a positive real number and a be a fixed real number. Then –

a) $|x-a|<r \Leftrightarrow a-r<x<a+r$ i.e $x \in (a-r, a+r)$

b) $|x-a| \le r \Leftrightarrow a-r \le x \le a+r$ i.e. $x \in [a-r,\ a+r]$

c) $|x-a|>r \Leftrightarrow x<a-r$ or $x>a+r$

d) $|x-a| \ge r \Leftrightarrow x \le a-r$ or $x \ge a+r$

iv) Let *a, b* positive real number, then –

a) $a<|x|<b \Leftrightarrow x \in (-b,-a) \cup (a,b)$

b) $a \le |x| \le b \Leftrightarrow x \in [-b,-a] \cup [a,\ b]$

c) $a \le |x-c| \le b \Leftrightarrow x \in [-b+c,\ -a+c] \cup [a+c,\ b+c]$

d) $a<|x-c|<b \Leftrightarrow x \in (-b+c,\ -a+c) \cup (a+c,\ b+c)$

7] **Graphical solution of linear inequalities in two variables :**

A line divides the cartesian plane into two parts. Each part is known as a half plane. A vertical line will divide the plane in left and right half planes and a non-vertical line will divide the plane into lower and upper planes.

i) The region containing all the solutions of an inequality is called the solution region.

ii) In order to indentify the half plane represented by an-equality, it is ust sufficient to take any point (*a, b*) (not on line) and check whether it satisties the inequality or not. If it satisfies, then the inequality represents the half plane and shade the region, which contains the point, otherwise the inequality represents that half plane which does not contain the point within it. For convenience; the point (0, 0) is preference.

iii) If an inequality is of the type $ax+by \ge c$ or $ax+by \le c$; then the points on the line $ax+by=c$ are also included in the solution region. So draw a dark line in the solution region.

iv) If an inequality is of the form $ax+by>c$ or $ax+by<c$; then the points on the line $ax+by=c$ are not to be included in the solution region. So draw a broken or dotted line in the solution region.

8] i) To represent $x<0$ (or $x>a$) on number line, put a circle on the number a and dark line to the let (or right) of the number a.

ii) To represent $x \le a$ (or $x \ge a$) on a number line; put a dark circle on the number a and dark the line to the left (or right) of the number a.

9] i) If an inequality is having $\le$ or $\ge$ symbol then the points on the line are also included in the solutions of the inequality and the graph of the inequality lies left (below) or right (above) of the graph of the equality represented by dark line that satisfies arbitary point in that on part.

ii) If an inequality is having < or > symbol, then the points on the line are not included in the solutions of the inequality and the graph of the inequality lies to the left (below) or right (above) of the graph of the corresponding equality represented by dotted lines that satisfies an arbitary point in that part.

10] The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.

11] Solving simultaneous linear inequations means finding the set of points $(x, y)$for which all the contraints are satisfied. The solution set of simultaneous linear inequations may be an empty set or it may be the region bounded by the straight lines corresponding to linear inequations or it may be an unbounded region with straight line boundaries.

12] Without change the symbol of inequality we can drop any term form one side of an inequation and put it on the other side with the opposite sign. This method is known as transposition.

13] Inequality and their solution set :

i) $\beta>\alpha$ then, $(x-\alpha)(x-\beta)\leq 0 \Leftrightarrow \alpha\leq x\leq\beta$

$(x-\alpha)(x-\beta)<0 \Leftrightarrow \alpha<x<\beta$

ii) $\beta>\alpha$ then, $(x-\alpha)(x-\beta)\geq 0 \Leftrightarrow x\leq\alpha$ or $x\geq\beta$

$(x-\alpha)(x-\beta)<0 \Leftrightarrow x<\alpha$ or $x>\beta$

iii) $\beta>\alpha$ then, $\Leftrightarrow \alpha\leq x<\beta$

$$\frac{x-\alpha}{x-\beta}<0 \Leftrightarrow \alpha<x<\beta$$

iv) $\beta>\alpha$ then, $\Leftrightarrow x\leq\alpha$ or $x>\beta$

$$\frac{x-\alpha}{x-\beta}>0 \Leftrightarrow x<\alpha \text{ or } x>\beta$$

Exercise – 6

Group - A

Objective Type Questions : [1 or 2 marks each]

1. Multiple choice type questions :

i) If $\frac{2x+3}{6}<x-1,$ then range of x will be

a) $\left(-\infty, \frac{9}{4}\right)$ b) $\left(\frac{9}{4}, \infty\right)$ c) $\left[\frac{9}{4}, \infty\right]$ d) $(-\infty 9]$

ii) If $x^2 - 5x - 6 \le 0$ then x belongs to

a) [2, 3] b) (2, 3) c) [-1, 6] d) $(6, \infty)$

iii) The region bounded by the inequation $| y - x | \le 3$ are lying in the quadrant –

a) 1st b) 2nd c) 3rd d) all of these

iv) If x is an integer, then the solution set O the inequation $-x^2+7x-6 > 0$ is

a) {2, 4} b) {3, 5} c) {2, 3, 4,5} d) {4, 5}

v) If $\frac{1}{a}+\frac{1}{b}>0$, where $a>0$, and $b<0$, then the relation is –

a) $a+b=0$ b) $a+b>0$ c) $a+b<0$ d) $a+b$*0

vi) $4(x-2) \le 5(x-4)$, then least value x *is*

a) 12 b) 10 c) –12 d) –10

vii) $p>q>1$, then maximum term among $\frac{1}{p-q}, \frac{1}{\sqrt{p-q}}, \frac{1}{\sqrt{p}-\sqrt{q}}$ is

a) $\frac{1}{p-q}$ b) $\frac{1}{\sqrt{p}+\sqrt{q}}$ c) $\frac{1}{\sqrt{p}-\sqrt{q}}$ d) $\sqrt{p}+\sqrt{q}$

viii) If x is real, then $\frac{(x-a)(x-b)}{(x-c)}$ has real value, if

a) $a>c>b$ b) $a>b>c$ c) $a<c<b$ d) $a<b<c$

ix) If $|x-1|>5$, then

a) $x\in(-4, 6)$ b) $x\in[-4, 6]$ c) $x\in(-\infty,-4)\cup(6,\infty)$ d) $x\in[-\infty,-4)\cup[6,\infty)$

x) Let x and b are real numbers, if $b>0$ and $|x|>b$, then

a) $x\in(-b, \infty)$ b) $x\in[-\infty, b]$ c) $x\in(-b, b)$ d) $x\in(-\infty,-b)\cup(b,\infty)$

2. Very short answer type questions : [1 or 2 marks each]

i) Find the solution set : $\left|\frac{3x-4}{2}\right| \le \frac{5}{12}$

ii) Solve the inequation $2y+1 \ge 0$ graphically in xy-plane.

iii) Solve : $-8 \le 4(x+1) \le 7, x\in R$

iv) Solve the inequation $| x +1 | \ge 3$ and draw the solution set in number line.

v) Solve : $| x-1 | \le 5, | x | \ge 2$

vi) Find the two consecutive odd natural numbers, both of which are larger than 10; such that their sum is less than 40.

vii) Solve : $\frac{1}{2}\left(\frac{3x}{5}+4\right)\geq\frac{1}{3}(x-6)$

viii) If r is a real number such that $|r|<1$ and if $a = 5(1-r)$; then show that $0 < a < 10$.

ix) Solve : 3 - | x | > |

x) If $x-y = 3$ and $x+y \geq 9$; then find the minimum value of x.

Group - B

3. Short answer type questions : [3 marks each]

i) Solve : $\frac{|x|-5}{|x|-3}>0\ (x\neq\pm3)$

ii) Solve : $\left|\frac{2}{x-4}\right|>1,\ x\neq4,\ x\in N$

iii) Solve the inequation by using graph and draw solution region : $x \geq 1, y \geq 0, x+y \leq 10$.

iv) The temperature (T ^{0}c) at a depth x km below the sunface of the earth is given by $T=32+25(x-3)$.

If the depth below the surface of earth is between 9.8 km and 13.8 km, find the range of temperature.

v) Solve : $\frac{x-4}{x+3}>0$, where $x\in R$ and $x\neq-3$. Show the solution set on real numbers axis.

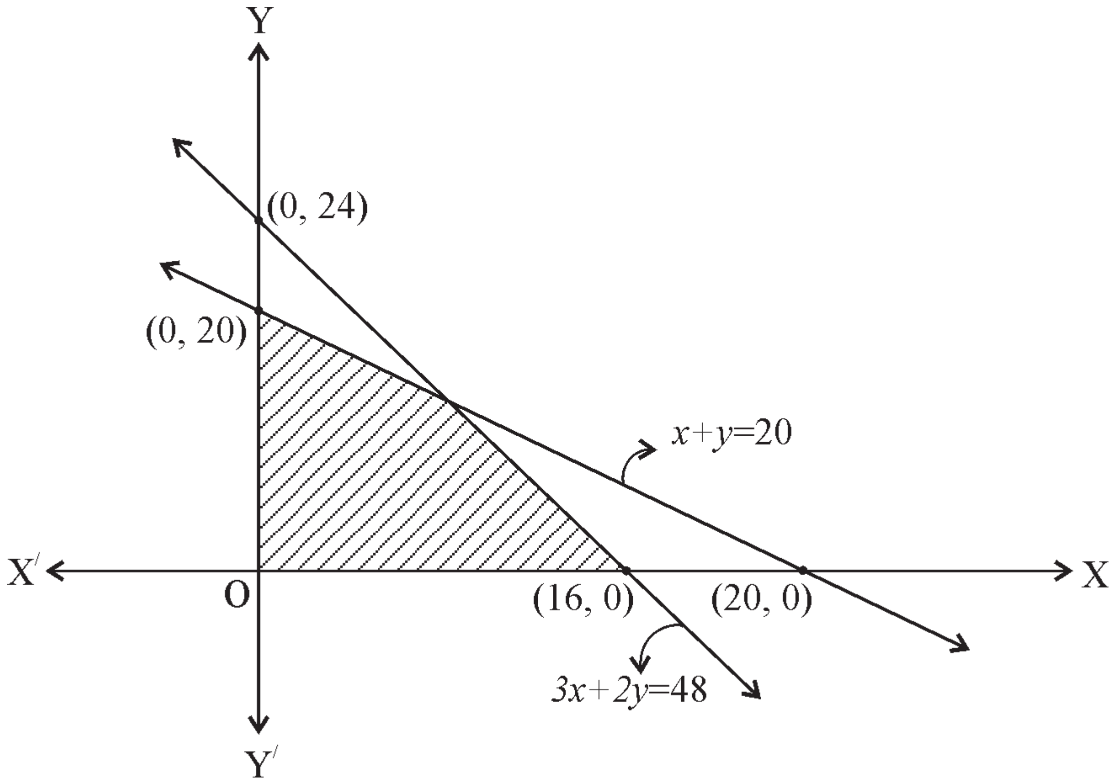
vi) Let x and $x+2$ be two consecutive even positive integers, such that $x > 12$ and the sum of the integers is less than 39.Find all possible pairs of such integers.

vii) Solve the inequation $\left|x+\frac{1}{4}\right|>\frac{7}{4}$ and draw the solution set in number line.

viii) Find the minimum value of x and maximum value of y from $2x+3y = 10$ and $x-2y \geq 12$.

ix) Maximum cost price of 4 shirts and 5-pants is Rs. 500. If shirts are selling 12% profit and pants are selling 10% profit then total profit is Rs. 54. Find the cost price of each shirt.

x) From the given graph of inequation, write the solution set :



Group - C

4. Long answer type questions : [4/6 marks each]

i) Solve : $\frac{|x+1|+2x+3}{x+3} > 2$,where $x \in R$ *and* $x \neq -3$

ii) How many liters of a 35%, acid solution must be added to 500 litres of 16% acid solution so that acid content in the resulting mixutre may be more than 25% but less than 30%.

iii) If *a, b, c* are three sides of a triangle then show that $\frac{(a+b+c)^2}{ab+bc+ca} \geq 3$.

iv) For all values of x if $x^2+4ax+2 > 0$ then show that $-\frac{1}{\sqrt{2}} < a < \frac{1}{\sqrt{2}}$.

v) If $x^2+2ax+10-3a > 0$, then show that $-5 < a < 2$, for all real values of x.

vi) A diet is to contain at least 400 units of carbohydrate, 500 units of fat and 300 units of protein. Foods F_1 contains 10 units of carbohydrate, 20 units of fat and 15 units of protein and food F_2 contains 25 unit of carbohydrate, 10 units of fat and 20 units of protein. Formulate the given data in the form of ineqations and show graphically the region representing the solution of these inequations.

vii) If $a, b, c > 0$ and if $abc = 1$, then show that $a+b+c+ab+bc+ca > 6$.

viii) If $x \in R$; then find the solution set of inequation $4^{-x+\frac{1}{2}} - 7(2^{-x}) - 4 < 0$.

ix) Solve : $|x-1|+|x-2|+|x-3| \geq 6$, where $x \in R$.

x) A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. If they operates his machines for at most 12 hours then formulate the given data in the form of inequations and show graphically the region representing the solution of these inequations.

ANSWERS

Group - A

1]. i) b ii) c iii) d iv) c v) c vi) a
vii) c viii) (a, d) ix) c x) d

2]. i) $\left[\frac{19}{18}, \frac{29}{18}\right]$ iii) $\left[-3, \frac{3}{4}\right]$ iv) $(-\propto, -4] \cup [2, \propto)$ v) $x \in (-\propto, -2) \cup [2, \propto)$
vi) (11,13), (17,19) vii) $-\propto < x \leq 120$ ix) (-2, 2)
x) 6

Group - B

3]. i) $(-\propto, -5) \cup (-3, 3) \cup (5, \propto)$ ii) 3, 5
iv) between 200^0c and 300^0c v) $x \in (-\propto, -3) \cup (4, \propto)$
vi) (14,16), (16,18) and (18,20) vii) $(-\propto, -2) \cup (1.5, \propto)$
viii) Minimum value of $x=8$, maximum value of $y=-2$
ix) Rs. 50 x) $3x+2y \leq 48$, $x+y \leq 20$, $x \geq 0$, $y \geq 0$

Group - C

4]. i) $(2, \propto) \cup (-4, -3)$
ii) more than 450 litres but less than 1400 litres.
vi) $10x+25y \geq 400$; $20x+10y \geq 500$; $15x+20y \geq 300$; $x \geq 0$, $y \geq 0$
viii) $-2 < x < \propto$
ix) $(-\propto, 0] \cup [4, \propto)$
x) $x+3y \leq 12$, $3x+y \leq 12$, $x \geq 0$, $y \geq 0$ where x and y are the number of package of nuts and package of bolts respectively.

Chapter - 7
Permutations and Combinations

Important Points and results :

- **Factorial Notation :**

  Let n be a positive integer. Then the continued product of first n natural numbers is called factorial n, to be denoted by n! or $\lfloor n$. Also, we define 0!=1.

  When n is negative or a fraction ; n! is not defined.

  Thus n! = n(n–1)(n–2).............3.2.1.

- Fundamental principal of counting or, simply, the multiplication principle :

  "If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the event in the given order is m×n.

  Fundamental Principle of counting can be generalised for any finite number of events.

  "If an event can occur in m different ways, following which another event occur in n different ways, following which a third event can occur in p different ways, then the total number of occurrance to the events in the given order is m×n×p.

- **Fundamental Principle of Addition :**

  If there are two operations such that they can be performed independently in m and n ways respectively then either of the two operations can be performed in (m+n) ways.

- **Permutions :**

  The different arrangements which can be made out of a given nuber of things by taking some or all at a times, are called permutations.

- **Permutations when all the objects are distinct :**
  Theorem 1 : The number of permutations of n different objects taken r at a time, where $0<r\leq n$ and the objects do not repeat is n(n–1)(n–2)............(n–r+1) which is denoted by ${}^{n}p_{r}$.

  ic. ${}^{n}p_{r} = \dfrac{n!}{(n-r)!}, \ 0 \leq r \leq n$

  Some Special Cases :

  ${}^{n}p_{n} = n!;\ {}^{n}p_{0} = 1,\ 0! = 1,\ {}^{n}p_{1} = n,\ {}^{n}p_{2} = n(n-1)$
  Theorem 2 : If an object can be used any number of times then number of ways to arrangements of n objects into r places = n^{r}.
  Theorem 3 : The number of permutations of n objects, where p objects are of the same kind and rest are all different = $\dfrac{n!}{p!}$.

Theorem 4 : The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ... p_k are of k^{th} kind and the rest, if any are of different kind is $\frac{n!}{p_1!\,p_2!\cdots\cdots\cdots p_k!}$.

- **Some Results :**

(i) ${}^nP_n = {}^nP_{n-1}$

(ii) ${}^nP_r = n.{}^{n-1}P_{r-1}$

(iii) ${}^{n-1}P_r + r.{}^{n-1}P_{r-1} = {}^nP_r$

(iv) ${}^nP_r = n.{}^{n-1}P_{r-1} = (n-r+1).{}^nP_{r-1}$

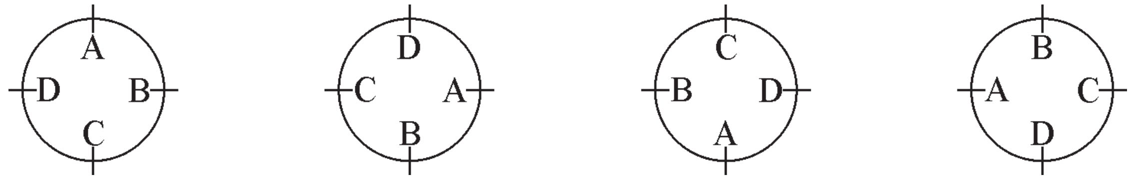
(v) $(2n)! = 2^n.n!\{1.3.5\cdots\cdots\cdots(2n-1)\}$

- Let there be n objects, of which m objects are alike of one kind, and the remaining (n–m) objects are alike of another kind. Then, the total number of mutually distinguishable permutations that can be formed from these object is $\frac{n!}{(m!)\times(n-m)!}$.
- The number of permutations of n different objects taken all at a time when each objects may be repeated any number of times in each arrangement is n^n.
- The number of all permutations (arrangments) of n distinct objects taken all at a time is n!.
- The number of mutually distinguishable permutations of n things, taken all at a time, of which p are alike of one kind, q alike of second such that p+q=n is $\frac{n!}{p!q!}$.
- The number of permutations of n things, of which p are alike of one kind, q are alike of second kind and remaining all are distinct, is $\frac{n!}{p!q!}$.

Circular Permutations : We have so far discussed arragements of things in discussed arrangments of things in a line or in a row. Such arrangements of things are known as linear permutations; arrangements of things in a circle or in a ring are known as circular permutations.

If we consider the linear permutations ABCD, BCDA, CDAB and DABC then clearly, they are distinct.

Now, we arrange A,B,C,D, along the circumference of a circle as shown below -



If we consider the position of a letter relative to others then we find that the above four arrangements are the same.

There are two types of circular permutations :

(i) The circular permutations in which the clockwise and the anticlockwise arrangements give rise to different permutations. e.g., seating arrangements of persons round a circle.

(ii) The circular permutations in which the clockwise and the anticlockwise arrangements give rise to same permutations, e.g. arranging of some beads to form a necklace.

- The number of circular permutations of n different objects is (n–1)!
- The number of ways in which n persons can be seated round a table is (n–1)!

- The number of ways in which n different beads can be arranged to form a necklace is $\frac{1}{2}(n-1)!$.

- **Combinations :**

Each of the different groups or selections that can be made out of a given number of things by taking some or all of them at a time, irrespective of their arrangements, is called a combination.

- **Notation :**

The number of all combinations of n things; taken r at a time, is denoted by ^nc_r, or c(n, r), where n and r integers such that n>0, r≥0 and n≥r and defined as.

$$^nc_r = \frac{n!}{(r!)\times(n-r)!}$$

$$= \frac{n(n-1)(n-2)............to\ r\ \ factors}{r!}$$

- **Remarks :**

(i) $^nc_n = 1$; since $^nc_n = \frac{n!}{n!(n-n)!} = \frac{1}{0!} = 1$

(ii) $^nc_0 = 1$; since $^nc_0 = \frac{n!}{0!(n-0)!} = \frac{n!}{n!} = 1$

[in reality, $^nc_0$ has no meaning]

(iii) $^nc_1 = n$; since $^nc_1 = \frac{n!}{1!(n-1)!} = \frac{n\cdot(n-1)!}{(n-1)!} = n$

(iv) $^np_r = \frac{n!}{(n-r)!}$ and

$$^{n}c_r = \frac{n!}{r!(n-r)!} = \frac{1}{r!}\cdot\frac{n!}{(n-r)!} = \frac{^{n}p_r}{r!}$$

$\Rightarrow \; ^{n}p_r = r! \times {}^{n}c_r$

- **Complementary Combinations :**

  The number of combinations of n different things taken r at a time is equal to the number of combinations of n different things taken (n–r) at a time.

- **Some results :**

  (i) $^{n}c_r = {}^{n}c_{n-r}$

  (ii) If $^{n}c_x = {}^{n}c_y$ then either x = y or x + y = n

  (iii) $^{n}c_r + {}^{n}c_{r-1} = {}^{n+1}c_r$

  (iv) $\frac{^{n}c_{r-1}}{^{n-1}c_{r-1}} = \frac{n}{(n-r+1)}$

  (v) $\frac{^{n}c_r}{^{n-1}c_{r-1}} = \frac{n}{r}$

  (vi) $\frac{^{n}c_r}{^{n}c_{r+1}} = \frac{r+1}{n-r}$

  (vii) $\frac{^{n}c_r}{^{n}c_{r-1}} = \frac{n-r+1}{r}$

- **Restricted combinations :**

  (i) The number of combinations of n different things taken r at a time in which p particular things always occur is $^{n-p}c_{r-p}$.

  (ii) The number of combinations of n different things taken r at a time in which p particular things never occur is $^{n-p}c_r$.

  The number of combinations of n different things taken one at a time is $^{n}c_1$; two at a time is $^{n}c_2$, three at a time is $^{n}c_3$ and so on, taken all at a time is $^{n}c_n$. Therefore, the required number of combinations of n different things taken any number at a time = $^{n}c_1 + {}^{n}c_2 + {}^{n}c_3 + ...{}^{n}c_n$.

  ∴ The total number of combinations of n different things taken any number at a time = $2^n - 1$.

  The total number of combinations of (p+q+r+....) things of which p things are alike of one kind, q things are alike of 2nd kind, r things are alike of third kind and so on, is [(p+1)(q+1)(r+1)....] –1.

If n is an even positive integer then ${}^nc_r$ is the greatest. When r=$n/2$; again, if n is an odd positive integer then ${}^nc_r$ is the greatest when $r=\frac{n-1}{2}$ or $r=\frac{n+1}{2}$.

Number of all straight lines obtained by joining two points out of n points = ${}^nc_2=\frac{1}{2}n(n-1)$.

Excrcise – 7

Group - A

A] Objective Type Questions : [1/2 marks each]

1. Multiple choice type questions :

(i) The number of words from the letters of the word 'BHARAT' in which B and H will never come together is

a) 360 b) 240 c) 120 d) None of these.

(ii) The number of five-digit telephone numbers having at least one of their digits repeated is

a) 90000 b) 100000 c) 30240 d) 69760

(iii) The number of permutations of n different things taking r at a time when 3 particular things are to be included is –

a) ${}^{n-3}p_{r-3}$ b) ${}^{n-3}p_r$ c) ${}^np_{r-3}$ d) $r!\ {}^{n-3}c_{r-3}$

(iv) The product of r consecutive positive integers is divisible by

a) r! b) r!+1 c) (r+1)! d) none of these

(v) If ${}^np_4=x.{}^{n-1}p_{r-1}$; then which of the following is the value of x ?

a) n b) n(n–1) c) $\frac{n-1}{n}$ d) $\frac{n}{n-r}$

(vi) m(m–1)(m–2)..........3.2.1 =

a) m! b) (m+1)! c) (m–1)! d) none of these

(vii) Given, ${}^np_5=x.{}^9p_3$; state which of the following is the value of x –

a) 56 b) 42 c) 30 d) 20

(viii) If ${}^{k+5}p_{k+1}=\frac{11(k-1)}{2}\cdot{}^{k+3}p_k$ then the value of k are

a) 7 and 11 b) 6 and 7 c) 2 and 11 d) 2 and 6

(ix) ${}^{n-1}c_r+{}^{n-1}c_{r-1}=$

a) ${}^nc_{r+1}$ b) ${}^{n+1}d_r$ c) ${}^nc_r$ d) n!

(x) If ${}^np_r=x.{}^np_r$ then x =

a) ${}^{n}p_{r-1}$ b) ${}^{n}c_{r-1}$ c) n! d) r!

(xi) If ${}^{n}C_{3} = K.n(n-1)(n-2)$ then K =

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{6}$

(xii) If $(n-r+1).{}^{n}C_{r-1} = m \times {}^{n}C_{r}$ then m =

a) r! b) 1 c) n d) r

(xiii) The number of combinations of n different things taken r at a time in which p particular things always occur is

a) ${}^{n-p}C_{r-p}$ b) ${}^{n-p}C_{r-p}$ c) pr d) (r–p)!

(xiv) The number of combinations of n different things taken r at a time in which p particular things never occur is –

a) ${}^{n-p}C_{r-p}$ b) ${}^{n-p}C_{r}$ c) ${}^{n}C_{r}$ d) $\frac{n-p!}{r!}$

(xv) If ${}^{16}C_{r} = {}^{16}C_{2r+1}$, then which of the following is the value of r ?

a) 6 b) 5 c) 4 d) 3

(xvi) The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is –

a) 432 b) 108 c) 36 d) 18

(xvii) A five digit number divisible by 3 is to be formed uisng the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is –

a) 216 b) 600 c) 240 d) 3135

(xviii) Everybody in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is

a) 11 b) 12 c) 13 d) 14

(xix) Given 5 different green dyes, four different blue dyes and three diffferent red dyes, the number of combinations of dyes which can be choosen taking at least one green and one blue dye is –

a) 3600 b) 3720 c) 3800 d) 3600

(xx) The numbers of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lies on the same line is

a) 105 b) 15 c) 175 d) 185

2. Very Short answer type questions : (each question caries 1 or 2 marks)

(i) If P(9, r) = 3024, find r.

(ii) If P(n–1, 3) : P(n, 4) = 1 : 9, find n.

(iii) If $P(15, r–1) : P(16, r–2) = 3:4$, find r

(iv) In how many ways can five children stand in a queqe ?

(v) In how many ways can 4 letters be posted in 5 letter boxes ?

(vi) Write the total number of possible outcomes in a throw of 3 dice in which at least one of the dice shows an even number.

(vii) Find the number of ways in which 7 men and 7 women can sit on a round table such that no two women sit together.

(viii) Find the remainder obtained when $1! + 2! + 3! + \dots\dots\dots + 200!$ is divided by 14.

(ix) In how many ways 4 women can collect water from 4 taps, if no tap remains unused ?

(x) Find the number of numbers that can be formed using all four digits 1, 2, 3, 4 without repetition.

Group - B

3. Short answer type questions : (each question carries 3 marks)

i. Prove that : $\dfrac{(2n+1)!}{n!} = 2^n\{1\cdot3\cdot5\cdots\cdots\cdot(2n-1)(2n+1)\}$

ii. Find the number of different words that can be formed from the letters of the word 'TRIANGLE', so that no vowels are together.

iii. If a convex polygon has 44 diagonals, then find the number of its sides.

iv. Find r, if $5\,{}^4p_r = 6\,{}^5p_{r-1}$

v. If ${}^9p_r + 5.{}^9p_4 = {}^{10}p_r$, then find r.

vi. If ${}^{(a^2-a)}C_2 = {}^{(a^2-a)}C_4$, then find a.

vii. Find the value of $\left({}^7C_0 + {}^7C_1\right) + \left({}^7C_1 + {}^7C_2\right) + \dots\dots\dots\dots\dots\dots + \left({}^7C_6 + {}^7C_7\right)$

viii. Find the number of ways in which a host lady can invite for a party of 8 out of 12 people of whom two do not want to attend the party together.

ix. How many four-letter words can be formed using the letters of word 'FAILURE', so that

(a) F is included in each word ?

(b) F is not included in any word ?

x. If ${}^np_r = 840$; ${}^nC_r = 35$, then find r .

Group - C

4. Long answer type questions : (each question carries 4 or 6 marks)

(i) If $\dfrac{{}^nP_{r-1}}{a} = \dfrac{{}^nP_r}{b} = \dfrac{{}^nP_{r+1}}{c}$, prove that $b^2 = a(b+c)$.

(ii) Four different letters are written and the corresponding addresses are correctly written on four envelopes. One letter can be replaced in one envelope. Find the number of ways so that all the letters are wrongly placed.

(iii) Find the total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together.

(iv) Find the number of students to be selected at a time from a group of 14 students so that the number of selections is greatest. Find the greatest number of selections. Also find the greatest number of selections when there are 15 students.

(v) If $\frac{{}^nC_{r-1}}{a} = \frac{{}^nC_r}{b} = \frac{{}^nC_{r+1}}{c}$ then prove that $n = \frac{ab+2ac+bc}{b^2-ac}$ and $r = \frac{a(b+c)}{b^2-ac}$

(vi) Find the number of permutations of the letters of the words FORECAST and MILKY taking 5 at a time of which 3 letters from the first word and 2 from the second.

(vii) How many different algebraic quantities can be formed by combining a, b, c, d, e with the '+' and '–' signs, all the letters taken together.

(viii) A bag contains 50 tickets numbered 1, 2, 3, ... 50, of which 5 are drawn at random and arranged in ascending order of their numbers $x_1<x_2<x_3<x_4<x_4<x_5$. Find the number of selections so that $x_3=30$.

(ix) Prove that : ${}^nC_r+3.{}^nC_{r-1}+3.{}^nC_{r-2}+{}^nC_{r-3} = {}^{n+3}C_r$

(x) Show that : $\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1\cdot3\cdot5\cdots\cdots(4n-1)}{\{1\cdot3\cdot5\cdots\cdots(2n-1)\}^2}$

(xi) A, B and C have respectively 4, 3 and 2 different books. In how many different ways can they interchange the books among themselves, without alterning the total numbers initially possessed by each ?

(xii) In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back ?

ANSWERS

Group - A

1]. i) b   ii) d   iii) d   iv) a   v) a   vi) a

vii) c   viii) b   ix) c   x) d   xi) d   xii) d

xiii) a   xiv) b   xv) b   xvi) b   xvii) a   xviii) b

xix) b   xx) d

2]. i) 4   ii) 9   iii) 14   iv) 120   v) 61   vi) 189

vii) $7! \times 6!$ viii) 6   ix) 4!   x) 24

Group - B

3]. ii) 14400   iii) n=11   iv) r = 3   v) 5   vi) 3

vii) $2^8 - 2$ viii) ${}^{12}c_8 - {}^{10}c_6$ ix) (a) ${}^{6}c_3 \times \underline{|4}$ (b) ${}^{6}c_4 \times \underline{|4}$ (x) 4

Group - C

4]. ii) 9   iii) 35   iv) 6435   vi) 67200   vii) 32

viii) 77140   xi) 1260   xii) 91.(12)!

Chapter - 8

Binomial Theorem

Important Points and results :

- If a and b are real numbers and n is a positive integer, then

  $(a+b)^n = {}^nc_0a^n+{}^nc_1a^{n-1}b+{}^nc_2a^{n-2}b^2+........+{}^nc_nb^n$, where ${}^nc_r = \dfrac{n!}{r!(n-r)!}$ for $0\leq r\leq n$.

- The general term in the expansion of $(a+b)^n$ in the $(r+1)^{th}$ term given by $T_{r+1}={}^nc_ra^{n-r}b^r$.
- The total number of terms in the binomial expansion of $(a+b)^n$ is n+1.
- The coefficients in the expansion of $(a+b)^n$ follow a certain pattern known as Pascals's triangle.
- If n is even, then the total number of terms in the expansion of $(a+b)^n$ is n+1 (odd). Hence, there is only one middle term, i.e. $\left(\dfrac{n}{2}+1\right)^{th}$ term is the middle term.
- If n is odd, then the total number of terms in the expansion of $(a+b)^n$ is n+1 (even). Hence, there are two middle terms i.e., $\left(\dfrac{n}{2}+1\right)^{th}$ and $\left(\dfrac{n+1}{2}+1\right)^{th}$ are two middle terms.
- The p^{th} term from the end in the expansion of $(a+b)^n$ is $(n-p+2)^{th}$ term from the beginning.
- In the Binomial expansion of $(a+b)^n$, the coefficents ${}^nc_0, {}^nc_1, {}^nc_2, ... {}^nc_n$ are known as binomial coefficients.
- If a=b=1, then the sum of all the binomial coefficients is equal to 2^n. i.e., ${}^nc_0+{}^nc_1+{}^nc_2+..........+{}^nc_n = 2^n$.

  Again if a=1, b=–1 then the sum of all the odd binomial coeffients is equal to the sum of all the even binomial coeffients and each is equal to 2^{n-1}, i.e. ${}^nc_0+{}^nc_2+{}^nc_4+........ = {}^nc_1+{}^nc_3+{}^nc_5+........=2^{n-1}$.

Exercise – 8

Group - A

Objective Type Questions : [1 or 2 marks each]

1. Multiple choice type questions :

i) The number of terms in the expansion of $\left(2x-\dfrac{3}{y}\right)^{15}$ is

a) 14 b) 15 c) 30 d) 16

ii) The number of terms in the expansion of $\left\{(2x+5y)^{13}-(2x-5y)^{13}\right\}$ is

a) 14 b) 7 c) 12 d) 28

iii) The 7^{th} term in the expansion of $\left(\sqrt{x}+\sqrt{y}\right)^{10}$ is

a) $120xy^2$ b) $210x^3y^4$ c) $210x^2y^3$ d) none of these.

iv) The 5^{th} term from the end in the expansion of $\left(x-\frac{1}{x}\right)^{12}$ is

a) $495x^{-4}$ b) $495x^4$ c) $99x^4$ d) $99x^{-4}$

v) If in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$, x^{-17} occurs in the r^{th} term, then

a) r = 10 b) r = 11 c) r = 12 d) r = 13

vi) The middle term in the expansion of $\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$ is

a) 250 b) 252 c) 251 d) none of these.

vii) The coefficent of x^{-3} in the expansion of $\left(x-\frac{m}{x}\right)^{11}$ is

a) $-924m^7$ b) $-792m^5$ c) $-792m^6$ d) $-330m^7$

viii) The term independent of x in the expansion $\left(x-\frac{1}{x}\right)^{10}$ is

a) 252 b) 210 c) 756 d) 504

ix) Which term contains x^7 in the expansion of $\left(2x^2-\frac{3}{x}\right)^{11}$?

a) 6^{th} b) 7^{th} c) 8^{th} d) 5^{th}

x) ${}^nc_0+{}^nc_2+{}^nc_4+{}^nc_6+$................= ?

a) 2^n b) 2^{n-1} c) 4^{n-1} d) 4^n

2. Very short answer type questions : (each question carries 1 or 2 marks)

i) Determine whether the expansion of $\left(x^2-\frac{2}{x}\right)^{18}$ will contain a term containing x^{10}?

ii) Find the relation of the coeffients of x^p and x^q in the expansion of $(1+x)^{p+q}$.

iii) Find the coefficient of x^n in the expansion of $(1+x)(1-x)^n$.

iv) Find the constant term in the expansion of $\left(3x+\frac{7}{x^3}\right)^{12}$.

v) Write the general term in the expansion of $(x^2-y)^6$.

vi) If the sum of coefficients in the expansion of $(x+y)^n$ is 4096, then find the value of n.

vii) In the binomial expansion of $(a+b)^n$, the coefficients of the 4^{th} and 13^{th} terms are equal to each other. Find the value of n.

viii) Write the 4^{th} term from the end in the expansion of $\left(\frac{3}{x^2}-\frac{x^3}{6}\right)^7$.

ix) Find the total number of terms in the expansion of $(a+b+c)^{10}$.

x) If the middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, find p.

Group - B

3. Short answer type questions : (each question carries 3 marks)

i) Expand $(2x-3y)^4$ using binomial theorem.

ii) Using binomial theorem, prove that $(101)^{50}>(100)^{50}+(99)^{50}$.

iii) Using binomial theorem, find the value of $(103)^4$.

iv) Determine which number is smaller $(1.2)^{4000}$ or 800 ?

v) Using binomial theorem, prove that $2^{3n}-7n-1$ is divisible by 49, where $n\in N$.

vi) Find the value of r when it is being given that the coefficients of $(2r+4)^{th}$ and $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal.

vii) If the 7^{th} terms from the beginning and end in the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ are equal, find the value of n.

viii) Find the value of a for which the coefficients of the middle terms in the expansions of $(1+ax)^4$ and $(1-ax)^6$ are equal.

xx) Find the middle terms in the expansion of $\left(\frac{p}{x}+\frac{x}{p}\right)^9$.

x) Find the value of a so that the term independent of x in the expansion $\left(\sqrt{x}+\frac{a}{x^2}\right)^{10}$ is 405.

xi) Find the coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+..........+(1+x)^{30}$.

xii) Find the value of $(0.999)^3$ correct to 3 places of decimal.

Group - C

Long answer type questions : [4 or 6 marks each]

i) Show that the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is $\frac{1\cdot3\cdot5\cdot\cdots\cdots(2n-1)}{n!}(-2)^n$, where $n\in N$.

ii) Find numerically the greatest term in the expansion of $(2+3x)^9$, where $x=\frac{3}{2}$.

iii) If x^p occours in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, then prove that its coefficient is $\frac{(2n)!}{\left\{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!\right\}}$.

iv) Find the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{bx}\right)^{11}$ and find the relation between a and b so that these coefficients are equal.

v) Find the coeffient of x^{50} after simplifying and collecting the like terms in the expansion of $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+..........+x^{1000}$.

vi) If in the expansion of $(1-x)^{2n-1}$, the coefficient of x^r is denoted by a_r, then prove that $a_{r-1}+a_{2n-r}=0$

vii) If a_1, a_2, a_3, a_4 be the coefficients of four consecutive terms in the expansion of $(1+x)^n$, then prove that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$.

viii) The 3rd, 4th and 5th terms in the expansion of $(x+a)^n$ are respectivley 84,280 and 560, find the values of x, a and n.

ix) If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^{2n}$ are in A.P, show that $2n^2-9n+7=0$.

x) If 3rd, 4th, 5th and 6th terms in the expansion of $(x+\alpha)^n$ be respectively a, b, c and d, prove that $\frac{b^2-ac}{c^2-bd}=\frac{5a}{3c}$.

xi) Find a, b and n in the expansion of $(a+b)^n$, if the first three terms in the expansion are 729, 7290 and 30375 respectively.

xii) Find the coefficient of the term independent of x in the expansion of $\left(\dfrac{x+1}{x^{2/3}-x^{1/3}+1}-\dfrac{x-1}{x-x^{1/2}}\right)^{10}$.

ANSWERS

Group - A

1.	i) d	ii) b	iii) c	iv) a	v) c
	vi) b	vii) d	viii) a	ix) a	x) b
2.	i) No	ii) Equal	iii) $(-1)^n(1-n)$	iv) ${}^{12}c_3.3^9.7^3$	v) $(-1)^r\ {}^6c_r x^{12-2r} y^r$
	vi) 12	vii) 15	viii) $\dfrac{35x^6}{48}$	ix) 66 x) ± 2	

Group - B

3.	i) $16x^4-96x^3y+216x^2y^2-216xy^3+81y^4$			iii) 112550881	iv) 800
	vi) r=6	vii) n=12	viii) $a=-\dfrac{3}{10}$	ix) $\dfrac{126p}{x},\dfrac{126x}{p}$	x) $a=\pm 3$
	xi) ${}^{31}c_6-{}^{21}c_6$	xii) 0.997			

Group - C

4.	ii) $T_7=\dfrac{7\times 3^{13}}{2}$	iv) ${}^{11}c_5a^6b^{-5}$, ${}^{11}c_6a^5b^{-6}$, ab=1		v) ${}^{1001}c_{50}$	viii) x=1, a=2, n=7
	xi) a=3, b=5, n=6		xii) 210		

Chapter - 9

Sequences and Series

Important Points and results :

- By a sequence, we mean an arrangement of number in definite order according to some rule.

  A sequence is a function whose domain is the set of natural numbers or some subsets of the type $\{1,2,3,..........n\}$. A sequence containing a finite number of terms is called a finite sequence. A sequence is called an infinite sequence if it is not a finite sequence.

- If $a_1, a_2, a_3, ... a_n, ...$ is a sequence, then the expression $a_1+a_2+a_3+.........+a_n+...........$ is called a series.

  A series is called a finite series if it has got finite number of terms, otherwise, it is called an infinite series.

- Those sequences whose terms follow certain patterns are called progressions.
- An arithmetic progression (A.P) is a sequence in which terms increase or decrease regularly by the same constant. This cosntant is called common difference of the A.P. Usually, we denote the first trm of A.P by a, the common difference by d and the last term by l. The general term or the n^{th} term of the A.P. is given by $a_n=a+(n-1)d$.
- If an A.P. consists of m terms, then n^{th} term from the end is equal to $(m-n+1)^{\text{th}}$ term from the beginning. The n^{th} term from the last is given by $a_n=l-(n-1)d$.
- The following ways of selecting terms of an A.P. are generally very convenient :

Number of terms	Terms	Common difference
3	$a-d,\ a,\ a+d$	d
4	$a-3d,\ a-d,\ a+d,\ a+3d$	$2d$
5	$a-2d,\ a-d,\ a,\ a+d,\ a+2d$	d
6	$a-5d,\ a-3d,\ a-d,\ a+d,\ a+3d,\ a+5d$	$2d$

- The sum S_n of the first n terms of an A.P. is given by $S_n=\frac{n}{2}[2a+(n-1)d]=\frac{n}{2}(a+l)$, where

  $l=a+(n-1)d$ is the last term of the A.P.. It is also in quadratic form n, $S_n=An^2+Bn$ where $A=\frac{d}{2}$,

  $B=a-\frac{d}{2}$ and the general term or n^{th} term is given by $a_n=S_n-S_{n-1}$.

- If a, A and b are in A.P. then A is called the arithmetic mean of numbers a and b where $A=\frac{a+b}{2}$.

- If the terms of an A.P. are increased, decreased, multiplied or divided by the same constant, they still remain in A.P.

  If a_1, a_2, a_3........... are in A.P. with common difference d, then

  i) $a_1 \pm k$, $a_2 \pm k$, $a_3 \pm k$, ... are also in A.P. with common difference d.

  ii) $a_1 k$, $a_2 k$, $a_3 k$,................. are also in A.P. with common difference dk ($k \neq 0$).

  iii) $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \cdots\cdots$ are also in A.P. with common difference $\frac{d}{k}$ ($k \neq 0$).

- If a_1, a_2, a_3, ... and b_1, b_2, b_3, ... are two A.P., then

  i) $a_1 \pm b_1$, $a_2 \pm b_2$, $a_3 \pm b_3$, ... are also in A.P.

  ii) $a_1 b_1$, $a_2 b_2$, $a_3 b_3$, ... and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \cdots\cdots$ are not in A.P.

- If n^{th} term of any sequence is linear expression in n, then the sequence is an A.P..
- If sum of n terms of any sequence is a quadratic expression in n, then sequence is an A.P.
- In an A.P. the sum of the terms equidistant from the beginning and the end is always same and is equal to the sum of first and last term.
- A sequence of non-zero numbers is called a geometric progression (G.P.) if the ratio of a term and the term preceding to it is always a constant quantity. The constant ratio is called the common ratio of the G.P.. Usually, we denote the first term of a G.P. by a and its common ratio by r. The general or the n^{th} term of G.P. is given by $a_n = ar^{n-1}$.
- If a G.P. consists of m terms, then n^{th} term from the end is $(m-n+1)^{th}$ term from the beginning.

  The n^{th} term from the end is given by $a_n = \frac{l}{r^{n-1}}$.

- In a G.P., the product of the terms equidistant from the beginning and the end is always same and is equal to the product of first and last term.
- It is always convenient to select the terms of a G.P. in the following manner :

Number of term	Terms	Common ratio
3	$\frac{a}{r}$, a, ar	r
4	$\frac{a}{r^3}$, $\frac{a}{r^3}$, ar, ar^3	r^2
5	$\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2	r

- The sum S_n of the first n terms of a G.P. is given by

$S_n = \frac{a(r^n - 1)}{r-1}$ or $\frac{a(1-r^n)}{1-r}$ if $r \neq 1$

$S_n = na$ if $r=1$

Also, $S_n = \frac{a - lr}{1-r}$ or $\frac{lr - a}{r-1}$, where l is the last term.

- If a, G and b are in G.P., then G is called the geometric mean of the numbers a and b and is given by $G=\sqrt{ab}$.
- If the terms of a G.P. are multiplied or divided by the same non-zero constant, then they still remain in G.P.

i) If a_1, a_2, a_3.............. are in G.P. then a_1k, a_2k, a_3k,........... and $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \cdots\cdots$ are also in G.P. where $k \neq 0$ is a constant.

ii) If a_1, a_2, a_3, ... are in G.P., then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$,............. are also in G.P.

iii) If a_1, a_2, a_3,............ and b_1, b_2, b_3............ are two G.P.s, then a_1b_1, a_2b_2, a_3b_3, ... and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \cdots\cdots$ are also in G.P.

- If A and G are respectively arithmetic and geometric means between two positive numbers a and b, then

i) $A>G$

ii) The quadratic equation having a, b as its roots is $x^2-2Ax+G^2=0$

iii) $a:b = \left(A + \sqrt{A^2 - G^2}\right) : \left(A - \sqrt{A^2 - G^2}\right)$

- **Some results on the sum of special sequences**

i) Sum of the first n natural numbers :

$$\sum n = 1 + 2 + 3 + \cdots\cdots + n = \frac{n(n+1)}{2}$$

ii) Sum of the squares of first n natural numbers :

$$\sum n^2 = 1^2 + 2^2 + 3^2 + \cdots\cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

iii) Sum of the cubes of first n natural numbers :

$$\sum n^3 = 1^3 + 2^3 + 3^3 + \cdots\cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

- **Sum of an infinite G.P.**

The sum of an infinite G.P. with first term a and common ratio r ($|r|<1$) is $S_n = \frac{a}{1-r}$

If $r \geq 1$, then the sum of an infinite G.P. tends to infinity

Exercise – 9
Group - A

Objective Type Questions : [1 or 2 marks each]

1. Multiple choice type questions :

i) A sequence may be defined as a

a) function whose range $\subseteq$ N b) function whose domain $\subseteq$ N

c) relation whose range $\subseteq$ N d) Progression having real values.

ii) The sides of a right triangle are in A.P. The ratio of their lengths is

a) 1:2:3 b) 2:3:4 c) 3:4:5 d) 5:8:3

iii) If 1+6+11+.........+x = 148 then x = ?

a) 36 b) 40 c) 48 d) 54

iv) If the sum of n terms of an A.P. be $3n^2-n$, then its common difference is

a) 2 b) 3 c) 4 d) 6

v) The sum of n terms of an A.P. is $3n^2+5n$. Which of its term is 164 ?

a) 28^{th} b) 27^{th} c) 26^{th} d) 29^{th}

vi) If the sum of p terms of an A.P. is q and the sum of q terms is p, then the sum of $p+q$ terms will be

a) $p-q$ b) $p+q$ c) pq d) $-(p+q)$

vii) If in an A.P. $S_n = n^2p$ and $S_m = m^2p$, where S_r denotes the sum of r terms of the A.P., then S_p is equal to

a) $\frac{1}{2}p^3$ b) mnp c) p^3 d) $(m+n)p^2$

viii) If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between two unequal numbers a and b, then the value of n is

a) 0 b) 1 c) 2 d) 4

ix) If the n^{th} term of a G.P. is 2^n, the sum of its first 6 terms is

a) 124 b) 126 c) 190 d) 254

x) If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$, then xyz are in

a) A.P b) G.P c) H.P d) None of these

xi) In a G.P., if the $(m+n)^{th}$ term is p and $(m-n)^{th}$ term is q, then its m^{th} term is

a) 0 b) pq c) $\sqrt{pq}$ d) $\frac{1}{2}(p+q)$

xii) The minimum value of the expression 3^x+3^{1-x}, $x \in R$ is

a) 0 b) $\frac{1}{3}$ c) 3 d)

xiii) If x be the A.M. and y, z be two G.M.s between two positive numbers, then $\frac{y^3+z^3}{xyz}$ is

a) 1 b) 2 c) $\frac{1}{2}$ d) none of these.

xiv) The sum of the series $1^2+3^2+5^2+$............ to n terms is

a) $\frac{n(n+1)(2n+1)}{2}$ b) $\frac{n(2n-1)(2n+1)}{3}$

c) $\frac{(n-1)^2(2n+1)}{6}$ d) $\frac{(2n+1)^3}{3}$

xv) If $b=a+a^2+a^3+$................$\propto$ then the value of a will be ?

a) $\frac{1}{a}$ b) $\frac{b}{1-b}$ c) $\frac{b}{1+b}$ d) $\frac{1}{b}$

2. Very short answer type questions : (each question carries 1 or 2 marks)

i) If 9 times the 9^{th} term of an A.P. is equal to 13 times the 13^{th} term, then find the 22^{nd} term of the A.P.

ii) If x, $2y$, $3z$ are in A.P., where the distinct numbers x, y, z are in G.P., then find the common ratio of the G.P.

iii) Find the value of n for which n^{th} terms of the A.P.s 3, 10, 17, ... and 63, 65, 67, ... are equal.

iv) If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in G.P., then find the value of x.

v) If second, third and sixth terms of an A.P. are consecutive terms of a G.P., then find the common ratio of the G.P.

vi) Write the product of n geometric means between two numbers a and b.

vii) Write the quadratic equation; the arithmetic and geometric means of whose roots are A and G respectively.

viii) Write the sum of first n odd natural numbers.

ix) The third term of a G. P. is 4. Find the product of its first five terms.

x) Find the sum upto infinity of the G.P. $-\frac{5}{4}, \frac{5}{16}, -\frac{5}{64}, \cdots\cdots$

Group - B

3. Short answer type questions : (each question carries 3 marks)

i) Which term of the sequence 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \cdots\cdots$ is the first negative term ?

ii) How many numbers of two digits are divisible by 7 ?

iii) Find the 10th common term between the arithmetic series 3+7+11+15+.................... and 1 + 6 + 11 + 16 + ...

iv) In the arithmetic progressions 2, 5, 8, ... upto 50 terms and 3, 5, 7, 9,......... upto 60 terms, find how many terms are identical ?

v) If x, y, z are in A.P. and A_1 is the A.M. between x and y while A_2 is the AM between y and z then prove that AM between A_1 and A_2 is y.

vi) In a G.P. of positive terms, if any term is equal to the sum of next two terms, find the common ratio of the G.P.

vii) If 5, x, y, z, 405 are the first five terms of a G.P., find the values of x, y, z.

viii) Find the 4th term of the G.P. whose 5th term is 32 and 8th term is 256.

ix) Insert three G.M. between 3 and 48.

x) The first term of an A.P. is a , the second term is b and the last term is c. Show that the sum of the A.P. is $\frac{(b+c-2a)(c+a)}{2(b-a)}$.

xi) If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P., prove that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P.

xii) If a is the A.M. of b and c and the two G.M. are G_1 and G_2, then show that $G_1^3+G_2^3=2abc$.

xiii) Prove that $6^{1/2} \times 6^{1/4} \times 6^{1/8} \times \cdots\cdots\infty = 6$

Group - C

Long answer type questions : [4 or 6 marks each]

i) If there are $(2n+1)$ terms in an A.P., then prove that the ratio of the sum of the terms in odd places and the sum of the terms in even places is n + 1 : n.

ii) If a, b, c, d are in G.P., prove that $a^2-b^2, b^2-c^2, c^2-d^2$ are also in G.P.

iii) If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in A.P., where $a_i>0$ for all i, show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\cdots\cdots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

iv) The ratio of the sum of n terms of two A.P.'s is $(7n+1) : (4n+27)$. Find the ratio of their m^{th} term.

v) If $(b-c)^2$, $(c-a)^2$, $(a-b)^2$ are in A.P., prove that $\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are in A.P.

vi) Show that (x^2+xy+y^2), (z^2+xz+x^2) and (y^2+yz+z^2) are consecutive terms of an A.P., if x, y and z are in A.P.

vii) The product of three numbers in A.P. is 224 and the largest number is 7 times the smallest. Find the numbers.

viii) The $(m+n)^{th}$ and $(m-n)^{th}$ terms of a G.P. are p and q respectively. Show that the m^{th} and the n^{th} terms of the G.P. are $\sqrt{pq}$ and $p.\left(\frac{q}{p}\right)^{\frac{m}{2n}}$.

ix) Find three numbers in G.P. where sum is 13 and the sum of whose square is 91.

x) The product of three numbers in G.P. is 216. If 2, 8, 6 be added to them, the results are in A.P.. Find the numbers.

xi) Find the sum of the series 5 + 5.5 + 5.55 + 5.555 +............ to n terms.

xii) If S_1, S_2 and S_3 be respectively the sum of n, $2n$ and $3n$ terms of a G.P.. Prove that $S_1(S_3-S_2) = (S_2-S_1)^2$.

xiii) If a, b, c, d are in G.P.. Prove that $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ are in G.P..

xiv) Find two positive numbers whose difference is 12 and whose A.M. exceeds the G.M. by 2.

xv) Find the sum of following series upto n terms.

a) $1^3 + 3^3 + 5^3 + 7^3 +$..................

b) $3 + 7 + 13 + 21 + 31 +$..............

c) $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots\cdots\cdots\cdots$

d) $1 \times 2 \times 4 + 2 \times 3 \times 7 + 3 \times 4 \times 10 +$............

xvi) Find the sum of the series whose n^{th} term is given by

a) $2n^2-3n+5$

b) $4n^3+6n^2+2n$

xvii) The sum of an infinite G.P. is 57 and the sum of their cubes is 9747, find the G.P..

xviii) If $|x| < 1$ and $|y| < 1$, find the sum to infinity of the following series

$(x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) +$..............

ANSWERS

Group - A

1. i) b ii) c iii) a iv) d v) b vi) d vii) c viii) a ix) b
   x) a xi) c xii) d xiii) b xiv) b xv) c

2. i) 0 ii) $\frac{1}{3}$ iii) 13 iv) $\log_a\left(\log_b^a\right)$ v) 3 vi) $(ab)^{n/2}$ vii) $x^2–2Ax+G^2 = 0$
   viii) n^2 ix) 4^5 x) –1

Group - B

3. i) 28^{th} term ii) 13 iii) 191 iv) 20 vi) $2\sin 18^0$ vii) 15, 45, 135 or –15, 45, –135
   viii) 16 ix) 6, 12, 24

Group - C

4. iv) (14m–6) : (8m+23) vii) 2,8,14 ix) 1,3,9 or 9,3,1 x) 18,6,2 or 2, 6, 18

   xi) $S=5n+\frac{5}{9}\left\{(n-1)-\frac{1}{9}\left(1-\frac{1}{10^{n-1}}\right)\right\}$ xiv) 16 and 4

   xv) a) $n^2(2n^2–1)$ b) $\frac{1}{3}n\left(n^2+3n+5\right)$ c) $\frac{1}{24}n\left(2n^2+9n+13\right)$ d) $\frac{1}{12}n(n+1)\left(3n^2+19n+14\right)$

   xvi) a) $\frac{1}{6}n\left(4n^2-3n+23\right)$ b) $n(n+1)^2(n+2)$

   xvii) $19, \frac{38}{3}, \frac{76}{9}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

   xviii) $\frac{x+y-xy}{(1-x)(1-y)}$

Chapter - 10

Straight Lines

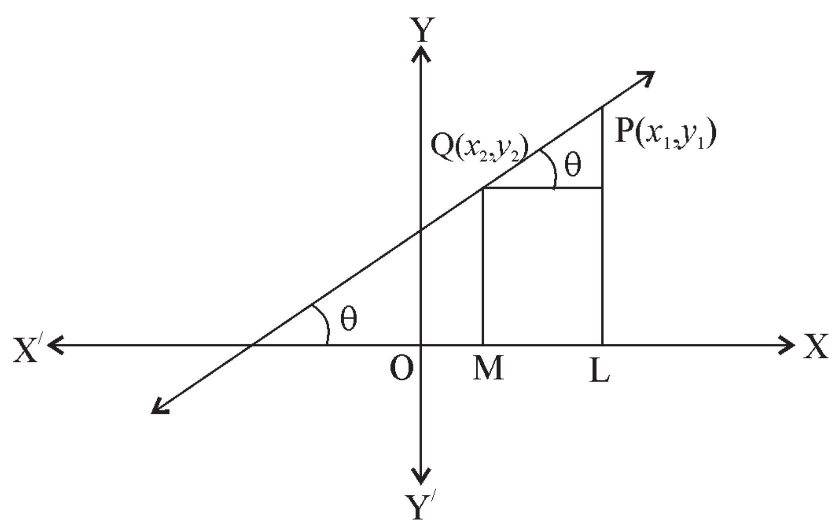
Important Points and results :

- **Definition of Straight Line :**

  If a point moves on a plane in a given direction then its locus is called a straight line and the equation of its locus is called the equation of the straight line.

- If a straight line makes an angle θ with the positive direction of the x-axis then the slope (or gradient) of the line = m = tanθ.

- If m be the slope of the line joining the points (x_1, y_1) and (x_2, y_2) then $m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$



- The quation of x-axis is y=0
- The equation of y-axis is x=0
- The equation of the line parallel ot x-axis and at a distance b unit from it is $y=b$.
- The equation of the line parallel to y-axis and at a distance a unit from it is $x=a$.
- **Slope-intercept form :**

  The equation of a straight line in slope-intercept form is $y=mx+c$, where m = slope of the line and c = y - intercept.

- **Point slope form :**

  The equation of a straight line in point-slope form is $y-y_1 = m(x-x_1)$, where m = slope of the line and (x_1, y_1) is a given point on the line.

- **Symmetrical form :**

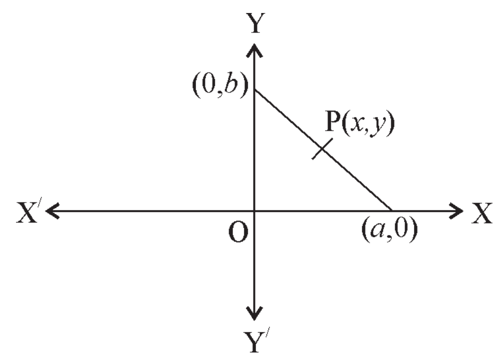
  The equation of a straight line in symmetrical form is $\dfrac{x-x_1}{\cos\theta}=\dfrac{y-y_1}{\sin\theta}=r$, where θ is the inclination of the line and r = distance between the points (x, y) and (x_1, y_1).

- **Two point form :**

  The equation of a straight line in two point form is $\dfrac{y-y_1}{x-x_1}=\dfrac{y_1-y_2}{x_1-x_2}$, where (x_1, y_1) and (x_2, y_2) are two given points on the line.

- **Intercept form :**

  The equation of a straight line in intercept form is $\dfrac{x}{a}+\dfrac{y}{b}=1$, where a = x-intercept and b = y-intercept of the line. The straight line intersects the x-axis at $(a, 0)$ and the y axis at $(0, b)$.



- **Normal form :**

  The equation of a straight line in normal form is $x\cos\alpha+y\sin\alpha = p$, where p (>0) is the perpendicular distance of the line from the origin and α $(0<\alpha<2\pi)$ is the angle that the drawn perpendicular on the line makes with the positive direction of the x-axis.

- **General form :**

  The equation of a straight line in general form is $ax+by+c = 0$, where a, b and c are real constants (a and b both are not zero simultaneously).

- To find the coordinates of the point of intersection of two given lines we solve the equations. The value of x is the abscissa and that of y is the ordinate of the point of intersection.

- The equation of any straight line through the point of intersection of the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ is $a_1x+b_1y+c_1+k(a_2x+b_2y+c_2) = 0$ where k ($\neq 0$ or ∞) is an arbitrary cosntant (or parameter)

- Three given straight lines are concurrent if the point of intersection of any two of them satisfies the equation of the third straight line.

- If θ be the acute angle between the straight lines $y=m_1x+c_1$ and $y=m_2x+c_2$ then,

  $$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right| \text{ or } \pm\frac{m_1-m_2}{1+m_1m_2}$$

- If two straight lines are parallel to each other, then their slopes would be equal, thus the condition of parallelism for the lines $y=m_1x+c_1$ and $y=m_2x+c_2$ is $m_1=m_2$.

- The equation of any straight line parallel to the line $ax+by+c=0$ is $ax+by=k$ where k is an arbitrary constant.

- Two straight lines are perpendicular to each other if the product of their slopes = –1. Thus, the condition of perpendicularity of the lines $y = m_1x+c_1$ and $y = m_2x+c_2$ is $m_1m_2 = -1$.
- The equation of any straight line perpendicular to the line $ax+by+c=0$ is $bx-ay=k$, where k is an arbitrary constant.
- The two equations $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$ represent the equation of the same straight lines where $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- Let P(x_1, y_1) be a point not lying on the straight line $ax+by+c=0$, then the length of the perpendicular drawn from P upon the line is $\pm\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$ or $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$.
- Let $y = mx+c_1$ and $y = mx+c_2$ be two lines, the distance between two parallel lines is given by $d = \frac{|c_1-c_2|}{\sqrt{1+m^2}}$ and if two parallel lines are $ax+by+c_1 = 0$ and $ax+by+c_2 = 0$, then distance $d = \frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$

Exercise – 10

Group - A

Objective Type Questions : [1 or 2 marks each]

I. Multiple choice type questions : (Choose the correct option)

1) The inclination of the straight line joining the points (6, 10) and (–8, –4) is

   a) 60^0 b) 120^0 c) 45^0 d) 135^0

2) The equation of the straight line parallel to y-axis and passing through the point (–4, 6) is

   a) $x+2 = 0$ b) $x+6 = 0$ c) $x-4 = 0$ d) $x+4 = 0$

3) A straight line passes through the point (–1,4) and makes an angle 60^0 with the positive direction of the x-axis then the equation of the straight line is

   a) $y+4 = \sqrt{3}(x-1)$ b) $y-4 = \sqrt{3}(x+1)$

   c) $x+4 = \sqrt{3}(y-1)$ d) $x-4 = \sqrt{3}(y+1)$

4) The slope of the line $3x+2y = 8$ and its intercept on y-axis is

   a) $\left(-\frac{3}{2}\right)$ and 4 units b) $\left(\frac{3}{2}\right)$ and 8 units

c) $\left(\frac{2}{3}\right)$ and 4 units   d) $\left(-\frac{2}{3}\right)$ and 8 units

5) If $y = mx+c$ represents the equation of a straight line parallel to x-axis, then

a) $m\neq 0, c\neq 0$ b) $m=0, c\neq 0$ c) $m\neq 0, c=0$ d) $m=0, c=0$

6) The perpendicular distance of the straight line 3x+4y+15 = 0 from the origin is

a) 4 unit   b) 3 unit   c) 9 unit   d) 18 unit

7) The condition for which the straight lines $l_1x+m_1y+n_1= 0$ and $l_2x+m_2y+n_2=0$ are perpendicular to each other is

a) $l_1l_2+m_1m_2= 0$ b) $l_1m_1+l_2m_2= 0$ c) $l_1m_2+l_2m_1= 0$ d) $l_1l_2-m_1m_2= 0$

8) Angle between the lines $x+\sqrt{3}y+7 = 0$ and $\sqrt{3}x-y+8 = 0$ is

a) 45^0 b) 30^0 c) 90^0 d) 60^0

9) Which of the following is the angle between the lines $x = 0$ and $ax+by+c = 0$

a) $\tan^{-1}\frac{a}{b}$ b) $\frac{\pi}{2}-\tan^{-1}\frac{a}{b}$ c) $\frac{\pi}{2}-\tan^{-1}\left(-\frac{a}{b}\right)$ d) $\tan^{-1}\left(-\frac{a}{b}\right)$

10) If the straight lines $2x-3y+5 = 0$ and $Px+2y = 6$ be parallel to each other, state which of the following is the values of P

a) $\frac{4}{3}$ b) $\frac{3}{4}$ c) $-\frac{4}{3}$ d) $-\frac{3}{4}$

11) The straight lines $5x-9y-12 = 0$ and $mx+10y = 2$ are perpendicular to each other, state which of the following is the value of m

a) 18   b) –9   c) 9   d) –18

12) If the distance between the lines $5x+12y-1 = 0$ and $10x+24y+K = 0$ be 2 units then the value of K is

a) 54   b) 50   c) 25   d) 100

II. Very short answer type questions : [1 or 2 marks each]

1) Find the point on the x-axis which is equidistant from the points (7,6) and (3,4).

2) If A is a point on the x-axis with abscissa –5 and B is a point on the y-axis with ordinate 8. Find the distance AB.

3) Find the coordinates of the point which divides the join of P(–5,11) and Q(4,–7) in the ratio 2:7.

4) Find the slope of a line which passes through the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$.

5) If the slope of the line joining the points A(x,2) and B(6,–8) is $-\frac{5}{4}$, find the value of x.

6) What is the value of y, so that the line through $(3, y)$ and $(2, 7)$ is paraller to the line through ↵ $(-1, 4)$ and $(0, 6)$?

7) Find the angle between the lines whose slopes are $(2-\sqrt{3})$ and $(2+\sqrt{3})$.

8) Find the equation of a line, which is parallel to y-axis and passes through $(-4, 3)$.

9) A line cutting off intercept -3 from the y-axis and the tangent at angle to the x-axis is $\frac{3}{5}$. Find its equation.

10) Find the equation of a line, which passes through the point $(2, 3)$ and makes an angle of 30^0 with the positive direction of x-axis.

11) If the line $\frac{x}{a}+\frac{y}{b}=1$ passes through the points $(2, -3)$ and $(4, -5)$, then find (a, b).

12) Find the equation of a line, which is equidistant from the lines $x=-2$ and $x=6$.

Group - B

Short answer type questions : (each question carries 3 marks)

1. If the slope of the line joining the points (2K, –2) and (1, –K) be (–2), find K.
2. Find the coordinates of the point on the line $7x-6y = 20$ for which the ordinate is double the abscissa.
3. Find the coordinates of the middle point of the portion of the straight line $4x+2y = 8$ intercepted between the x and the y-axis.
4. The perpendicular distance of the straight line $3x+4y+\mathrm{m} = 0$ from the origin is 2 unit find m.
5. A straight line is perpendicular to the straight line $6x-8y = 12$ and pases through $(3, 2)$. Find its equation.
6. Prove that the points $(3, 1)$, $(5, -5)$ and $(-1, 13)$ are collinear. Find the equation of the straight line on which the points lie.
7. Find the angle between the straight lines $x-\sqrt{3}y=3$ and $\sqrt{3}x-y+1=0$.
8. Find the equation of the straight line passing through the points $(-3,4)$ and parallel to the straight line $2x-3y = 5$.
9. Find the equation of the straight line equidistant from the point $(-2,3)$ and the line $8y = 9x-12$.
10. The perpendicular distance of the line $y+mx = 13$ from the origin is 12 unit. Find m.

Group - C

Long answer type questions : [4 or 6 marks each]

1. Find the equations to the straight lines which are at a distance of 3 unit from the origin and which passes through the intersection of the lines $4x+3y+1 = 0$ and $5x-y = 13$.

2. If $ab+bc+ca = 0$, show that the lines $\frac{x}{a}+\frac{y}{b}=\frac{1}{c}$, $\frac{x}{b}+\frac{y}{c}=\frac{1}{a}$ and $\frac{x}{c}+\frac{y}{a}=\frac{1}{b}$ are concurrent.

3. Show that the area of the traingle formed by the straight lines $y = m_1x+c_1$, $y = m_2x+c_2$ and $x = 0$ is $\frac{1}{2}\frac{(c_1-c_2)^2}{|m_1-m_2|}$ sq. units.

4. The equation of the sides AB and AC of a triangle ABC are $3x+4y+9 = 0$ and $4x-3y+16 = 0$ respectively. The third side passes through the point D(5, 2) such that BD : DC = 4 : 5. Find the equation of the third side.

5. The coordinates of the vertices A, B and C of a triangle ABC are (0,5), (–1,–2) and (11,7) respectively. Find the coordinates of the foot of the perpendicular from B on AC.

6. Find the angle between the lines $3x = 4y+7$ and $5y = 12x+6$ and find also the equations of the lines through the point (4,5) making equal angles with the given lines.

7. If lx+my+n = 0 be the perpendicular bisector of the line segment joining (α, β) and (γ, δ) then prove that $\frac{\gamma-\alpha}{l}=\frac{\delta-\beta}{m}=\frac{2(l\gamma+m\delta+n)}{l^2+m^2}$.

8. Find the equation of the straight line which passes through the point of intersection of the two lines $2x-y+5 = 0$ and $5x+3y-4 = 0$ and is perpendicular to the line $x-3y+21 = 0$.

9. Find the equation of the straight line which is parallel to the straight line $3x+2y-6 = 0$ and which forms a triangle of area 21 square units with the straight lines $x-2y = 0$ and $y-2x = 0$.

10. Show that the four lines $y = 0$, $y = 2$, $y-\sqrt{3}x = 0$ and $y+\sqrt{3}x = 8\sqrt{3}$ form a cyclic trapezium. Find the coordinates of its vertices and also its area.

11. If one vertex of an equilateral triangle is (2, 3) and equation of a side is x+y = 2, find the equations of other two sides.

12. Find the distance of (5, 3) from the line 3x-2y-1 = 0 making an angle 45^0 with it.

13. Find the coordinate of the foot of the perpendicnlar from (-2, 6) upon the line 2x+3y = 1. Also find coordinate of image of the point (-2, 6) with respect to the line 2x+3y = 1.

14. If equation of one side of rectangle is 4x+7y+5 = 0 and coordinates of two vertices are (-3, 1) and (1, 1) then find equations of other three sides.

15. Find the equation of straight line that passes throngh the point of intersection of 3x+4y = 4 and 2x+5y+2 = 0 and whose distance from origin in 2 units.

ANSWER

Group - A

I. Multiple choice type questions :

1.(c) 2.(d) 3.(b) 4.(a) 5.(b) 6.(b) 7.(a) 8.(c) 9.(c) 10.(c) 11.(a) 12.(b)

II. Very short answer type questions :

(1) $\left(\frac{15}{2},0\right)$ (2) $\sqrt{89}$ (3) $(-3,7)$ (4) $\frac{2}{t_2+t_1}$ (5) $x=-2$ (6) $y=9$ (7) 60^0 or 120^0

(8) $x=-4$ (9) $5y-3x+15=0$ (10) $x-\sqrt{3}y=2-3\sqrt{3}$ (11) $(a,b)=(-1,-1)$ (12) $x=2$

Group - B

Short answer type questions :

(1) $k=\frac{4}{5}$ (2) $(-4,-8)$ (3) $(1,2)$ (4) $m=\pm 10$ (5) $8x+6y=36$ (6) $3x+y=10$ (7) 30^0

(8) $2x-3y+18=0$ (9) $8y-9x-15=0$ (10) $m=\pm\frac{5}{12}$

Group - C

Long answer type questions :

(1) $y+3=0$ and $12x-5y=39$ (4) $x=5$ (5) $\left(-\frac{11}{5},\frac{23}{5}\right)$ (6) $7x+9y=73$ and $9x-7y=1$ (8) $3x+y=0$

(9) $3x+2y=\pm 28$ (10) $(0,0)$, $(8,0)$, $\left(\frac{8\sqrt{3}-2}{\sqrt{3}},2\right)\left(\frac{2}{\sqrt{3}},2\right)$ and area$=\frac{4}{3}\left(12-\sqrt{3}\right)$ square unit.

11) $(2+\sqrt{3})x-y=1+2\sqrt{3}$ and $(2-\sqrt{3})x-y=1-2\sqrt{3}$

12) $\frac{8\sqrt{26}}{13}$ unit.

13) (-4, 3), (-6, 0)

14) 7x-4y+25=0, 7x-4y=3 and 4x+7y=11

15) y+2=0 and 4x+3y=10

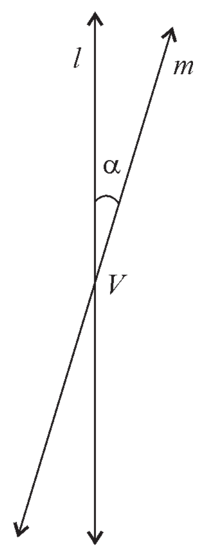
Chapter - 11

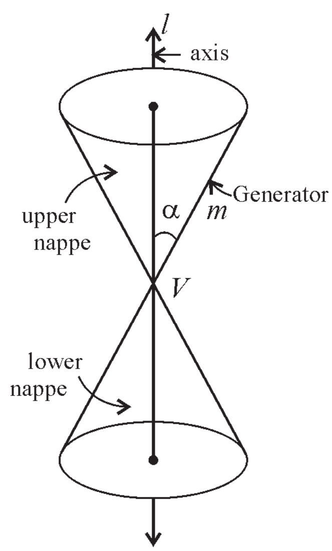
Conic Sections

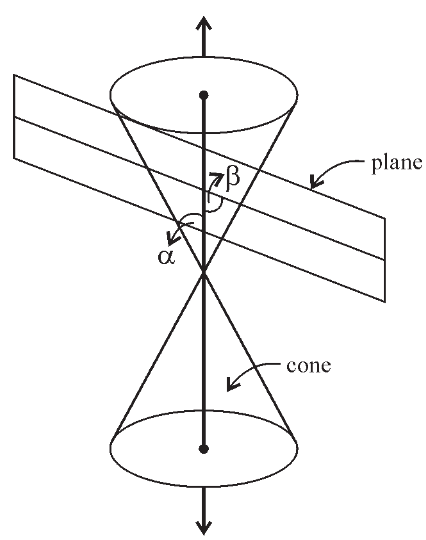
Important points and Results :

- **Sections of a Cone :**

  Let l be a fixed vertical line and m be another line intersecting it at a fixed point V and inclined to it at an angle α.







The point V is called the **vertex**. The line l is the **axis** of the cone. The rotating line m is called the **generator** of the cone. The vertex separates the cone into two parts called nappes.

If we take the intersection of a plane with a cone, the section so obtained is called a conic section. Thus, conic section are the curves obtained by intersecting a right circular cone by a plane.

- **Circle, Ellipse, Parabola and Hyperbola :**

When the plane cuts the nappe (other than the vertex) of the cone, we have the following situations:

Let β be the angle made by the intersecting plane with the vertical axis of the cone.

a) When $\beta = 90^0$; the section is a **circle (fig-1).**

b) When $\alpha<\beta<90^0$; the section is a ellipse **(fig-2).**

c) When $\alpha=\beta$, then section is parabola **(fig-3).**

d) When $0\leq\beta<\alpha$; the plane cuts through both the nappes and the curves of intersection is a **hyperbola (fig-4).**

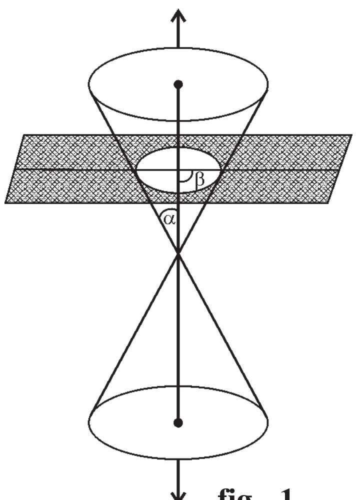


fig - 1

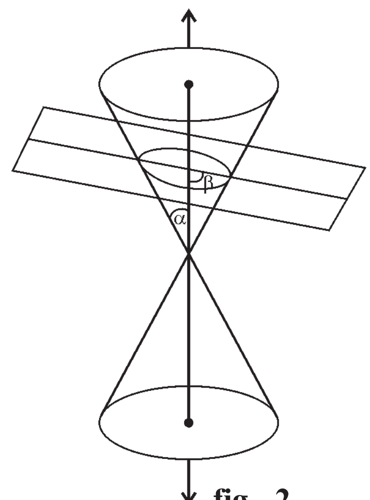


fig - 2

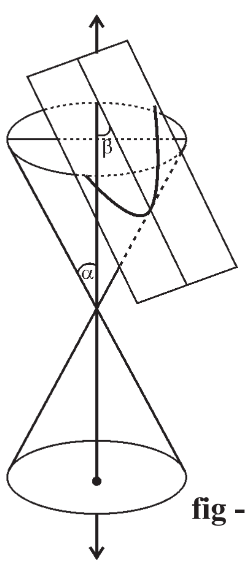


fig - 3

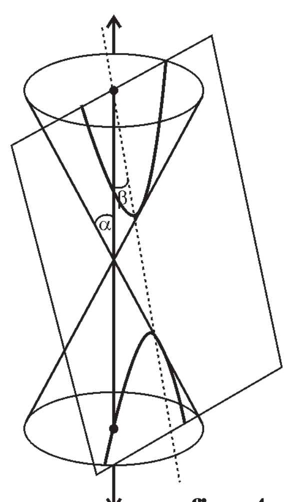


fig - 4

- **Degererated conic sections :**

  When the plane cuts at the vertex of the cone, we have the following different cases :

  a) When $\alpha<\beta\leq 90^0$; then the section is a point. **(fig - 5)**

  b) When $\beta=\alpha$; the plane contains a generator of the cone and the section is a straight line. It is the degenerated case of a parabola. **(fig - 6)**

  c) When $0\leq\beta<\alpha$; the section is a pair of intersecting straight lines. It is the degenarated case of hyperbola. **[fig - 7(a) & 7(b)]**

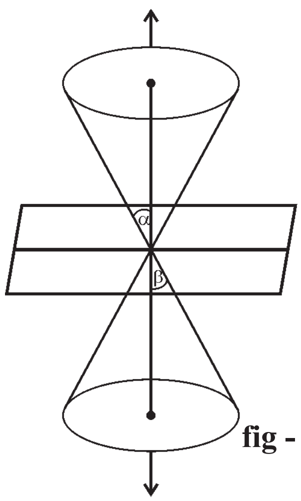


fig - 5

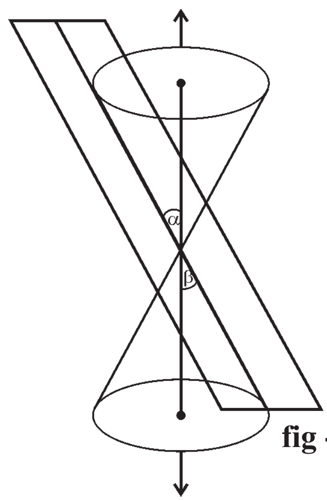


fig - 6

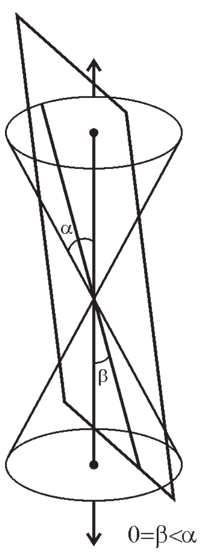


fig - 7(a)

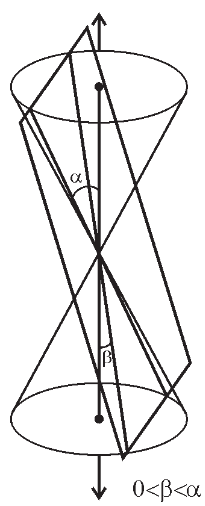
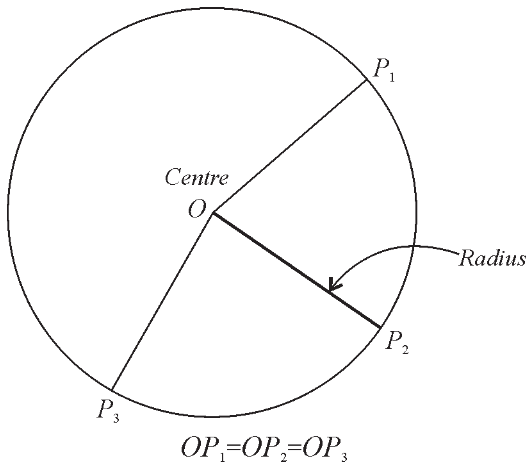


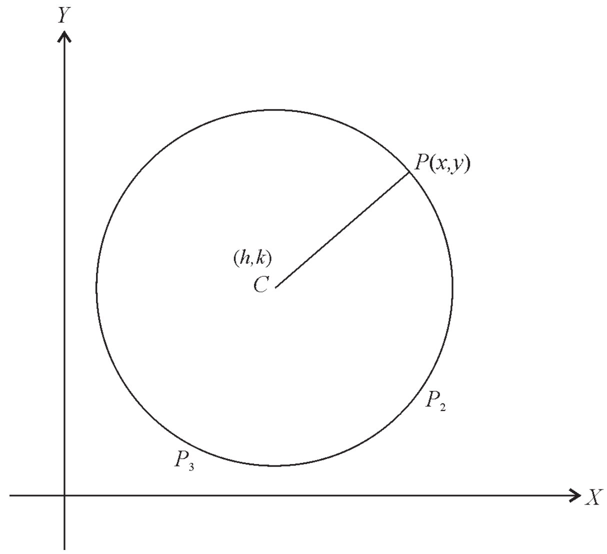
fig - 7(b)

- **Circle :**

  A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

  The fixed point is called the centre of the circle and the distance from the centre to a point on the circle is called the radius of the circle.





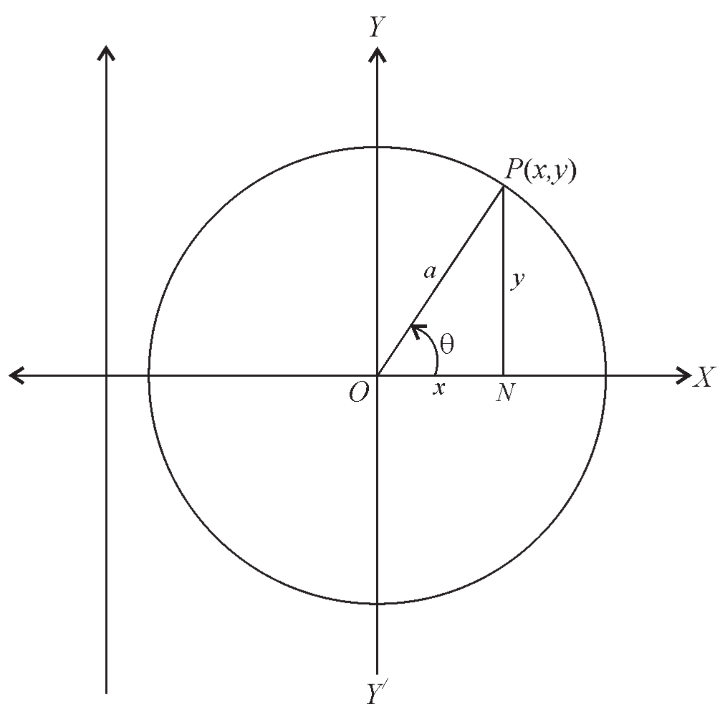
C (h, k) be the centre and r the radius of circle. Let $P(x,y)$ be any point on the circle.Then by the definition $|CP|=r$. By the distance formula we have,

$$\sqrt{(x-h)^2+(y-K)^2}=r$$

i.e $\quad (x-h)^2+(y-K)^2 = r^2$

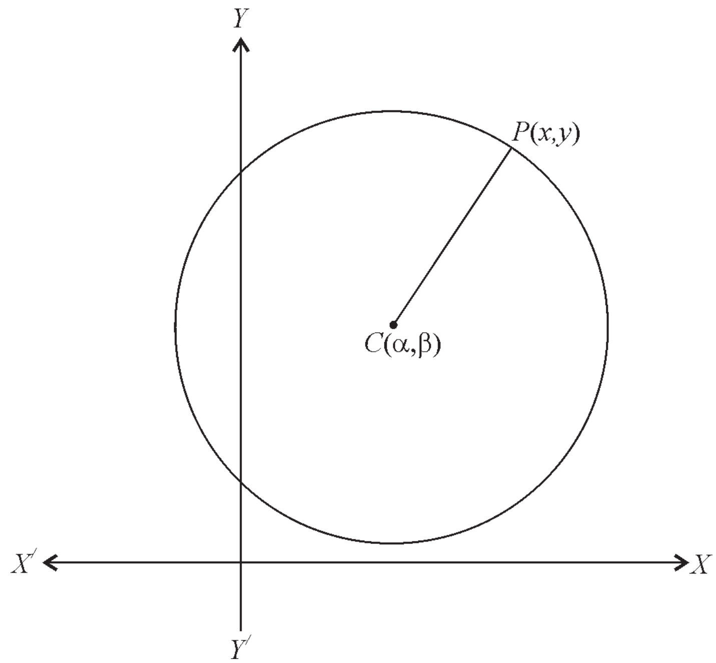
- The equation of the circle having centre at the origin O and radius a unit (i.e. the equation of the circle in standard form).

  $x^2 + y^2 = a^2$



The parametric form of the equation of the above circle is $x=a\cos\theta$, $y=a\sin\theta$.

- The equation of the circle having centre at (α, β) and radius a unit is $(x-\alpha)^2 + (y-\beta)^2 = a^2$



- The general form of the equation of a circle is $x^2+y^2+2gx+2fy+c = 0$.

  The coordinates of the centre of the circle are $(-g, -f)$ and its radius = $\sqrt{g^2+f^2-c}$ unit. The above circle makes an intercept $2\sqrt{g^2-c}$ on the x-axis and $2\sqrt{f^2-c}$ on the y-axix.

  Remark : If $g^2+f^2>c$, the equation represent a real circle.

  But, if $g^2+f^2<c$, then the radius of the circle becomes imaginary; hence, in this case the coordinates of any real point do not satisfy above equation. In other words, equation does not represent any real circle.

  If $g^2+f^2=c$ then the radius of the circle becomes zero. In this case, the above circle reduces to the point $(-g, -f)$. In other words, the equations represents a **point circle**.

- The general quadratic eqution in x and y viz, $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a circle if the coefficient of x^2 = the coefficient of y^2 (i.e. $a=b\neq0$) and the coefficient of xy is zero (i.e. $h=0$).

- **Equation of circles in some special cases :**

  i) The equation of a circle passing through the origin is

  $x^2+y^2+2gx+2fy = 0$ ....(1)

  Or, $(x-\alpha)^2+(y-\beta)^2 = \alpha^2+\beta^2$ .... (2)

  See that both the euations (1) and (2) are satisfied by (0, 0)

ii) If the centre of a circle be on the x-axis, then the y coordinate of the centre will be zero. Hence, the equation of the circle will be of the form :

$$x^2+y^2+2gx+c=0$$

Or, $(x-\alpha)^2+y^2=a^2$

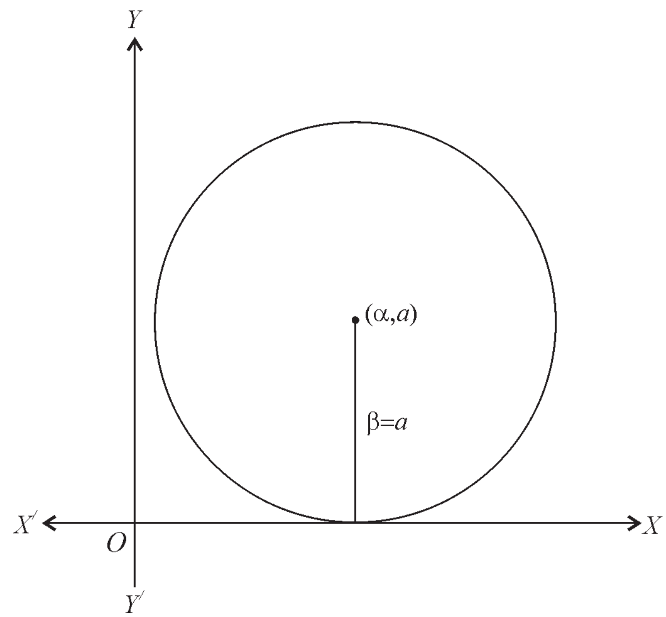
Again, if the centre be on the y-axis then the x coordinate of the centre will be zero. Hence, the equation of the circle will be of the form :

$$x^2+y^2+2fy+c=0$$

Or, $x^2+(y-\beta)^2=a^2$

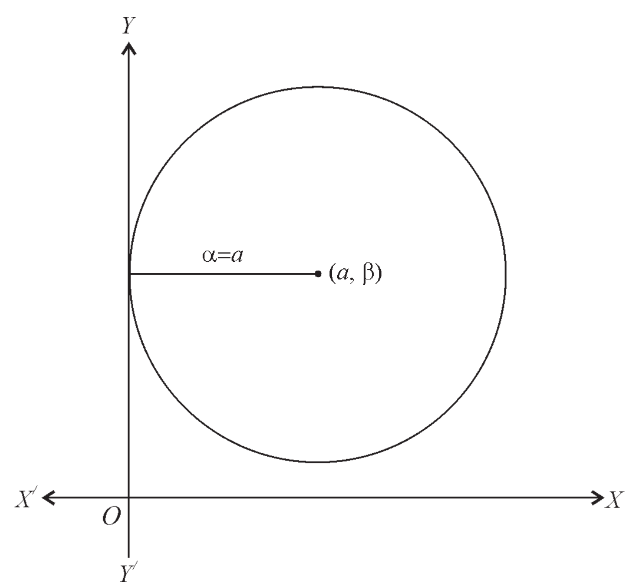
iii) If a circle touches the x-axis; the y coordinate of the centre will be equal to the radius of the circle. Hence, the equation of the circle will be of the form :

$(x-\alpha)^2+(y-a)^2=a^2$



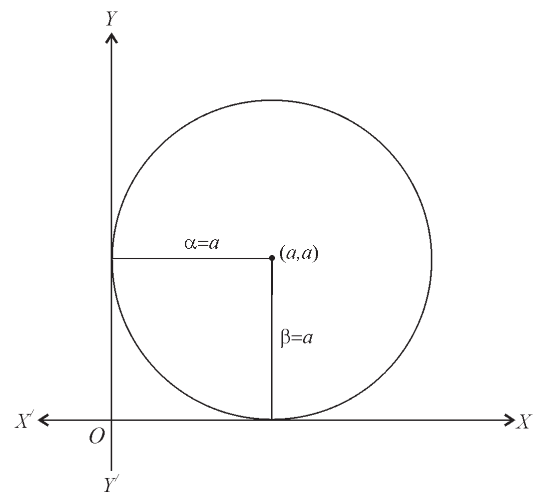
iv) If a circle touches the y-axis then the x coordinate of the centre will be equal to the radius of the circle. Hence, the equation of the circle will be of the form :

$(x-a)^2+(y-\beta)^2=a^2$

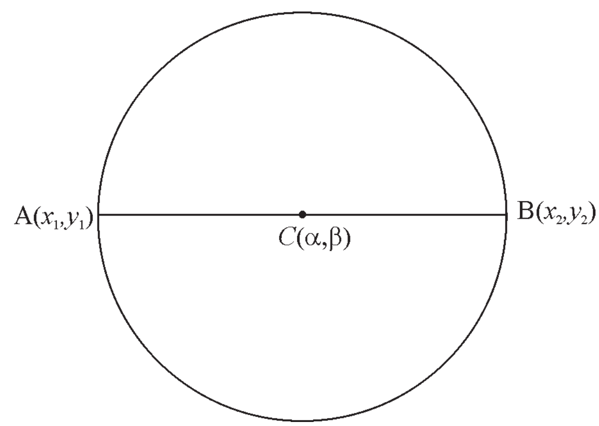


v) If a circle touches both the coordinate axis then the abscissa as well as ordinate of the centre will be equal to the radius of the circle. Hence the equation of the circle will be of the form :

$(x-a)^2 + (y-a)^2 = a^2$



- The equation of the circle drawn on the line segment joining the points (x_1, y_1) and (x_2, y_2) as diameter is $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$



- $x^2+y^2+2g_1x+2f_1y+c_1 = 0$ ....(1)

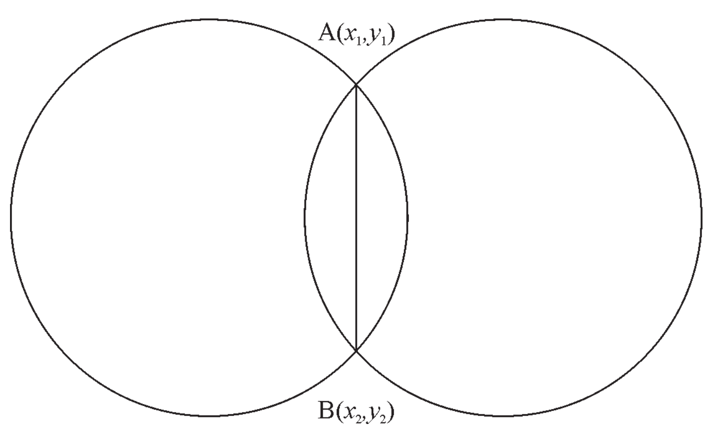
  $x^2+y^2+2g_2x+2f_2y+c_2 = 0$ ....(2)

  i) The equation of the common chord of the interesting circles (1) and (2) is

  $2(g_1-g_2)x + 2(f_1-f_2)y + c_1 - c_2 = 0$

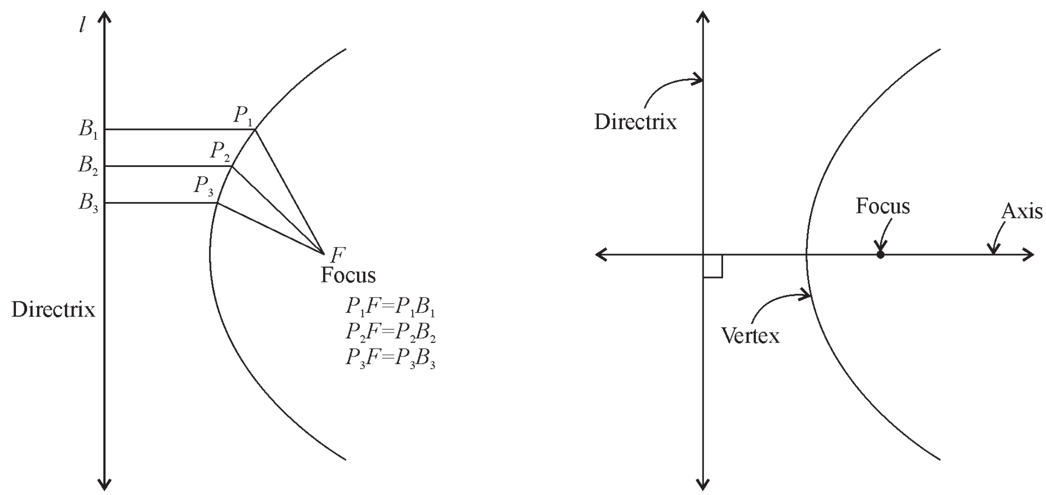
  ii) The equation of any circle through the points of intersection of the circle (1) and (2) is

  $x^2+y^2+2g_1x+2f_1y+c_1 + K(x^2+y^2+2g_2x+2f_2y+c_2) = 0$ $[K \neq -1]$



- The equation of a circle concentric with the circle $x^2+y^2+2gx+2fy+c=0$ is $x^2+y^2+2gx+2fy+c'=0$.
- The point (x_1, y_1) lies outside, on or within the circle $x^2+y^2+2gx+2fy+c=0$ accordingly as $x_1^2+y_1^2+2gx_1+2fy_1+c >, =$ Or < 0.
- **Parabola :**

  A parabola is the set of all points in a plane that are equidistant form a fixed line and a fixed point (not on the line) in the plane.

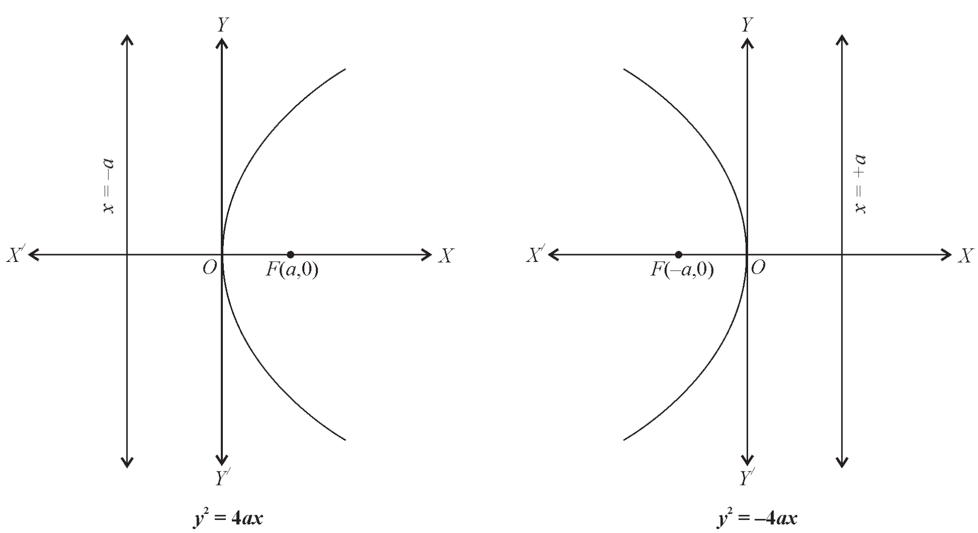


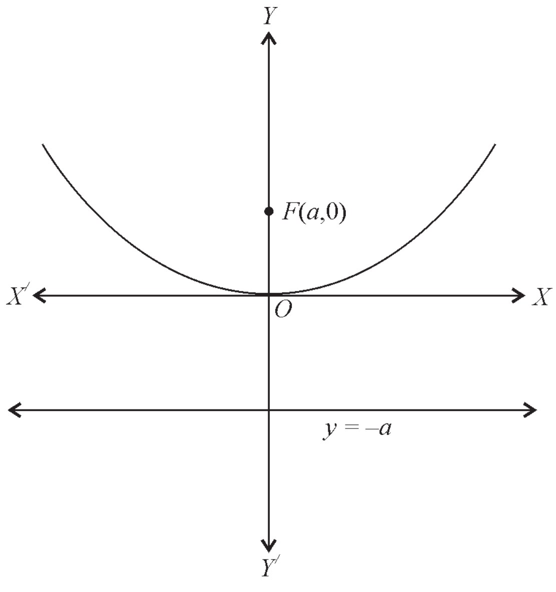
  The fixed line is called the directrix of the parabola and the fixed point F is called the focus.

  A line through the focus and perpendicular to the directrix is called the axis of the parabola. The point of intersection of parabola with the axis is called the vertex of the parabola.

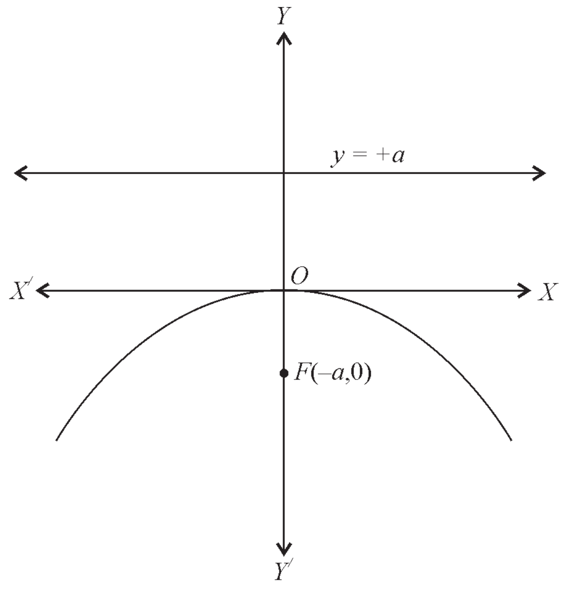
- **Standard equations of a Parabola :**

  The equation of a parabola is simplest, if the vertex is at orgin and the axis of symmetry is along the x-axis or y-axis. The four possible such orientations of parabola are shown below –



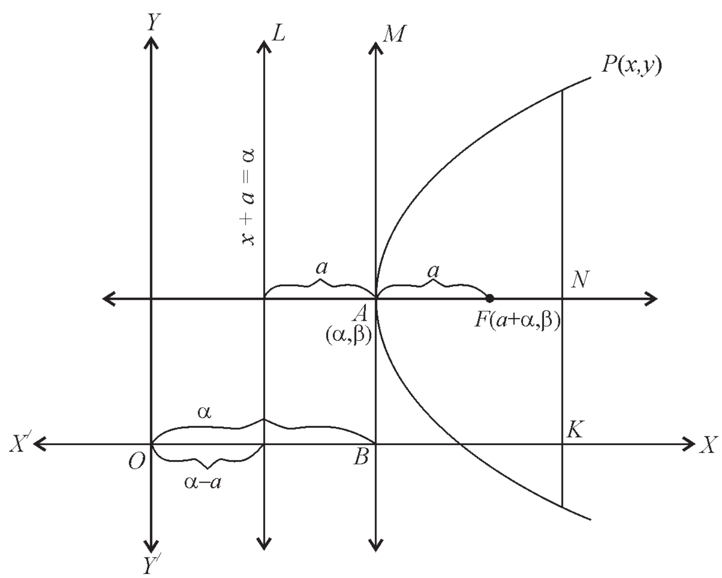


$x^2 = 4ay$



$x^2 = -4ay$

- The equation of the parabola with its vertex at the point (α, β) and axis parallel to x-axis is $(y-\beta)^2 = 4a(x-\alpha)$.



Note : Above equation represents the equation of a parabola whose

i) axis is parallel to positive x-axis and its equation is $y=\beta$.

ii) coordinates of vertex are (α, β)

iii) coordinates of focus are $(a+\alpha, \beta)$

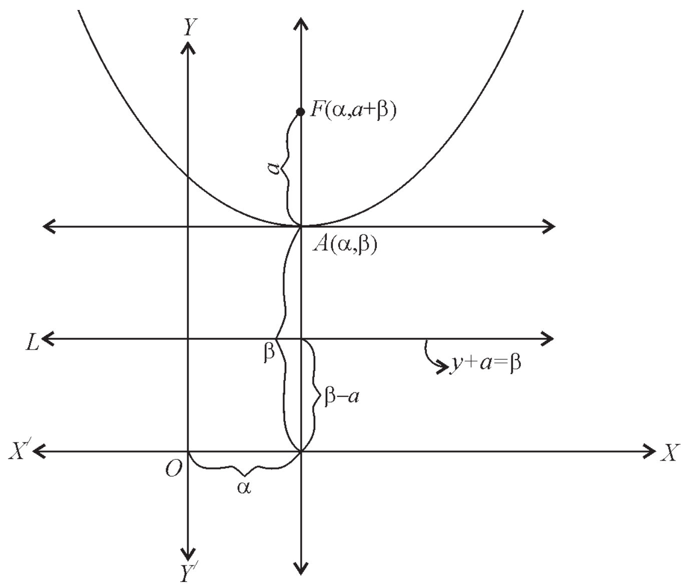
iv) equation of directrix is $x-\alpha = -a$ or $x+a = \alpha$.

v) length of Latus rectum = $4a$.

vi) equation of tangent at the vertex is $x = a$

vii) coordinates of the ends of latus rectum are $(\alpha+a, \beta+2a)$ and $(\alpha+a, \beta-2a)$

- The equation of the parabola with its vertex at the point (α, β) and axis parallel to y-axis is $(x-\alpha)^2 = 4a(y-\beta)$ $[a>0]$.



Above equation represents the equation of a parabola whose

i) axis is parallel to positive y-axis and its equation is $x = \alpha$.

ii) coordinates of vertex are (α, β)

iii) coordinates of focus are $(\alpha, a + \beta)$

iv) equation of directrix is $y - \beta = -a$ or $y+a = \beta$.

v) length of Latus rectum $= 4a$.

vi) equation of tangent at the vertex is $y = \beta$

vii) coordinates of the ends of latus rectum are $(\alpha+2a, \beta+a)$ and $(\alpha-2a, \beta+a)$

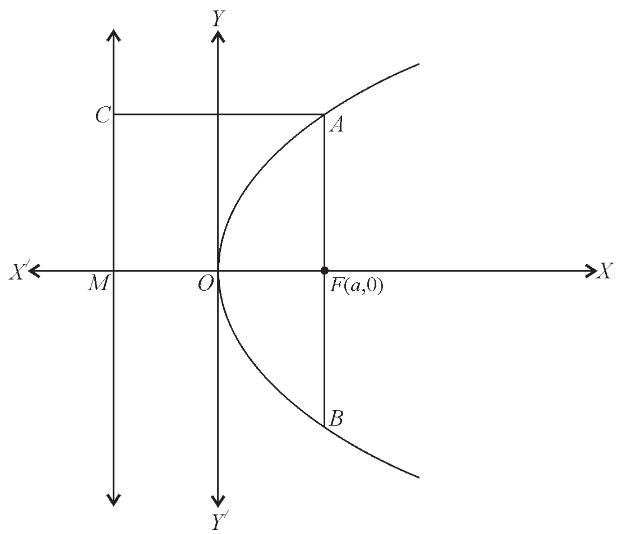
- $x = ay^2+by+c$ $(a\neq0)$ represents the equation of a parabola whose axis is parallel to x-axis.
- $y = px^2+qx+r,$ $(p\neq0)$ represents the equation of a parabola whose axis is parallel to y-axis.
- The point $P(x_1,y_1)$ lies outside, on or inside the parabola $y^2=4ax$ according as $y_1^2-4ax_1>,$ = or <0.

$x = at^2$, $y = 2at$ (for all values of t) safisty the equation $y^2 = 4ax$. The equations $x = at^2$, $y = 2at$ are called the parametric equations of the parabola $y^2 = 4ax$. By the point 'p' on $y^2 = 4ax$ we mean the point $(at^2, 2at)$ and are called the parametric coordinates, t being the parameter.

- **Latus Rectum :**

Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola

AB = length of the latus rectum = $4a$



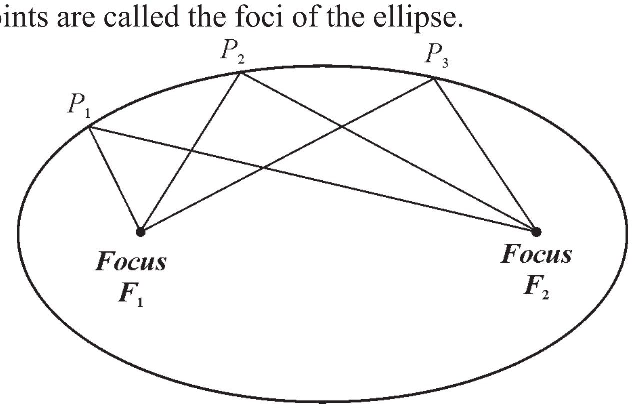
- **a = distance between the vertex and focus of the parabola :**

Equation of parabola	Coordnates of vertex	Axis of the parabola	Coordinates of focus	Length of latus rectum	Equation of directrix
i) $y^2=4ax$ $(a>0)$	(0,0)	Positive x-axis	$(a, 0)$	$4a$	$x+a=0$
ii) $y^2=-4ax$ $(a>0)$	(0,0)	Negative x-axis	$(-a, 0)$	$4a$	$x-a=0$
iii) $x^2=4ay$ $(a>0)$	(0,0)	Positive y-axis	$(0, a)$	$4a$	$y+a=0$
iv) $x^2=-4ay$ $(a>0)$	(0,0)	Negative y-axis	$(0, -a)$	$4a$	$y-a=0$
v) $(y-\beta)^2=-4a(x-\alpha)$	(α, β)	Parallel to x-axis	$(a+\alpha, \beta)$	$4a$	$x+a=\alpha$
vi) $(x-\alpha)^2=-4a(y-\beta)$	(α, β)	Parallel to y-axis	$(a, \alpha+\beta)$	$4a$	$y+a=\beta$

- **Ellipse :**

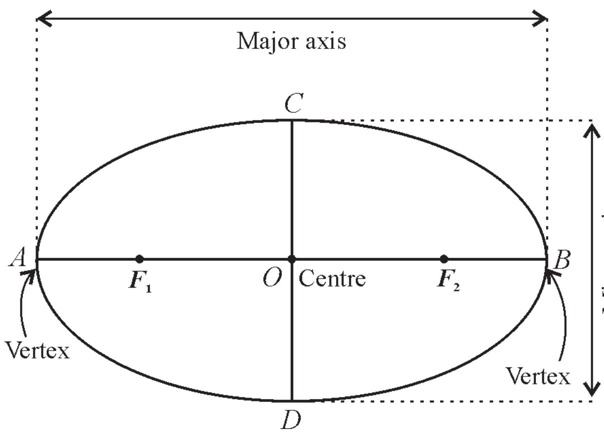
An ellepse is the set of all points in a plane, the sum of whose distance from two fixed points in the plane is a contstant.

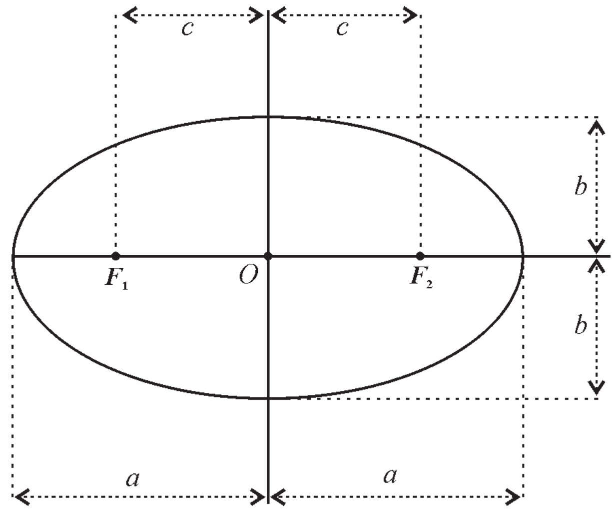
The two fixed points are called the foci of the ellipse.



$$P_1F_1+P_1F_2 = P_2F_1+P_2F_2 = P_3F_1+P_3F_2$$

- The mid point of the line segment joing the foci is called the **centre** of the ellipse.The line segment through the foci of the ellipse is called the **major** axis and the line segment through the centre and perpendicular to the major axis is called the **minor** axis. The end point of the major axis are called the **vertices** of the ellipse.

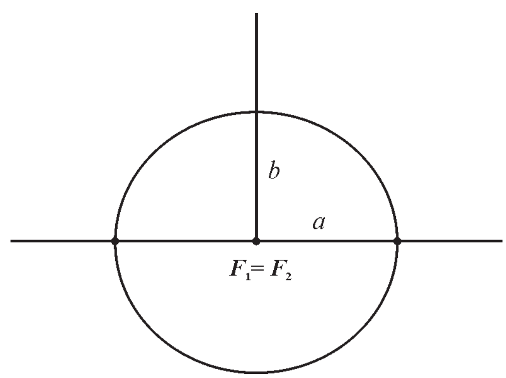




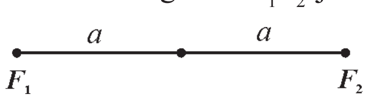
- We denote the length of the major axis by $2a$, the length of the minor axis by $2b$ and the distance between the foci by $2c$. Thus, the length of the semi major axis is a or semi-minor axis is b.
- Relationship between semi-major axis, semi-minor axis and the distance of focus from the centre of the ellipse is $a^2 = b^2+c^2$ i.e. $c = \sqrt{a^2 - b^2}$.
- **Special cases of an ellipse :**

  In the equation $c^2 = a^2–b^2$, obtained above, if we keep a fixed and vary c from 0 to a, the resulting ellipse will vary in shape.

  Case-(i) : When c=0, both foci merge together with the centre of the ellipse and $a^2 = b^2$ i.e a=b and so the ellipse becomes circle. Thus, circle is a special case of an ellipse.



  Case-(ii) : When c=a, then b=0. The ellipse reduce to the line segment F_1F_2 joining the two foci.

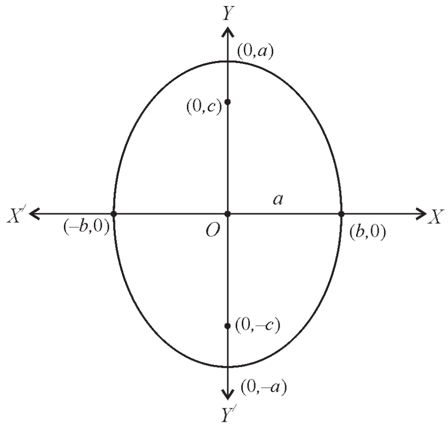


- **Ecentricity :**

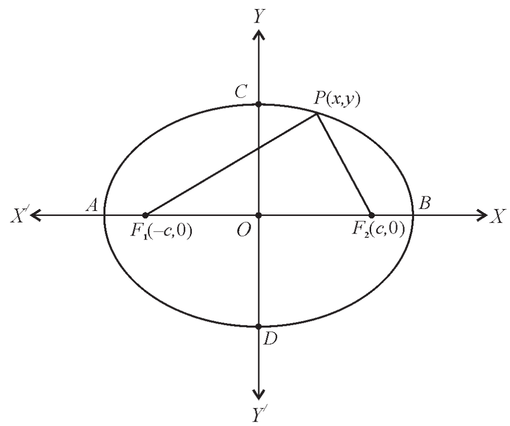
  The ecentricity of a ellipse is the ratio of the distances from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse (eccentricity is denoted by e) i.e. $e = \frac{c}{a}$.

- **Standard equations of an ellipse :**

The equation of an ellipse is simplest if the centre of the ellipse is at the origin and the foci are on the x-axis or y-axis. The two such possible orientations are shown –



$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Remark : The standard equations of ellipse have centre at the origin and the major and minor axis are coordinate axes. However, the study of the ellipse with centre at any other point, and any line through the centre as major and the minor axes passing through the centre and perpendicular to major axis and are beyond the scope here.

- **From the standard equations of the ellipse we have the following observations :**

i) Ellipse is symmetric with respect to both the coordinate axes, so if (x, y) is a point on the ellipse, then $(-x, y)$, $(x, -y)$ and $(-x, -y)$ are also points on the ellipse.

ii) The foci always lie on the major axis. The major axis can be determined by finding the intercepts on the axes of symmetry. That is major axis is along the x-axis if the coeffient of x^2 has the larger denominator and it is along the y-axis if the cofficient of y^2 has the larger denominator.

- **Other form of the equation of Ellipse :**

i) If the centre of the ellipse is at (α, β) and the major and minor axes are parallel to x and y axes respectively, then equation of the ellipse will be

$$\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1 \quad \left[a^2>b^2\right]$$

where $2a$ = length of major axis and $2b$ = length of minor axis.

ii) If the centre of the ellipse is at (α, β) and the major and minor axes are parallel to y and x axes respectively, then the equation of the ellipse will be

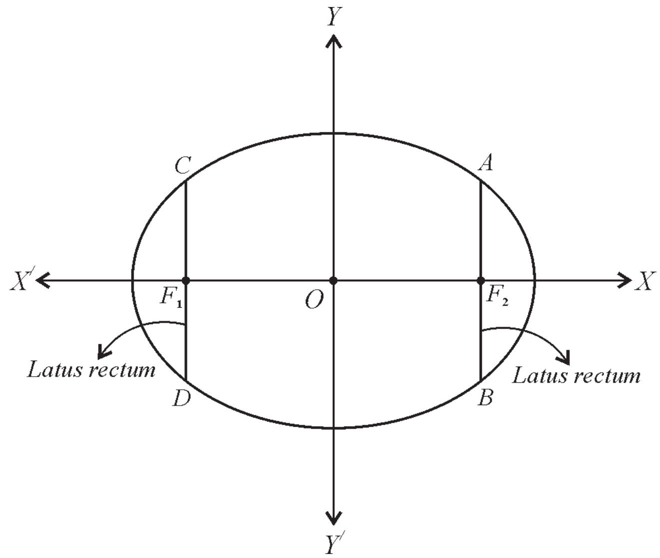
$$\frac{(y-\beta)^2}{a^2}+\frac{(x-\alpha)^2}{b^2}=1 \quad \left[a^2>b^2\right]$$

where $2a$ = length of major axis and $2b$ = length of minor axis.

- **Latus Rectum :**

  Latus rectum of an ellipse is line segment perpendicular to the major axis through any of the foci and whose end points lie on the ellipse.

  Length of the Latus rectum = $\dfrac{2b^2}{a}$



- **Axis, Coordinates of centre, vertices, foci etc. of different types ellipse are given in the following table :**

	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $[a^2>b^2]$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ $[a^2>b^2]$	$\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1$ $[a^2>b^2]$	$\frac{(x-\alpha)^2}{b^2}+\frac{(y-\beta)^2}{a^2}=1$ $[a^2>b^2]$
Major axis	x-axis	y-axis	parallel to x-axis	parallel to y-axis
Minor axis	y-axis	x-axis	parallel to y-axis	parallel to x-axis
Equation of major axis	y=0	x=0	y=β	x=α
Equation of minor axis	x=0	y=0	x=α	y=β
Length of major axis	2a unit	2a unit	2a unit	2a unit
Length of minor axis	2b unit	2b unit	2b unit	2b unit
Coordinates of centre	(0, 0)	(0, 0)	(α, β)	(α,β)
Coordinates of vertices	(±a, 0)	(0, ±a)	(α±a, β)	(a,β±a)
Ecentricity	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{b^2}{a^2}}$
Coordinates of foci	(±ae, 0)	(0, ±ae)	(α±ae, β)	(a, β±ae)
Distance between two foci	2ae unit	2ae unit	2ae unit	2ae unit
Length of latus rectum	$\frac{2b^2}{a}$ unit	$\frac{2b^2}{a}$ unit	$\frac{2b^2}{a}$ unit	$\frac{2b^2}{a}$ unit

Coordinates of the four ends of latus rectam	$\left(ae, \frac{b^2}{a}\right), \left(ae, -\frac{b^2}{a}\right)$ $\left(-ae, \frac{b^2}{a}\right), \left(-ae, -\frac{b^2}{a}\right)$	$\left(\frac{b^2}{a}, ae\right), \left(-\frac{b^2}{a}, ae\right)$ $\left(\frac{b^2}{a}, -ae\right), \left(-\frac{b^2}{a}, -ae\right)$	$\left(\alpha \pm ae, \beta \pm \frac{b^2}{a}\right)$	$\left(\alpha \pm \frac{b^2}{a}, \beta \pm ae\right)$
Equation of latus rectam	$x=\pm ae$	$y=\pm ae$	$x=\alpha\pm ae$	$y=\beta\pm ae$
Equation of directrices	$x = \pm \frac{a}{e}$	$x = \pm \frac{a}{e}$	$x = \pm \frac{a}{e}$	$y = \beta \pm \frac{a}{e}$
Distance between two directrices	$\frac{2a}{e}$ unit	$\frac{2a}{e}$ unit	$\frac{2a}{e}$ unit	$\frac{2a}{e}$ unit

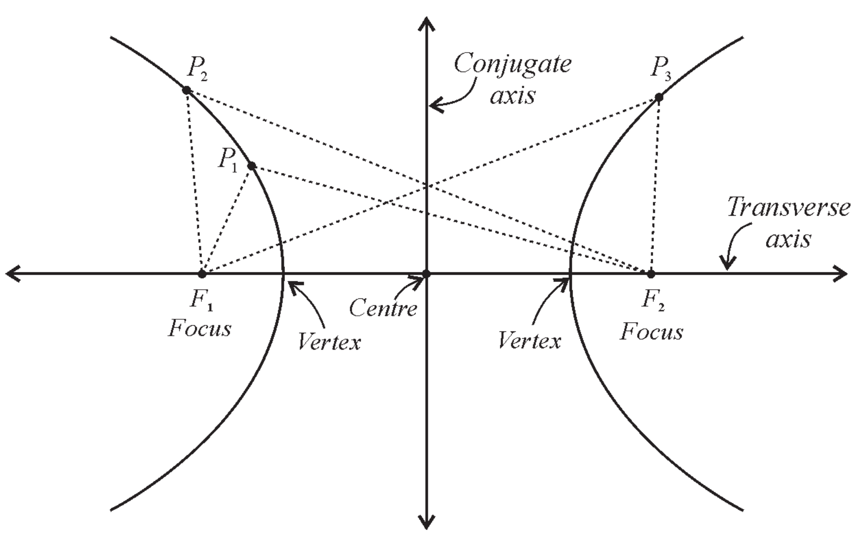
The auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2+y^2=a^2$.

The parametric equations of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are $x=a\cos\phi$, $y=b\sin\phi$.

The coordinates of any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are $P(a\cos\phi, b\sin\phi)$, where f is the eccentric angle of the point ***P***.

- **Hyperbola :**

A hyperbola is the set of all point in a plane, the difference of whose distance from two fixed points in the plane is a constant.



$$P_1F_2 - P_1F_1 = P_2F_2 - P_2F_1 = P_3F_1 - P_3F_2$$

The two fixed points are called the **foci** of the hyperbola.

The mid-point of the line segment joining the foci is called the **centre** of the hyperbola.

The line through the foci is called the **transverse axis** and the line through the centre and perpendicular to the transverse axis is called the **conjugate axis**.

The points at which the hyperbola intersects the transverse axis are called the vertices the hyperbola.

The distance between the two foci by $2c$. The distance between two vertices (the length of the transverse axis) by $2a$ and we define the quantity b as $b = \sqrt{c^2 - a^2}$.

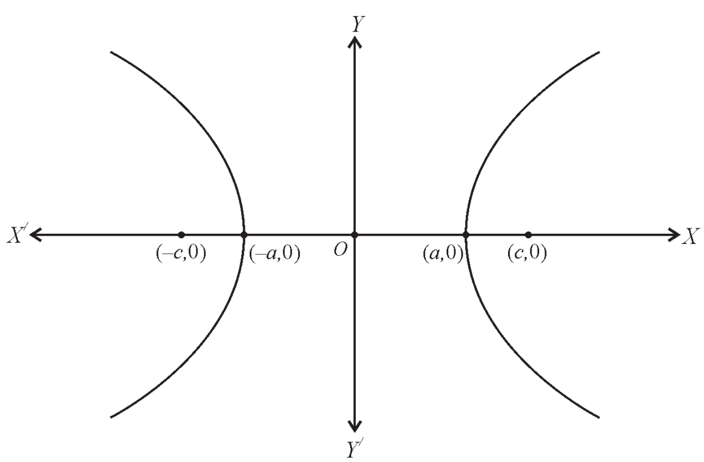
The length of the conjugate axis is $2b$.

- **Eccentricity :**

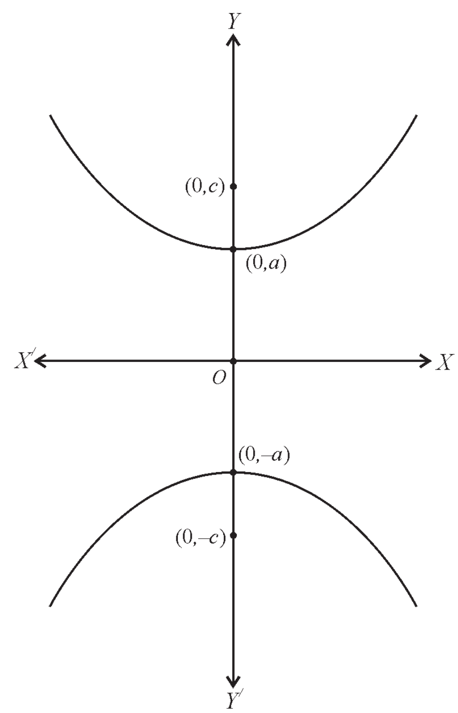
  The ratio $e = \dfrac{c}{a}$ is called the eccentricity of the hyperbola since $c \geq a$, the eccentricity is never less than one.

- **Standard equation of Hyperbola :**

  The equation of a hyperbola is simplest if the centre of the hyperbola is at the origin and the foci are on the x-axis or y-axis. The two such possible orientation are shown –



(a) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$



(b) $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

- A hyperbola in which $a=b$ is called on rectangular hyperbola.
- **Remark :** The standard equations of hyperbolas transverse and conjugate axes as the co-ordinate axes and the centre at the origin. However, there are hyperbolas with any two perpendicular line as transverse and conjugate axes.

- **From the standard equations of hyperbolas,we have the following observations :**

  i) Hyperbola is symmetric with respect to both the axes; since if (x, y) is a point on the hyperbola, then $(-x, y)$, (x, y) and $(-x, -y)$ are also points on the hyperbola.

  ii) The foci are always on the transverse axis.

- **Latus rectum :**

  Latus rectum of hyperbola is a line segment perpendicular to the transvers axis through any of the foci and whose end points lie on the hyperbola.

  The length of the latus rectum in hyperbola is $\dfrac{2b^2}{a}$.

- **Rectangular Hyperbola :**

  A hyperbola whose transverse axis is equal to its conjugate axis is called a rectangular or **equilateral hyperbola**.

  The equation of rectangular hyperbola is $x^2-y^2=a^2$.

- **Other forms of the Equations of Hyperbola :**

  i) If the centre of the hyperbola is at (α, β) and transverse and conjugat axes are parallel to x and y axes respectively, then the equation of the hyperbola is

  $$\frac{(x-\alpha)^2}{a^2}-\frac{(y-\beta)^2}{b^2}=1$$

  Where, $2a$ = length of transverse axis and

  $2b$ = length of conjugate axis

  ii) If the centre of the hyperbola is at (α, β) and transverse and conjuage axes are parallel to y and x axes respectively. Then the equation of the hyperbola is

  $$\frac{(y-\beta)^2}{a^2}-\frac{(x-\alpha)^2}{b^2}=1$$

  Where, $2a$ = length of transverse axis and

  $2b$ = length of conjugate axis

- The point $P(x_1, y_1)$ lies outside, on or inside the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ according as $\dfrac{x_1^2}{a^2}-\dfrac{y_1^2}{b^2}-1<0,$ OR >0

- The parametric equation of the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ are $x=a\sec\phi$, $y=b\tan\phi$ and the parametric coordinates of any point P on the parametric coordinates of any point P on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are ($a\sec\phi$, $b\tan\phi$).

- **Characteristics of different types of hyperbolas are given in the following table :**

	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$	$\frac{(x-\alpha)^2}{a^2}-\frac{(y-\beta)^2}{b^2}=1$	$\frac{(y-\beta)^2}{a^2}-\frac{(x-\alpha)^2}{b^2}=1$
Transverse axis	x-axis	y-axis	parallel to x-axis	parallel to y-axis
Conugate axis	y-axis	x-axis	parallel to y-axis	parallel to x-axis
Equation of transverse axis	$y=0$	$x=0$	$y=\beta$	$x=\alpha$
Equation of conugate axis	$x=0$	$y=0$	$x=\alpha$	$y=\beta$
Length of transverse axis	$2a$ unit	$2a$ unit	$2a$ unit	$2a$ unit
Length of conugate axis	$2b$ unit	$2b$ unit	$2b$ unit	$2b$ unit
Coordinates of centre	(0, 0)	(0, 0)	(α, β)	(α,β)
Coordinates of vertices	$(\pm a, 0)$	$(0, \pm a)$	$(\alpha\pm a, \beta)$	$(a, \beta\pm a)$
Eccentricity	$e=\sqrt{1+\frac{b^2}{a^2}}$	$e=\sqrt{1+\frac{b^2}{a^2}}$	$e=\sqrt{1+\frac{b^2}{a^2}}$	$e=\sqrt{1+\frac{b^2}{a^2}}$
Coordinates of foci	$(\pm ae, 0)$	$(0, \pm ae)$	$(\alpha\pm ae, \beta)$	$(a, \beta\pm ae)$
Distance between two foci	$2ae$ unit	$2ae$ unit	$2ae$ unit	$2ae$ unit
Length of latus rectum	$\frac{2b^2}{a}$ unit	$\frac{2b^2}{a}$ unit	$\frac{2b^2}{a}$ unit	$\frac{2b^2}{a}$ unit
Coordinates of the four ends of latus rectum	$\left(ae, \frac{b^2}{a}\right), \left(ae, -\frac{b^2}{a}\right)$ $\left(-ae, \frac{b^2}{a}\right), \left(-ae, -\frac{b^2}{a}\right)$	$\left(\frac{b^2}{a}, ae\right), \left(-\frac{b^2}{a}, ae\right)$ $\left(\frac{b^2}{a}, -ae\right), \left(-\frac{b^2}{a}, -ae\right)$	$\left(\alpha \pm ae, \beta \pm \frac{b^2}{a}\right)$	$\left(\alpha \pm \frac{b^2}{a}, \beta \pm ae\right)$
Equation of latus rectum	$x=\pm ae$	$y=\pm ae$	$x=\alpha\pm ae$	$y=\beta\pm ae$
Equation of directrices	$x=\pm\frac{a}{e}$	$x=\pm\frac{a}{e}$	$x=\alpha\pm\frac{a}{e}$	$y=\beta\pm\frac{a}{e}$
Distance between two directrices	$\frac{2a}{e}$ unit	$\frac{2a}{e}$ unit	$\frac{2a}{e}$ unit	$\frac{2a}{e}$ unit

Exercise – 11

Group - A

Objective Type Questions : [1 or 2 marks each]

1. Multiple choice type questions :

i) Which of the following points lies on the circumference of the circle $x^2+y^2 = 16$?

a) (0, 2) b) (0,3) c) (–4, 0) d) (2, 3)

ii) The area (in sq. units) of an equilateral triangle inscribed in the circle $x^2+y^2-4x-6y-23 = 0$ is–

a) $27\sqrt{2}$ b) $27\sqrt{3}$ c) $27\sqrt{5}$ d) $25\sqrt{3}$

iii) The circle $(x-4)^2+(y-3)^2 = 9$

a) touches the x-axis b) touches the y-axis

c) touches both the coordinates axes d) does not touch any of the two axes.

iv) The equation $x^2+y^2+2gx+2fy+c = 0$ represent a point-circle when –

a) g^2+f^2-c b) $g^2-f^2=c$ c) $g^2+f^2=c$ d) $f^2-g^2=c$

v) If the equation of a circle is $\lambda x^2+(2\lambda-3)y^2-4x+6y-1=0$ then the coordinates of centre are

a) $\left(\frac{2}{3}, -1\right)$ b) $\left(\frac{4}{3}, -1\right)$ c) $\left(-\frac{2}{3}, 1\right)$ d) $\left(\frac{2}{3}, 1\right)$

vi) If the points of intersection of the parabola $y^2=4ax$ and the circle $x^2+y^2+2gx+2fy+c = 0$ are (x_1,y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) respectively, then

a) $y_1+y_2+y_3+y_4 = 0$ b) $\sqrt{x_1}+\sqrt{x_2}+\sqrt{x_3}+\sqrt{x_4} = C$

c) $y_1-y_2+y_3-y_4 = 0$ d) $y_1-y_2-y_3+y_4 = 0$

vii) Let $y^2 = 4ax$ be a parabola and $x^2+y^2+2bx = 0$ be a circle. If the parabola and the circle touch each other externally, then –

a) $a>0, b>0$ b) $a>0, b=0$ c) $a<0, b>0$ d) $a<0, b<0$

viii) Coordinate of focus of parabola $y^2 = - 16x$ is –

a) (-4,0) (b) (4,0) (c) (0,4) (d) (0,-4)

ix) The coordinate of focus of the parabola $x^2 = \frac{ab}{a+b} \cdot y$ are –

a) $\left(\frac{ab}{4(a+b)}, 0\right)$ b) $\left(\frac{-ab}{4(a+b)}, 0\right)$ c) $\left(0, \frac{ab}{4(a+b)}\right)$ d) $\left(0, \frac{-ab}{4(a+b)}\right)$

x) The parametric equations of the parabola $y^2=12x$ are –

a) $x=6t^2, y=3t$ b) $x=3t^2, y=6t$ c) $x=t^2, y=6t$ d) $x=3t^2, y=t$

xi) The eccentricity of the ellipse $4x^2+25y^2=100$ is

a) $\frac{\sqrt{21}}{5}$ b) $\frac{3\sqrt{7}}{5}$ c) $\frac{7\sqrt{3}}{5}$ d) $\frac{\sqrt{23}}{5}$

xii) The eccentric angle of the point $\left(2, \frac{3\sqrt{3}}{2}\right)$ on the ellipse $9x^2+16y^2 = 144$ is –

a) 90^0 b) 60^0 c) 30^0 d) 45^0

xiii) The equation of auxiliary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a^2 > b^2)$ is

a) $x^2+y^2 = 4a^2$ b) $x^2+y^2 = 2a^2$ c) $x^2+y^2 = a^2$ d) none of these

xiv) The length of the latus rectum of the ellipse $9x^2+25y^2 = 225$ is –

a) $\frac{18}{5}$ unit b) $\frac{16}{5}$ unit c) $\frac{9}{5}$ unit d) $\frac{8}{5}$ unit

xv) Let P be a point on the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. If the distance of P from centre of the ellipse be equal with the average value of semi major axis and semi minor axis, then the co-ordinates of P is –

a) $\left(\frac{2\sqrt{91}}{7}, \frac{3\sqrt{105}}{14}\right)$ b) $\left(\frac{2\sqrt{91}}{7}, \frac{-3\sqrt{105}}{14}\right)$

c) $\left(\frac{-2\sqrt{105}}{7}, \frac{-3\sqrt{91}}{7}\right)$ d) $\left(-\frac{2\sqrt{105}}{7}, \frac{3\sqrt{91}}{14}\right)$

xvi) If e_1 is the eccentricity of the hyperbola $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ the $e_1 =$

a) $\sqrt{1+\frac{a^2}{b^2}}$ b) $\sqrt{1-\frac{a^2}{b^2}}$ c) $\sqrt{1+\frac{b^2}{a^2}}$ d) $\sqrt{1-\frac{b^2}{a^2}}$

xvii) The length of the transverse axis of the hyperbola $9y^2–4x^2 = 36$ is –

a) 2 unit b) 3 unit c) 4 unit d) 5 unit

xviii) The coordinates of foci of the hyperbola $x^2–y^2 = 4$ are

a) $(\pm 2\sqrt{2}, 0)$ b) $(2\sqrt{2}, 0)$ c) $(-2\sqrt{2}, 0)$ d) $(0, \pm 2\sqrt{2})$

xix) If the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ coincide with those of the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ then $b^2 =$

a) 6 b) 7 c) 8 d) 9

xx) If e_1 and e_2 be the eccentricities of the hyperbola $9x^2–16y^2 = 576$ and $9x^2–16y^2 = 144$ respectively. Then –

a) $e_1 = 2e_2$ b) $e_2 = 2e_1$ c) $2e_1 = 3e_2$ d) $e_1 = e_2$

2. Very short answer type questions : (each question caries 1 or 2 marks)

i) Find the radius of the circle which passes through the origin and the points $(a,0)$ and $(0,b)$.

ii) The straight line $3x–4y+7 = 0$ is a tangent to the circle $x^2+y^2+4x+2y+4 = 0$ at P, find the equation of its normal at the same point.

iii) The parametric equations of a circle are $x=\frac{1}{2}(-3+4\cos\theta)$, $y=\frac{1}{2}(1+4\sin\theta)$. Find the equation of the circle.

iv) The parabola $y^2=2ax$ goes through the point of intersection of $\frac{x}{3}+\frac{y}{2}=1$ and $\frac{x}{2}+\frac{y}{3}=1$. Find its focus.

v) Find the point on the parabola $y^2=4ax$ $(a>0)$ which forms a triangle of area $3a^2$ with the vertex and focus of the parabola.

vi) For what values of a will the point (8, 4) be an inside point of the parabola $y^2=4ax$?

vii) Find the distance between the foci of the ellipse $3x^2+4y^2 = 12$.

viii) Find the equation of an ellipse, the distance between the foci is 8 units and the distance between the directriecs is 18 units.

ix) If the ellipse $\frac{x^2}{a_1^2}+\frac{y^2}{b_1^2}=1\left(a_1^2>b_1^2\right)$, has same eccentricity, as that of the ellipse $\frac{x^2}{a_2^2}+\frac{y^2}{b_2^2}=1\left(a_2^2>b_2^2\right)$. Prove that $a_1b_2 = a_2b_1$.

x) If the length of conjugate axis and the length of latus rectum of a hyperbola are equal, find its ecentricity.

Group - B

3. Short answer type questions : (each question carries 3 marks)

i) If $x = \frac{3}{K}$ be the equation of directrix of the parabola $y^2+4y+4x+2 = 0$, then find the value of K.

ii) Find the locus of middle points of a family of focal chords of the parabola $y^2 = 4ax$.

iii) The straight line $y = 2t^2$ intersects the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ at a real points, if $|t|\le K$, then find the value of K.

iv) Find the equation of the ellipse which passes through the points (1, 3) and (2, 1).

v) Find the lengths of axes of the ellipse whose eccentricity is $\frac{3}{5}$ and the distance between focus and directrix is 16.

vi) Find the parametric coordinates of the point $\left(\frac{1}{\sqrt{3}}, \frac{1}{2}\right)$ on the hyperbola $12x^2–4y^2 = 3$.

vii) Find the equation of the hyperbola whose coordinates of foci are $\left(\pm\frac{5}{2}, 0\right)$ and the length of latus rectum is $\frac{9}{4}$.

viii) The coordinates of the centre of the circle $2x^2+2y^2+ax+by+c=0$ are $(3, –4)$; find a and b.

ix) $3x+y = 5$ and $x+y+1 = 0$ are two diameters to the circle which passes through the point $(–2,2)$. Find its equation.

x) Find the centre and radius of the circle $(x–a)^2+(y+b)(y–b) = 0$.

Group - C

Long answer type questions : [each question carries 4/6 marks]

i) Find the equation of the circle whose diameter is the join of the origin and the point $\left(a^3, \frac{1}{a^3}\right)$. Prove that the circle passes through the point $\left(\frac{1}{a}, a\right)$.

ii) Show that the circle $x^2+y^2+6(x–y)+9 = 0$ touches the coordinate axes. Also find the equation of the circle which passes through the common points of intersection of the above circle and the straight line $x–y+4 = 0$ and which also passes through the origin.

iii) Prove that, sum of distances of any point on $9x^2 + 16y^2 = 144$ from its foci in constant.

iv) The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the point of intersection of the lines $7x+13y-87=0$ and $5x-8y+7=0$ and its length of latus rectum is $\frac{32\sqrt{2}}{5}$; find a and b.

v) The vertices of an ellipse are (–1, 2) and (9,2). If the distance between its foci be 8, find the equation of the ellipse and the equation of its directrices.

vi) If the extremities of a focal chord of the parabola $y^2=4ax$ be $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$. Prove that $t_1t_2=-1$.

vii) Prove that the length of any chord of the parabola $y^2=4ax$ passing through the vertex and making an angle θ with the positive direction of x-axis is $4a\operatorname{cosec}\theta.\cot\theta$.

viii) Find the equations of the parabola whose coordinates of vertex are (–2, 3) and the equation of the directirx is $2x+3y+8=0$.

ix) The numerical value of the product of the perpendicular distances of a moving point form the lines $4x-3y+11=0$ and $4x+3y+5=0$ is $\frac{144}{25}$. Find the equation to the locus of the moving point.

x) $P(a\sec\phi, a\tan\phi)$ is a variable point on the hyperbola $x^2-y^2=a^2$, and $A(2a, 0)$is a fixed point. Prove that locus of the middle point of AP is a rectangular hyperbola.

xi) Show that the equation of the circle described on the chord $x\cos\alpha+y\sin\alpha=p$ of the circle $x^2+y^2=a^2$ as diameter is $x^2+y^2-a^2-2p(x\cos\alpha+y\sin\alpha-p)=0$.

xii) A chord $\overline{PQ}$ of the parabola $y^2=4ax$, subtends a right angle at the vertex, show that the mid point of $\overline{PQ}$ lies on the parabola $y^2=2a(x-4a)$.

ANSWERS

Group - A

1]. i) c ii) b iii) b iv) c v) a vi) a

vii) a viii) a ix) c x) b xi) a xii) b

xiii) c xiv) a xv) d xvi) a xvii) c xviii) a

xix) b xx) d

2]. i) $\frac{1}{2}\sqrt{a^2+b^2}$ unit ii) $4x+3y+11=0$ iii) $2x^2+2y^2+6x-2y-3=0$

iv) $\left(\frac{3}{10}, 0\right)$ v) $(9a, 6a), (9a, -6a)$ vi) $a > \frac{1}{2}$

vii) 2 unit viii) $\frac{x^2}{36}+\frac{y^2}{20}=1$ x) $\sqrt{2}$

Group - B

3]. i) 2 ii) $y^2=2ax$ iii) 1

iv) $8x^2+3y^2=35$ v) 30 and 24 vi) $\left(\frac{1}{2}\sec 30^0, \frac{\sqrt{3}}{2}\tan 30^0\right)$

vii) $9x^2-16y^2=36$ viii) $a=-12$, $b=16$ ix) $x^2+y^2-6x+8y-36=0$, $\sqrt{61}$ unit

x) $(a, 0)$ and b unit.

Group - C

4]. i) Equation is $x^2+y^2-a^3x-\frac{y}{a^3}=0$

ii) $4(x^2+y^2)+15(x-y)=0$

iv) $a=5\sqrt{2},\ b=4\sqrt{2}$

v) $9x^2+25y^2-72x-100y+19=0$; $4x=16\pm 25$

viii) $13\left[x^2+(y-6)^2\right]=(2x+3y+8)^2$

ix) $16(x+2)^2-9(y-1)^2=144$

Chapter - 12

Three Dimensional Geometry

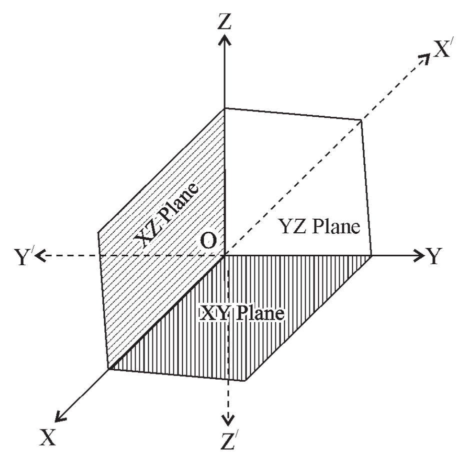
Important points and Results :

- **Rectangular Cartesian Co-ordinate system in three Dimension :**

  $\overrightarrow{OX}$, $\overrightarrow{OY}$ and $\overrightarrow{OZ}$ are the positive directions of the three axes while $\overrightarrow{OX}$, $\overrightarrow{OY}$ and $\overrightarrow{OZ}$ are their negative directions.

  XOY is the plane containing the coordinate axes $\overleftrightarrow{XOX}$ and $\overleftrightarrow{YOY'}$.

  YOZ is the plane containig the coordinate axes $\overleftrightarrow{YOY'}$ and $\overleftrightarrow{ZOX}$ and ZOX is the plane containing the co-ordinate axes $\overleftrightarrow{ZOZ'}$ and $\overleftrightarrow{XOX'}$.



  These three planes which are mutually perpendicular to each other, are called the coordinate planes and usually known as XY plane, YZ plane and ZX plane respectively.

  Three mutually perpendicular lines XOX', YOY' and ZOZ' divide the space into eight parts known as octants.

  The sign of coordinates of he points in the octants in which the space is divided are given in the following table :

Octants	I	II	III	IV	V	VI	VII	VIII
Co-ordinate	OXYZ	OX′YZ	OX′Y′Z	OXY′Z	OXYZ′	OX′YZ′	OX′Y′Z′	OXY′Z′
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

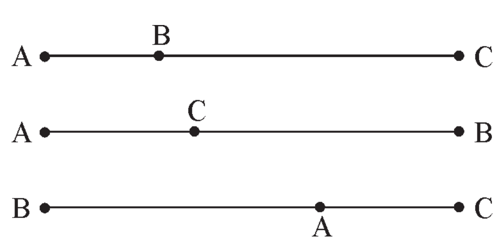
- The equation of the YZ plane is $x = 0$
- The equation of the ZX plane is $y = 0$
- The equation of the XY plane is $z = 0$
- The coordinates of any point on XY plane are $(x, y, 0)$
- The coordinates of any point on YZ plane are $(0, y, z)$

- The coordinates of any point on ZX plane are $(x, 0, z)$
- The coordinates of any point on the x-axis are $(x, 0, 0)$
- The coordinates of any point on the y-axis are $(0, y, 0)$
- The coordinates of any point on the z-axis are $(0, 0, z)$
- $y=0$, $z=0$ represent the equation of x-axis.
- $z=0$, $x=0$ represent the equation of y-axis.
- $x=0$, $y=0$ represent the equation of z-axis.
- The equation of a plane parallel to YZ plane is $x=a$.
- The equation of a plane parallel to ZX plane is $y=b$.
- The equation of a plane parallel to XY plane is $z=c$.
- $y=b$ and $z=c$ represent the equation of a line parallel to x-axis.
- $z=c$ and $x=a$ represent the equation of a line parallel to y-axies.
- $x=a$ and $y=b$ represent the equation of a line parallel to z-axis.
- The distance of the point $P(x,y,z)$ from the origin 0 is $\overline{OP} = \sqrt{x^2 + y^2 + z^2}$.
- **Distance between two points :**

  The distance between the point $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

  $\overline{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
- **Condition for collinearity :**

  The points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are collinear if $\overline{AB} + \overline{BC} = \overline{AC}$ or $\overline{AC} + \overline{CB} = \overline{AB}$ or $\overline{BA} + \overline{AC} = \overline{BC}$

- **Section Formulae :**

  Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two given points in space.

  For internal division :

  If the point R divides the line-segment $\overline{PQ}$ internally in the ratio $m:n$, then the coordinates of

R are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

- **For External division :**

  If the point R divides the line segment $\overline{PQ}$ externally in the ratio m:n, then the coordinates of R are $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$.

- **Coordinates of the Mid-point :**

  If the point R is the mid point of the line segment $\overline{PQ}$, then m:n = 1:1 so that the coordinates of R are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

  If the point R divides the line segment $\overline{PQ}$ in the ratio K:1, then the coordinates of R are $\left(\frac{Kx_2+x_1}{K+1}, \frac{Ky_2+y_1}{K+1}, \frac{Kz_2+z_1}{K+1}\right)$

  Coordinates of the point which trisect the line segment :

  If R_1 and R_2 trisect the line segment $\overline{PQ}$ then $PR_1 = R_1R_2 = R_2Q$.

  Thus, for finding the coordinates of R_1, take ratio 1:2 and for finding the coordinates of R_2, take ratio 2:1.

- **Coordinates of Centroid of a Triangle :**

  If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are the vertices of a triangle, then the coordinates of its centroid are

  $$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

- **Properties of Triangles :**
- Scalene triangle : All three sides are unequal.
- Right angles triangle : The sum of squares of any two sides of a triangle is equal to the square of the third side.
- Isosceles triangle : Any two sides of a triangle are equal.
- Equilateral triangle : All three sides of a triangle are equal.

- **Properties of Quadrilaterals :**
- Rectangle : Opposite sides are equal and diagonals are equal.
- Parallelogram : Opposite sides are equal and diagonals are unequal. Also diagonals bisect each other.
- Rhombus : All four sides are equal and diagonal are unequal.
- Square : All four sides are equal and diagonals are equal.

Exercise – 12

Group - A

Objective Type Questions : [1 or 2 marks each]

I. Multiple choice type questions :

1) The coordinates of any point in YZ plane are of the form

   a) $(x, 0, z)$ b) $(x, y, 0)$ c) $(0, y, z)$ d) None of these

2) $x=b$ and $z=c$ represent the equation of a line parallel to –

   a) x-axis b) y-axis c) z-axis d) None of these

3) $(0, a, 0)$ are the coordinates of any point on –

   a) x-axis b) y-axis c) z-axis d) None of these

4) The points (5, 2, 4), (6, –1, 2) and (8, –7, k) are collinear if k is –

   a) 3 b) –3 c) 2 d) –2

5) If the distance between the points (–1, 1, c) and (2, 1, 1) is 3, then the value of c is

   a) 3 b) 2 c) 1 d) –1

6) The equation of xy plane is –

   a) $x=0$ b) $y=0$ c) $z=0$ d) None of these

7) The ratio in which the line segment joing the points (2, –3, 4) and (3, 4, –1) is divided by the zx plane is –

   a) 3 : 4 b) 4 : 3 c) –2 : 3 d) 1 : 4

8) If the coordinates of two extremities of a diagonal of a square are (4, 4, 7) and (0, 6, 3), then the length of a side is

   a) 3 unit b) 4 unit c) $3\sqrt{2}$ unit d) $2\sqrt{6}$ unit

9) The equation of z-axis in three dimensional space is

   a) $y=0, z=0$ b) $x=0, y=0$ c) $x=0, z=0$ d) None of these

10) YOZ plane divides the line segment joining the points (3, –2, –4) and (2, 4, –3) in the ratio –

a) 1 : 2   b) –4 : 2   c) –2 : 3   d) –3 : 2

11) The coordinates of the vertices of a triangle are (4, 6, 0), (0, –3, 7) and (–4, 0, –1), then the coordinates of the centroid of the triangle are –

a) (0, 1, 2)   b) (–1, 1, 2)   c) (0, 2, 1)   d) None of these

12) The equation of YZ plane is –

a) $y+z = 0$ b) $yz = 0$ c) $y = 0$ d) $x = 0$

II. Very short answer type questions : [1 or 2 marks each]

1) Find the distance between the points A(2,3,1) and B(1,–2,0)

2) Find the values of x, if the distance between two points (x,–8,4) and (3,–5,4) is 5.

3) Find the octants in which the points (–2,–3,5) lie.

4) Where do the points (3, 0, –4) lie ?

5) If the distance between the points (–1,–3,c) and (2,1,–2) is $5\sqrt{2}$ unit, find c.

6) Prove that the triangle formd by joining the points (2,3,4), (3,4,2), (4,2,3) is an equilateral triangle.

7) Prove that the points (4,7,–6), (2,5,–4) and (1,4,–3) are collinear.

8) Prove that the poins (1,–3,1), (0,1,2) (2,–1,3) is a right angled triangle.

9) Find the coordinates of the point in the xy plane which is equidistant from the points A(0,0,1), B(2,0,3) and C(0,3,2).

10) Find the co-ordinates of the points on y-axis which are at a distance $\sqrt{41}$ unit from the point (3,2,–4).

Group - B

Short answer type questions : (each question carries 3 marks)

1) Three consecutive vertices of a parallelogram ABCD are A(6,–2,4), B(2,4,–8) and C(–2,2,4). Find the coordinates of the fourth vertex.

2) Find the third vertex of triangle whose centroid is origin and two vertices are (2,4,6) and ↵ (0,–2,5).

3) If the origin is the centroid of a ΔABC having vertices A(a,1,3), B(–2,b,–5) and C(4,7,c), then find the values of a, b, c.

4) Using section formula, show that the three points A(–2,3,5), B(1,2,3) and C(7,0,–1) are collinear.

5) Find the image of the point (3, 2, –4) in the ZX plane.

6) Find the equation of the locus of a moving point which is always equidistant from the points (3,4,–5) and (–2,1,4).

7) Prove that the triangle formed by joining the points (–4,9,6), (0,7,10), (–1,6,6) is an isosceles right angled triangle.

8) Find the perimeter of the triangle whose vertices are (0,1,2), (2,0,4) and (–4,–2,7).

Group - C

Long answer type questions : [4 or 6 marks each]

1) Find the coordinates of the point equidistant from the four points (2,1,2), (–1,1,3), (0,5,6) and (3,2,2).

2) Show that the point O(0,0,0), P(a,a,0), Q(a,0,a) and R(0,a,a) from a regular tetrahedron.

3) Prove that the points (–1,–3,4), (1,–6,10), (7,–4,7) and (5,–1,1) are the vertices of the rhombus.

4) Two vertices of a parallelogram are (2,5,–3) and (3,7,–5), if its diagonals meet at (4,3,3), find the coordinates of the other two vertices.

5) The coordinates of the mid points of the sides of a triangle are (2,3,4), (1,5,–1) and (0,4,–2). Find the coordinates of the vertices and also find the centroid of the triangle.

6) Show that the points (0,0,0), (–2,0,0), (0,2,0) and (0,0,4) lie on a sphere whose centre is (–1,1,2)

7) Find the coordinates of the points of trisection of the line-segment joining the points (2,1,–3) and (5,–8,3) that is nearer to (2,1,–3)

8) Find the centroid of a triangle, the midpoint of whose sides are D(1,2,–3), E(3,0,1) and F(–1,1,–4).

9) Find the coordinate of the points which trisect the line segment joing the points A(2,1,–3) and B(5,–8,3)

10) Find the ratio in which the line segment joining the points (2,1,3) and (1,–3,–4) is divided by the plane $3x–2y–3z=3$. Also find the coordinates of the point of division.

ANSWER

Group - A

Objective type question :

I. Multiple choice questions :

1.(c) 2.(b) 3.(b) 4.(d) 5.(c) 6.(c) 7.(a) 8.(c) 9.(b) 10.(d)
11.(a) 12.(d)

II. Very short answer type questions :

1) $3\sqrt{3}$ units 2) 7 or (–1) 3) $OX'Y'Z$ 4) ZX plane 5) 3 or –7 9) (3,2,0)
10) (0,–2,0) or (0,6,0)

Group - B

Short answer type questions :

1) (2,–4,16) 2) (–2, –2, –1) 3) a=–2, b=–8 and c=2 5) (3, –2, –4)

6) $10x+6y-18z = 29$ 8) $(10+5\sqrt{2})$ unit

Group - C

Long answer type questions :

1) (1,3,4) 4) (6,1,9) and (5,–1,11) 5) (–1,6,–7), (1,2,3) and (3,4,5 and $\left(1,4,\frac{1}{3}\right)$

7) (3,–2,1) 8) (1,1,–2) 9) (3,–2,–1) and (4,–5,1)

10) 4 : 9 and $\left(\frac{22}{13},\frac{-3}{13},\frac{11}{13}\right)$

Chapter - 13

Limits and Derivatives

Important points and Results :

- **Limits of a function :**
- Let f be a function defined in a domain which we take to be an interval, Say, I. We shall study the concept of limit of f at a point 'a' in I.
- We say $\lim_{x\to a^-} f(x)$ is the expected value of f at $x=a$ given the value of f near x to the left of a. This value is called the left hand limit of f at a.
- We say $\lim_{x\to a^+} f(x)$ is the expected value of f at $x=a$ given the value of near x to the right of a. This value is called the right hand limit of $f(x)$ at a.
- If the right and left hand limits coincide, we call that common value as the limit of $f(x)$ at $x=a$ and denoted it by $\lim_{x\to a} f(x)$.
- **Some properties of limits –**
- Let f and g be two functions such that both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Then

i) $\lim_{x\to a}\left[f(x)+g(x)\right]=\lim_{x\to a} f(x)+\lim_{x\to a} g(x)$

ii) $\lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a} f(x)-\lim_{x\to a} g(x)$

iii) For every real number α, $\lim_{x\to a} \alpha f(x)=\alpha\lim_{x\to a} f(x)$

iv) $\lim_{x\to a}\left[f(x).g(x)\right]=\lim_{x\to a} f(x).\lim_{x\to a} g(x)$

v) $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, where $g(x)\neq 0$

- For a function $f(x)$ and a real number a, $\lim_{x\to a} f(x)$ and $f(a)$ may not be same.

Infact

i) $\lim_{x\to a} f(x)$ exist but $f(a)$ (the value of $f(x)$ at x=a) may not exist.

ii) The value of $f(a)$ exist but $\lim_{x \to a} f(x)$ does not exist.

iii) $\lim_{x \to a} f(x)$ and $f(a)$ both exist but are unequal.

iv) $\lim_{x \to a} f(x)$ and $f(a)$ both exist and are equal.

- **Followings are some standard limit :**

i) $\lim_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$

ii) $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

iii) $\lim_{x \to 0} \dfrac{\tan x}{x} = 1$

iv) $\lim_{x \to 0} \dfrac{\log(1+x)}{x} = 1$

v) $\lim_{x \to 0} \dfrac{a^x - 1}{x} = \log_e^a,\ a \neq 0,\ a > 1$

vi) $\lim_{x \to 0} \dfrac{e^x - 1}{x} = 1$

- **Derivatives :**
- Suppose *f* is a real valued function and *a* is a point in its domain of defination. The derivative of f at *a* is defined by $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ provided this limit exists. Derivative of *f(x)* at a is denoted by $f^{/}(a)$.
- Geometrically the derivative of a function *f(x)* at a point *x=a* is the slope of the tangent to the curve *y=f(x)* at the point $(a,\ f(a))$.

Algebra of derivative of function :

- Let *f* and *g* be two functions such that their derivatives are defined in a common domain. Then,

i) Derivative of the sum of two functions is the sum of the derivatives of the functions.

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

ii) Derivative of the difference of two functions is the difference of the derivatives of the functions. $\dfrac{d}{dx}\left[f(x) - g(x)\right] = \dfrac{d}{dx} f(x) - \dfrac{d}{dx} g(x)$

iii) Derivative of the product of two functions is given by the following product rule.

$$\frac{d}{dx}\left[f(x).g(x)\right] = \left(\frac{d}{dx} f(x)\right).g(x) + f(x).\left(\frac{d}{dx} g(x)\right)$$

iv) Derivative of quotient of two functions is given by the following quotient rule (wherever the denominator in non-zero).

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{\left(\frac{d}{dx}f(x)\right).g(x)-f(x).\left(\frac{d}{dx}g(x)\right)}{(g(x))^2}$$

- Defferentiation of a constant function is zero, i.e. $\frac{d}{dx}(c)=0$
- Let *f(x)* be a differentiable function and let *c* be a cosntant. Then, *cf(x)* is also differentiable such that $\frac{d}{dx}(cf(x))=c\frac{d}{dx}f(x)$.
- Let *f(x), g(x), h(x)* be three differentiable functions. Then,

$$\frac{d}{dx}\{f(x).g(x).h(x)\}=\left(\frac{d}{dx}f(x)\right).g(x).h(x)+f(x).\left(\frac{d}{dx}g(x)\right).h(x)+f(x).g(x).\left(\frac{d}{dx}h(x)\right)$$

- **Derivative of a function of function :**

Chain rule : If $y=f(t)$ and $t=g(x)$ then

$$\frac{dy}{dt}=f'(t),\ \frac{dt}{dx}=g'(x)$$

$$\therefore \frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=f'(t)\times g'(x)$$

- **Following are some standard derivatives :**

i) $\frac{d}{dx}(x^n)=nx^{n-1}$

ii) $\frac{d}{dx}(a^x)=a^x\log_e^a,\ a>0,\ a\neq 1$

iii) $\frac{d}{dx}(e^x)=e^x$

iv) $\frac{d}{dx}(\log_e x)=\frac{1}{x}$

v) $\frac{d}{dx}(\sin x)=\cos x$

vi) $\frac{d}{dx}(\cos x)=-\sin x$

vii) $\frac{d}{dx}(\tan x)=\sec^2 x$

viii) $\frac{d}{dx}(\cot x)=-\cos ec^2 x$

ix) $\frac{d}{dx}(\sec x)=\sec x\ \tan x$

x) $\frac{d}{dx}(\cos ecx)=-\cos ecx\ \cot x$

Exercise – 13

Group - A

Objective Type Questions : [1 or 2 marks each]

1. Multiple choice type questions : (Choose correct answer)

i) $\lim_{x \to 1} \frac{x^m - 1}{x^n - 1}$ is

a) 1 b) $\frac{m}{n}$ c) 0 d) $-\frac{m}{n}$

ii) $\lim_{y \to 0} \frac{(1+y)^m - 1}{y}$ is

a) m b) $-m$ c) 1 d) 0

iii) $\lim_{x \to \pi} \frac{\sin x}{x - \pi}$ is

a) 1 b) 2 c) –1 d) –2

iv) $\lim_{x \to 1} \frac{\sin \pi x}{x - 1}$ is equal to

a) $-\frac{1}{\pi}$ b) $\frac{1}{\pi}$ c) π d) $-\pi$

v) If $f(x) = \begin{cases} x \sin\frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, then $\lim_{x \to 0} f(x)$ equals

a) 1 b) 0 c) –1 d) none of these

vi) $\lim_{x \to 0} \frac{|\sin x|}{x}$ is equals to

a) 1 b) –1 c) does not exist d) none of these

vii) If $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & ,[x] \neq 0 \\ 0 & ,[x] = 0 \end{cases}$, where [.] denotes the greatest integer function, then $\lim_{x \to 0} f(x)$ is

a) 1 b) 0 c) –1 d) none of these

viii) Let $f(x)=x-[x]$, $x \in R$, then $f'\left(\frac{1}{2}\right)$ is

a) 1 b) –1 c) 0 d) $\frac{3}{2}$

ix) If $f(x)=x\sin x$, then $f'\left(\pi/2\right)=?$

a) -1 b) 1 c) 0 d) $\frac{1}{2}$

x) If $f(x)=\frac{x-4}{2\sqrt{x}}$, then $f'(1)$ is equals to

a) $\frac{5}{4}$ b) $\frac{4}{5}$ c) 1 d) 0

xi) If $f(x)=\frac{x^n-a^n}{x-a}$, then $f'(a)$ is

a) 1 b) 0 c) $\frac{1}{2}$ d) does not exist

xii) $\lim\limits_{x\to 2}[x-2]$, where [.] is greatest integer function, is equal to

a) 1 b) 2 c) 0 d) does not exist.

2. Very short answer type questions : (each question caries 1 or 2 marks)

i) Find the value of $\lim\limits_{x\to 0}\frac{\sin x^0}{x}$

ii) Find the value of $\lim\limits_{n\to\infty}\frac{1+2+3+..........+n}{n^2}$

iii) If $\lim\limits_{x\to 2}\frac{x^n-2^n}{x-2}=80$ and $n\in N$, find n.

iv) Find the value of $\lim\limits_{x\to 0}\frac{\sin x}{x(1+\cos x)}$.

v) Find the value of $\lim\limits_{x\to 0}\frac{a^{\sin x}-1}{\sin x}$.

vi) Find the derivative of $f(x)=3|2+x|$ at $x=-3$.

vii) If $y=\log_5 x$, then find $\frac{dy}{dx}$.

viii) Find the value of $\frac{d}{dx}(x|x|)$.

ix) If $y=\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\cdots\infty\right)$, show that $\frac{dy}{dx}=y$.

Group - B

3. Short answer type questions : (each question cares 3 marks)

Evaluate the following limits in Exercise (i) to (xviii)

i) $\lim\limits_{x\to 0} \dfrac{2x^2+3x+4}{x^2+3x+2}$

ii) $\lim\limits_{x\to 2} \left\{\dfrac{x}{x-2} - \dfrac{4}{x^2-2x}\right\}$

iii) $\lim\limits_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

iv) $\lim\limits_{x\to a} \dfrac{(x+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{x-a}$

v) $\lim\limits_{x\to 4} \dfrac{x^3-64}{x^2-16}$

vi) $\lim\limits_{x\to 1} \dfrac{x^2-\sqrt{x}}{\sqrt{x}-1}$

vii) $\lim\limits_{x\to 1} \dfrac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$

viii) $\lim\limits_{n\to \infty} \dfrac{n!}{(n+1)!-n!}$

ix) $\lim\limits_{x\to 0} \dfrac{1-\cos 2x}{x}$

x) $\lim\limits_{x\to 0} \dfrac{1-\cos 2mx}{1-\cos 2nx}$

xi) $\lim\limits_{x\to \frac{\pi}{2}} (\sec x - \tan x)$

xii) $\lim\limits_{x\to 0} \dfrac{\tan 3x - 2x}{3x-\sin^2 x}$

xiii) $\lim\limits_{x\to \pi} \dfrac{1+\cos^3 x}{\sin^2 x}$

xiv) $\lim\limits_{x\to 0} \dfrac{a^x-b^x}{x}$

xv) $\lim\limits_{x\to 0} \dfrac{3^x+3^{-x}-2}{x^2}$

xvi) $\lim\limits_{x\to 0} \dfrac{\sin 3x}{3^x-1}$

xvii) $\lim\limits_{x\to 0} \dfrac{e^{3x}-e^{2x}}{x}$

xviii) $\lim\limits_{x\to 5} \dfrac{\log x - \log 5}{x-5}$

Differentiate each of the functions w.r.t x in exercise (xix) to (xxx).

xix) $\dfrac{ax^2+bx+c}{\sqrt{x}}$

xx) $\log_3 x$

xxi) $\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)^3$

xxii) $x^3.e^x$

xxiii) $\frac{x+e^x}{1+\log x}$

xxiv) $\frac{10^x}{\sin x}$

xxv) $\frac{1+\log x}{1-\log x}$

xxvi) $x^5e^x + x^6 logx$

xxvii) $e^{\sqrt{\cot x}}$

xxviii) cos^2x^3

xxix) $\sqrt{x\sin x}$

xxx) $cos(x+a)$

Group - C

4. Long answer type questions : [4 or 6 marks]

i) If $f(x)=\begin{cases}\frac{x-|x|}{x}, & x\neq 0\\ 2, & x=0\end{cases}$ show that $\lim_{x\to 0} f(x)$ does not exist.

ii) Let *f(x)* be a function defined by $f(x)=\begin{cases}4x-5, & if\ x\leq 2\\ x-\lambda, & if\ x>2\end{cases}$. Find λ, if $\lim_{x\to 2} f(x)$ exists.

iii) If *f* is an odd function and if $\lim_{x\to 0} f(x)$ exists. Prove that this limit must be zero.

iv) Evaluate $\lim_{x\to 2} f(x)$ (if it exists), where $f(x)=\begin{cases}x-[x] & , x<2\\ 4 & , x=2\\ 3x-5 & , x>2\end{cases}$

v) Let $f(x)=\begin{cases}\frac{K\cos x}{\pi-2x}, & where\ x\neq\frac{\pi}{2}\\ 3 & , where\ x=\frac{\pi}{2}\end{cases}$,

and if $\lim_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right)$, find the value of *K*.

Evaluate the following limits. (vi to xix)

vi) $\lim_{h\to 0}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$

vii) $\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)$

viii) $\lim_{x\to 0}\frac{e^x-e^{\sin x}}{x-\sin x}$

ix) $\lim_{x\to 0}\frac{a^x+b^x-c^x-d^x}{x}$

x) $\lim\limits_{x\to 0}\dfrac{\log(5+x)-\log(5-x)}{x}$

xi) $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^3}$

xii) $\lim\limits_{x\to 0}\dfrac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}$

xiii) $\lim\limits_{y\to 0}\dfrac{(x+y)sex(x+y)-x\sec x}{y}$

xiv) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{x-\frac{\pi}{4}}$

xv) $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

xvi) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\cos x-1}{\cot x-1}$

xvii) $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}$

xviii) $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cot x-\cos x}{(\pi-2x)^3}$

xix) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$

xx) If $\lim\limits_{x\to a}\dfrac{x^3-a^3}{x-a}=\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1}$, find all possible values of a.

xxi) Find the derivatives of the following functions by first principles :

a) $\sqrt{ax+b}$ b) ax^2+bx+c c) x^n

d) $\dfrac{2x+3}{3x+2}$ e) sin^2x f) $sinx^2$

g) xe^x h) $\tan\sqrt{x}$ i) $\sqrt{\cot x}$

j) $log\,sinx$ k) $e^{\sqrt{\tan x}}$ l) $e^{\sqrt{2x}}$

xxii) Differentiate the following functions w.r.t. x :

a) $(xsinx+cosx)\,(xcosx-sinx)$

b) $e^x\log\sqrt{x}\tan x$

c) $\dfrac{\sec x+\tan x}{\sec x-\tan x}$

d) $\dfrac{x\sin x}{1+\cos x}$

xxiii) If $y=\sqrt{\dfrac{x}{a}}+\sqrt{\dfrac{a}{x}}$, prove that $2xy\dfrac{dy}{dx}=\left(\dfrac{x}{a}-\dfrac{a}{x}\right)$

xxiv) If $y=\frac{2-3\cos x}{\sin x}$, find $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$

xxv) If $y=\frac{\cos x-\sin x}{\cos x+\sin x}$, show that $\frac{dy}{dx}+y^2+1=0$

xxvi) If $y=\sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2)\frac{dy}{dx}+y=0$.

xxvii) If $f(x)=\lambda x^2+\mu x+12$, $f'(4)=15$ and $f'(2)=11$, then find λ and μ.

ANSWERS

Group - A

1. i) b ii) a iii) c iv) d v) b vi) c
   vii) d viii) a ix) b x) a xi) d xii) d

2. i) $\frac{\pi}{180}$ ii) $\frac{1}{2}$ iii) 5 iv) $\frac{1}{2}$ v) *loga* vi) –3

   vii) $\frac{1}{x}\log_5 e$ viii) 2x, when $x>0$; $-2x$, when $x<0$.

Group - B

3. i) 2 ii) 2 iii) $\frac{1}{2\sqrt{2}}$ iv) $\frac{3}{2}(a+2)^{\frac{1}{2}}$ v) 6 vi) 3

   vii) $\frac{1}{4}$ viii) 0 ix) 0 x) $\frac{m^2}{n^2}$ xi) 0 xii) $\frac{1}{3}$

   xiii) $\frac{3}{2}$ xiv) $\log\frac{a}{b}$ xv) $(\log 3)^2$ xvi) $\frac{3}{\log 3}$ xvii) 1 xviii) $\frac{1}{5}$

   xix) $\frac{3a}{2}\sqrt{x}+\frac{b}{2\sqrt{x}}-\frac{c}{2x\sqrt{x}}$ xx) $\frac{1}{x\log 3}$ xxi) $\frac{3}{2}x^{\frac{1}{2}}-\frac{3}{2}x^{\frac{-5}{2}}+\frac{3}{2}x^{\frac{-1}{2}}-\frac{3}{2}x^{\frac{-3}{2}}$

   xxii) $x^2e^x(3+x)$ xxiii) $\frac{x\log x(1+e^x)-e^x(1-x)}{x(1-\log x)^2}$

   xxiv) $10^x\operatorname{cosec}x(\log 10-\cot x)$ xxv) $\frac{2}{x(1-\log x)^2}$ xxvi) $x^4(5e^x+xe^x+x+6x\log x)$

xxvii) $\dfrac{-\operatorname{cosec}^2 x}{2\sqrt{\cot x}} e^{\sqrt{\cot x}}$ xxviii) $-3x^2\sin(2x^3)$ xxix) $\dfrac{x\cos x+\sin x}{2\sqrt{x\sin x}}$

xxx) $-\sin(x+a)$

Group - C

ii) $\lambda=-1$ iv) 1 v) $K=6$ vi) $\dfrac{-1}{2x^{3/2}}$ vii) $-\dfrac{1}{3}$ viii) 1

ix) $\log\dfrac{ab}{cd}$ x) $\dfrac{2}{5}$ xi) $\dfrac{1}{2}$ xii) 4 xiii) $\sec x(x\tan x+1)$

xiv) $\sqrt{2}$ xv) $\dfrac{1}{4\sqrt{2}}$ xvi) $\dfrac{1}{2}$ xvii) $2\sqrt{a}\cos a$ xviii) $\dfrac{1}{16}$ xix) -4

xx) $\pm\dfrac{2}{\sqrt{3}}$

xxi) a) $\dfrac{a}{2\sqrt{ax+b}}$ b) $2ax+b$ c) nx^{n-1} d) $-\dfrac{5}{(3x+2)^2}$ e) $\sin 2x$

f) $2x\cos x^2$ g) $(x+1)e^x$ h) $\dfrac{1}{2\sqrt{x}}\sec^2\sqrt{x}$ i) $-\dfrac{\operatorname{cosec}^2 x}{2\sqrt{\cot x}}$ j) $\cot x$

k) $e^{\sqrt{\tan x}}.\dfrac{\sec^2 x}{2\sqrt{\tan x}}$ l) $\dfrac{1}{\sqrt{2x}}e^{\sqrt{2x}}$

xxii) a) $x\{x\cos 2x-\sin 2x\}$ b) $\dfrac{1}{2}e^x\left\{\log x.\tan x+\dfrac{\tan x}{x}+\log x.\sec^2 x\right\}$

c) $\dfrac{2\cos x}{(1-\sin x)^2}$ d) $\dfrac{x+\sin x}{1+\cos x}$

xxiv) $6-2\sqrt{2}$ xxvii) $\lambda=1,\ \mu=7$

Chapter - 14

Mathematical Reasoning

Important points and Results :

In mathematical language, there are two kinds of reasoning – inductive and deductive.

- **Statements :**

  A statement is a sentence which is either true or false, but not both.

  Note : No sentence can be called a statement if

  i) It is an exclamation
  ii) It is an order or request
  iii) It is a question
  iv) It involves variable time such as 'today', 'tomorrow', 'yesterday' etc.
  v) It involves variable places such as 'here', 'there', 'every where' etc.
  vi) It involves pronouns such as 'she', 'he', 'they' etc.

- **Simple statements :**

  A statement is called simple if it can not be broken down into two or more statements.

- **Compound statements :**

  A compound statement is the one which is made up of two or more simple statements. These simple tatements are called component statement.

- **Negation of a statement :**

  The denial of a statement is called the negation of statement.

  If P is a statement, then the negation of p is also a statement and is denoted by ~p, and read as 'not p'.

- **Connectives :**

  There are many ways of combining simple statements to form new statements. The words which combine or change simple statements to form new statements or compound statements are called connectives. The basic connectives 'conjunction' corresponds to the word 'and'; 'disjunction' corresponds to the word 'or'. Throughout we use the symbol '$\wedge$' to denote conjunction; '$\vee$' to denote disjunction.

- **Conjunction :**

  If two simple statements p and q are connected by the word 'and', then the resulting compound statement 'p and q' is called a conjunction of p and q and is written in symbolic form as '$p\wedge q$'.

  Truth table for $p\wedge q$:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- **Disjunction :**

  If two simple statements p and q are connected by the word 'or', then the resulting compound statement 'p or q' is called disjunction of p and q and is written in symbolic form as '$p \vee q$'.

  Truth table for $p \vee q$:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation of Conjunction : The negation of a conjunction $p \wedge q$ is the disjunction of the negation of p and the negation of q. Equivalently, we write $\sim(p \wedge q) = \sim p \vee \sim q$.

Negation of disjunctoin : The negation of a disjunction $p \vee q$ is the conjunction of the negation of p and the negation of q. Equivalently, we write $\sim(p \vee q) = \sim p \wedge \sim q$.

Negation of a negation : Negation of negation of a statement is the statement itself. Equivalently, we write $\sim(\sim p) = p$.

- **Conditional statement :**

  If p and q are any two statements then the compound statements "if p then q" formed by joining p and q by a connective "if....then" is called a conditoinal statment or an implication and is written in symbolic form as $p \rightarrow q$ or $p \Rightarrow q$.

  Truth table for $p \Rightarrow q$

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The conditional statement $p \Rightarrow q$ can be expressed in several different ways, these are

i) p implies q

ii) p only if q

iii) q is a necessary condition for p

iv) p is a sufficient conditon for q.

v) $\sim q \Rightarrow \sim p$.

- **Converse and contrapositive of a conditional statement :**
  The conditional statement '$q \rightarrow p$' is called the converse of the conditional statement '$p \rightarrow q$'.

  The statement '$(\sim q) \rightarrow (\sim p)$' is called the contrapositive of the statement '$p \rightarrow q$'.

- **Biconditional statement :**
  If two statements p and q are connected by the connective "if and only if" then the resulting compound statement "p if and only if q" is called a biconditional of p and q and is written in symbolic form as $p \leftrightarrow q$ or $p \Leftrightarrow q$.

  Truth table for $p \Leftrightarrow q$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- **Quantifiers :**

  Quantifiers are phrases like 'There exists' and 'For all'. The symbol '$\forall$' stands 'For all' and '$\exists$' stands 'There exists'.

  'For all' is called universal quantifiers.

  'There exists' is called existential quantifiers.

- **Validity of a statement :**

  Validity of a statement means checking when the statement is true and when it is not true. This depends upon which of the connectives, quantifiers and implication is being used in statement.

- **Rule–1 : Statements with 'and'**
  If p and q are mathematical statements, then in order to show that the statement 'p and q' is true, the following steps are followed.

  Step-1 : Show that the statement p is true.

  Step-2 : Show that the statement q is true.

- **Rule-2 : Statements with 'or'**
  If p and q are mathematical statements, then in order to show that the statement 'p or q' is true, one must consider the following.

Case-1 : By assuming that p is false, show that q must be true.

Case-2 : By assuming that q is false, show that p must be true.

- **Rule-3 : Statements with 'If - then'**

  In order to prove the statement 'if p then q' we need to show that any one of the following case in true.

  Case-1 : By assuming that p is true, prove that q must be true. (Direct method)

  Case-2 : By assuming that q is false, prove that p must be false. (Contrapositive method)

  Case-3 : By assuming that p is true and q is false and obtain a contradiction from assumption. (contradiction method)

- **Rule-4 : Statements with 'if and only if'**

  In order to prove the statement "p if and only if q"; we need to show

  i) If p is true, then q is true and

  ii) If q is true, then p is true.

- **Invalidity of statements by counter examples :**

  In order to show that a statement is false, we may give an example of a situation where the statement is not valid. Such an example is called a counter example.

Exercise – 14

Gropu - A

Objective Type Questions : [1 or 2 marks each]

1. Multiple choice type questions :

i) Which of the following is a statement ?

a) y is a real number b) Switch off the fan.

b) 2 is a natural number c) Let me go.

ii) Which of the following is not a statement ?

a) Smoking is injurious to health b) 2+3=5

c) 2 is the only even prime number d) Come here.

iii) The connective in the statement $3+4>7$ or $3+4<7$ is

a) and b) or c) $<$ d) $>$

iv) The negation of the statement "A circle is an ellipse" is

a) A circle is not an ellipse. b) An ellipse is a circle.

c) A circle is an ellipse. d) An ellipse is not a circle.

v) The negation of the statement '5 is greater than 9' is

a) 5 is less than 9. b) 5 is not greater than 9.

c) 9 is less than 5. d) 5 is equal to 9.

vi) The contrapositive of the statement "If 7 is greater than 5, then 8 is greater than 6" is

a) If 8 is greater than 6, then 7 is not greather than 5.

b) If 8 is not greater than 6, then 7 is greater than 5.

c) If 8 is not greater than 6, then 7 is not greater than 5.

d) If 8 is greater than 6, then 7 is greater than 5.

vii) The converse of the statement "If $x>y$, then $x+b>y+b$" is

a) If $x<y$, then $x+b<y+b$ b) If $x+b>y+b$, then $x>y$.

c) If $x<y$, then $x+b>y+b$. d) If $x>y$, then $x+b<y+b$.

viii) Which of the following is the conditional $p \rightarrow q$?

a) p only if q. b) If q, then p.

c) q is sufficient for p. d) p is necessary for q.

ix) Which of the following statement is a conjunction ?

a) Amal and Kamal are friends b) Both Amal and Kamal are tall.

c) Both Amal and Kamal are enemies d) None of these.

x) Which of the following is not a negation of "A natural number is greater than zero" ?

a) A natural number is not greater than zero.

b) It is false that a natural number is greater than zero.

c) It is false that a natural number is not greater than zero.

d) None of these.

Very short answer type questions :

2. Which of the following sentences are statements ? Give reasons for your answer.

i) The set of prime integers is infinite.

ii) A triangle has three sides.

iii) Go to your class.

iv) Every relation is a function.

v) Where is your bag ?

vi) Every rhombus is a square.

vii) $y + 6 = 5$

viii) May God bless you !

ix) The number x is an even number.

x) $x^2+5|x|+6 = 0$ has no real roots.

xi) $\left(2+\sqrt{3}\right)$ is a complex number.

3. Find the component statements of the following compound statements :

i) Number 5 is prime or odd.

ii) $\sqrt{11}$ is an irrational number or a rational number.

iii) The number 60 is divisible by 3, 9 and 5.

iv) (1+i) is a real or a complex number.

v) Two lines in a plane either intersect at one point or they are parallel.

vi) A square is a quadrilateral or a 5-sided polygon.

vii) 0 is less than every positive integer and every negative integer.

viii) Chandigarh is the capital of Haryana and Bihar.

4. Write the negation of the following statements :

i) Mumbai is a city.

ii) All mathematicians are man.

iii) The number 7 is greater than 5.

iv) All cats scratch.

v) $3+6 = 8$

vi) A leap year has 366 days.

vii) 2 is not a prime number.

Group - B

Short answer type questions : (3 marks each)

5. Find the component statements of the following compound statements and check whether they are true or false.

i) 50 is a multiple of 2 and 5.

ii) The school is closed, if it is a holiday or a Sunday.

iii) Square of an integer is positive or negative.

iv) $x=5$ and $x=2$ are the roots of the equation $3x^2-x-10=0$.

6. Translate the following statements into symbolic form :

i) Ruma passed in Mathematics and Science.

ii) x and y are odd integers.

iii) Either x or x+1 is an even integer.

iv) If x=5 and y=3, then x+y=8.

v) ABC is an equilateral triangle if and only if its each interior angle is 60^0.

vi) If it is raining today, then 3+4>6.

7. Write the negation of the following compound statements :

i) Ramesh lives in Assam or he lives in Tripura.

ii) The sand heats up quickly in the Sun and does not cool down fast at night.

iii) x+y = y+x and 30 is an even number.

iv) A triangle has either 3-sides or 4-sides.

v) |x| is equal to either x or –x.

8. Identify the quantifiers and write the negation of the following statements :

i) For all even integers x, x^2 is also even.

ii) There exists a number which is a multiple of 6 and 9.

iii) Every living person is not 120 years old.

v) For all positive integers x, we have x+3>7.

9. Write the converse and contrapositive of the following statements.

i) If a ΔABC is right angled at B, then $AB^2+BC^2=AC^2$.

ii) If ΔABC and ΔDEF are congruent, then they are equiangular.

iii) If she works, then she will earn money.

iv) If A and B are subsets of x such that $A\subseteq B$, then $(X-B)\subseteq(X-A)$.

v) If the two lines are parallel, then they do not intersect in the same plane.

vi) If p(3)=0, then p(x) is divisible by (x–3).

vii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

viii) If x:y=3:2, then 2x=3y.

10. Write each of the following statements in the form 'if - then'.

i) A rhombus is a square only if each of its angles measures 90^0.

ii) When a number is a multiple of 9, it is necessarily a multiple of 3.

iii) The game is cancelled only if it is raining.

11. Rewrite each of the following statements in the form 'if and only if'.

i) p : Today is 14th of August.

q : Tomorrow is Independence day.

ii) p : In ΔABC, $\angle B = \angle C$.

q : In ΔABC, $AC = AB$

iii) p : A and B are two sets such that $A \subseteq B$ and $B \subseteq A$.

q : $A = B$

iv) p : If $f(a)=0$, then $(x-a)$ is a factor of polynomial $f(x)$.

q : If $(x-a)$ is a factor of polynomial $f(x)$, then $f(a)=0$.

Group - C

4. Long answer type questions : [4 or 6 marks each]

12. Rewrite the following statement in five different ways conveying the same meaning.

If a given number is a multiple of 6; then it is a multiple of 3.

13. Show that the following statement is true :

For any real numbers x, y if $x=y$, then $2x+a=2y+a$ where $a \in z$.

14. Prove by direct method that any integer 'n', n^3-n is always even.

15. Check the validity of the following statements :

i) p : 125 is divisible by 5 and 7.

ii) q : 100 is a multiple of 4 and 5.

16. Show that the statement :

P : If x is a real number such that $x^3+x=0$, then $x=0$ is true by

i) direct method ii) method of contrapositive iii) method of contradiction.

17. By giving a counter example, show that the following statements are not true.

i) If n is an odd integer, then n is prime.

ii) For any real number a and b; a^2-b^2 implies $a=b$.

18. Use contradiction method to prove that p : $\sqrt{3}$ is an irrational number is a true statement.

ANSWERS

Group - A

1. i) c ii) d iii) b iv) a v) b vi) c vii) b viii) a ix) d x)c

2. i) Statement ii) Statement iii) Not a statement iv) Statement v) Not a statement vi) Statement vii) Not a statement viii) Not a statement ix) Not a statement x) Statement xi) Statement

3. i) p : Number 5 is prime

q : Number 5 is odd

ii) p : $\sqrt{11}$ is an irrational number

q : $\sqrt{11}$ is a rational number

iii) p : The number 60 is divisible by 3.

q : The number 60 is divisible by 9.

r : The number 60 is divisible by 5.

iv) p : (1+i) is a real number.

q : (1+i) is a complex number.

v) p : Two lines in a plane are intersect at one print.

q : Two lines in a plane are parallel.

vi) p : A square is a quadrilateral.

q : A square is a 5-sided polygon.

vii) p : 0 is less than every positive integer.

q : 0 is less then every negative integer.

viii) p : Chandigarh is the capital Haryana.

q : Chandigarh is the capital of Bihar.

4. i) Mumbai is not a city

ii) Some mathematicians are not man.

iii) The number 7 is not greater than 5.

iv) Some cats donot scratch.

v) $3+6\neq 8$

vi) A leap year does not have 366 days.

vii) 2 is a prime number.

Group - B

5. i) P : 50 is a multiple of 2,

   q : 50 is a multiple of 50, true

   ii) p : The school is closed if it is a holiday

   q : The school is closed if it is a Sunday, true

   iii) p : Square of an integer is positive

   q : Square of an integer is negative, false

   iv) p : x=5 is the root of the equation $3x^2-x-10=0$

   q : x=2 is the root of the equation $3x^2-x-10=0$, false

6. i) p : Ruma passed in Mathematics.
   q : Ruma passed in Science. } symbolic form : $p \wedge q$

   ii) p : x is odd integers
   q : y is odd integers. } symbolic form : $p \wedge q$

   iii) p : x is an even integer
   q : x+1 is an even integer. } symbolic form : $p \vee q$

   iv) p : x=5 and y=3
   q : x+y=8 } symbolic form : $p \Rightarrow q$

   v) p : ABC is an equilateral triangle.
   q : Its each interior angle is 60^0. } symbolic form : $p \Leftrightarrow q$

   vi) p : It is raining today
   q : 3+4>6 } symbolic form : $p \Rightarrow q$

7. i) Ramesh does not live in Assam and he does not live in Tripura

   ii) Either the sand does not heat up quickly in the Sun or it cools down fast at night.

   iii) $x+y \neq y+x$ or 30 is not an even number.

   iv) A triangle has neithr 3-sides nor 4-sides.

   vi) $|x|$ is not equal to x and it is not equal to –x.

8. i) Quantifier : For all there exist an even integer x such that x^2 is not even.

   ii) Quantifier : There exist. There does not exist a number which is a multiple of both 6 and 9.

   iii) Quantifier : For every. There exist a living person who is 120 years old.

   iv) Quantifier : For all. There exist a positive integer x such that $x+2 \leq 8$.

9. i) Converse : In a ΔABC, if $AB^2+BC^2=AC^2$, then it is not right angled at B.

Contrapositive : In a ΔABC, if $AB^2+BC^2 \neq AC^2$, then it is not right angled at B.

ii) Converse : If ΔABC and ΔDEF are equiangular, then they are congruent.

Contrapositive : If ΔABC and ΔDEF are not equiangular, then they are not congruent.

iii) Converse : If she earns money, then she works.

Contrapositive : If she does not earn money, then she does not work.

iv) Converse : If A and B ae subsets of X such that $(X-B) \subseteq (X-A)$, then $A \subseteq B$.

Contrapositive : If A and B ae subsets of X such that (X–B) is not a subset (X–A), then A is not a subset of B.

v) Converse : If the two lines do not intersect in the same plane, then they are parallel.

Contrapositive : If the two lines intersect in the same plane, then they are not parallel.

vi) Converse : If p(x) is divisible by (x–3), then $p(3)=0$.

Contrapositive : If p(x) is not divisible by (x–3), then $p(3) \neq 0$.

vii) Converse : If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Contrapositive : If a quadrilateral is not a parallelogram, then its diagonals do not bisect each other.

viii) Converse : If $2x=3y$, then $x:y=3:2$

Contrapositive : If $2x \neq 3y$, then $x:y \neq 3:2$

10. i) If each angle of a rhombus measures 90^0, then it is a square.

ii) If a number is a multiple of 9, then it is a multiple of 3.

iii) If it is raining, then the game is cancelled.

11. i) "Today is 14th of August if and only if tomorrow is Independence Day".

ii) In ΔABC, $\angle B = \angle C \Leftrightarrow AC = AB$.

iii) For any sets A and B, $A=B \Leftrightarrow (A \subseteq B$ and $B \subseteq A)$

iv) (x–a) is a factor of polynomial f(x) if and only if $f(a)=0$.

Chapter - 15

Statistics

Important points and Results :

- **Data and its types :**

  A group of information that represents the qualitative or quantitative attributes of a variable or set of variables is called data.

  There are two types of data. These are

  i) Ungrouped data : In an ungrouped data, data is listed in a series e.g., 2,4,6,..........etc.

  ii) Grouped data : It is of two types –

  a) Discrete data : In this type, data is presented in such a way that exact measurements of items are clearly shown.

  b) Continuous data : In this type, data is arranged in groups or classes but they are not exactly measurable, they form a continuous series.

- **Measures of Central Tendency :**

  A certain value that represent the whole data and signifying its characteristics is called measure of central tendency. Mean, Median and Mode are the measure of central tendency.

  Mean

  For Raw or ungrouped data :

  The mean of n observations $x_1, x_2, \ldots\ldots\ldots\ldots, x_n$ is given by

$$\text{Mean } (\bar{x}) = \frac{x_1 + x_2 + \ldots\ldots\ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

  For ungrouped data with frequencies :

  Let $x_1, x_2, \ldots\ldots\ldots\ldots, x_n$ be n observations with respective frequencies $f_1, f_2, \ldots\ldots\ldots\ldots\ldots, f_n$. Then

$$\text{Mean } (\bar{x}) = \frac{\sum_{i=1}^{n} f_i x_i}{\sum f_i}$$

  Median

  For ungrouped data :

i) Let n be the number of observations. First, arrange the data in ascending or descending order.

Case-I : If n is odd, Median = value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

Case-II : If n is even

$$\text{Median} = \frac{Value\ of\ \left(\frac{n}{2}\right)^{th} + Value\ of\ \left(\frac{n}{2}+1\right)^{th}\ observation}{2}$$

ii. 1. If data is discrete, then first arrange the data in ascending or descending order and find cumulative frequency (c.f.).

Now, find $\frac{N}{2}$, where $N=\sum f_i$ =Total frequency.

If $\sum f_i = N$ is even, then,

$$\text{Median} = \frac{Value\ of\ \left(\frac{N}{2}\right)^{th} + Value\ of\ \left(\frac{N}{2}+1\right)^{th}\ observation}{2}$$

If $\sum f_i = N$ is odd, then, Median = value of the $\left(\frac{N+1}{2}\right)^{th}$ observation.

- **For grouped data :**

For continuous data, first arrange the data in ascending or descending order and then find the cumulative frequencies of all classes. Now, find $\frac{N}{2}$, where $N = \sum f_i$. Afterward, find the class interval, whose cumulative frequencey is just greater than or equal to $\frac{N}{2}$ and it is called the median class.

Then, Median = $l + \frac{\left(\frac{N}{2} - c_f\right)}{f_m} \times h$

where, l = lower limit of median class

N = Total frequency.

cf = Cumulative frequency of class preceding the median class.

f_m = Frequency of the median class.

h = Width of the class.

Mode

For ungrouped data : The observation with maximum frequency is called the mode.

For grouped data : First identify the class with maximum frequency. It is called the modal class. Then, calculate mode using the formula given by,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where, l = lower limit of modal class.

f_1 = frequency of the modal class.

f_0 = frequency of the class preceding the modal class.

f_2 = frequency of the class succeeding the modal class.

h = width of the class inverval.

Note : Relation between Mean, Median and Mode is Mode = 3 Median – 2 Mean.

Dispersion :

- Dispersion means scatteredness around the central value.
- There are four measures of dispersion. These are –

i) Range

ii) Quartile Deviation

iii) Mean Deviation

iv) Standard Deviation

Range is the difference between the greatest and least values of the variable.

Quartile deviation : Quartile deviation is based on the difference between first quartile (Q_1) and the third quartile (Q_3) in the frequency distribution and the difference (Q_3–Q_1) is known as the interquartile range. The difference divided by two is known as quartile deviation or semi interquartile range.

$$\therefore \text{Quratile deviation (Q.D)} = \frac{Q_3 - Q_1}{2}$$

Where, $Q_1 = l_1 + \dfrac{\frac{N}{4} - F_1}{f_1} \times h_1$

l_1 = lower bounday of the Q_1 class (i.e., the class in which cum. frequency just greater than $\frac{N}{4}$)

F_1 = cum. Frequency below l_1.

f_1 = freq. of the Q_1-class

h_1 = width of the Q_1-class

Similarly, $Q_3 = l_3 + \left(\dfrac{\frac{3N}{4} - F_3}{f_3}\right) \times h_3$

Mean Deviation (M.D.) :

Mean deviation is the arithmetic mean of the absolute values of deviations about some point (mean or median or mode)

a) For individual observation, we have $\frac{1}{n}\sum_{i=1}^{n}|x_i - a|$, where a=mean, median, mode.

Also, M.D. = $a + h\left\{\frac{1}{n}\sum_{i=1}^{n}|u_i|\right\}$, where, $u_i = \dfrac{x_i - a}{h}$

b) For a discrete frequency distribution, we have

M.D. = $\frac{1}{N}\sum_{i=1}^{n} f_i|x_i - a|$, a = mean, median, mode

Also, M.D. = $a + h\left\{\frac{1}{N}\sum_{i=1}^{n} f_i u_i\right\}$, where, $u_i = \dfrac{x_i - a}{h}$

Coefficient of Mean Deviation :

For comparing two or more series for variability, the corresponding releative measure "Coefficient of mean deviation" is calculated.

$\therefore$ Coefficient of Mean deviation = $\dfrac{\text{Mean Deviation}}{\text{Arithmetic Mean}} = \dfrac{\text{M.D}}{\bar{x}}$

Variance :

The mean of squares of deviations from mean is called the variance and it is denoted by the symbol 'σ^2'.

The variance of 'n' observations $x_1, x_2, \ldots\ldots\ldots\ldots x_n$ is given by $\sigma^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

Significance of deviation :

i) If the deviation is zero, then all observations are equal to mean.

ii) If the deviation is small, this indicates that the observations are close to the mean.

iii) If the deviation is large, there is a high degree of dispersion of the observation from mean.

Standard Deviation (S.D.) :

Standard deviation is the positive square root of variance i.e. $\sqrt{\sigma^2} = \sigma$. It is also known as root mean square deviation.

Variance and standard deviation of ungrouped data:

Variance of n observations $x_1, x_2, \ldots\ldots, x_n$ is given by

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} x_i^2}{n} - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2$$

and S.D.(σ) = $\sqrt{\text{variance}} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$

Also, *variance* = $h^2\left[\frac{1}{n}\sum u_i^2 - \left(\frac{1}{n}\sum u_i\right)^2\right]$ where $u_i = \frac{x_i - a}{h}$

Variance and standard deviation of grouped data :

i) For discrete frequency distribution :

Let the discrete frequencey distribution be $x_1, x_2, \ldots\ldots\ldots, x_n$ and $f_1, f_2, \ldots\ldots\ldots, f_n$. Then by direct method –

$$\textit{Variance}\ (\sigma^2) = \frac{1}{N}\sum_{i=1}^{n}(x_i - \bar{x})^2 \,.\, fi$$

$$= \frac{1}{N}\sum f_i x_i^2 - \left(\frac{\sum f_i x_i}{N}\right)^2$$

and S.D.(σ) = $\sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}$

By shortcut method,

$$\text{Variance}(\sigma^2) = \frac{1}{\mathrm{N}}\sum f_i d_i^2 - \left(\frac{\sum f_i d_i}{\mathrm{N}}\right)^2$$

and S.D.(σ) = $\sqrt{\frac{1}{N}\sum f_i d_i^2 - \left(\frac{\sum f_i d_i}{N}\right)^2}$

where $d_i = x_i - a$, a = assumed mean.

ii) For continuous frequency distribution –

By step-deviation method,

$$\textit{Variance}\ (\sigma^2) = h^2\left[\frac{1}{\mathrm{N}}\sum f_i u_i^2 - \left(\frac{\sum f_i u_i}{\mathrm{N}}\right)^2\right]$$

and S.D.(σ) = $h\sqrt{\frac{1}{\mathrm{N}}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{\sum f_i u_i}{\mathrm{N}}\right)^2}$

Where $u_i = \frac{x_i - a}{h}$, a = assumed mean, h=width of the class.

Note :

(i) The ratio of S.D.(σ) and the AM$(\bar{x})$ is called the co-efficient of standard deviation $\left(\frac{\sigma}{\bar{x}}\right)$.

(ii) Standard deviation of first n natural numbers can be calculated as $\mathrm{SD} = \sqrt{\frac{n^2 - 1}{12}}$

(iii) If each observation is multiplied by a constant 'K', the variance of the resulting observations become 'K^2' times the original variance.

(iv) On adding (or subtracting) a positive number to (or from) each observation of a group does not affect the variance.

(v) In order to compare two or more frequency distributions, we compare their co-efficients of variations. The co-efficient of variation is defined as $C.V = \frac{\sigma}{\bar{x}} \times 100$

(vi) The distribution having greater C.V has more variability around the central value than the distribution having smaller value of the C.V

Exercise – 15

Group - A

Objective type questions : **[1 or 2 marks each]**

1. Multiple choice type questions :

i) The mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is

a) 2 b) 2.57 c) 3 d) 3.75

ii) Following are the marks obtained by 9 students in a mathematics test 50, 69, 20, 33, 53, 39, 40, 65, 59. The mean deviation from the median is

a) 9 b) 10.5 c) 12.67 d) 14.76

iii) The standard deviation of data 6, 5, 9, 13, 12, 8 and 10 is

a) $\frac{52}{7}$ b) $\sqrt{\frac{52}{7}}$ c) $\sqrt{6}$ d) 6

iv) For a frequency distribution, the standard deviation is computed by applying the formula.

a) $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$ b) $\sigma = \sqrt{\left(\frac{\sum fd}{\sum f}\right)^2 - \frac{\sum fd^2}{\sum f}}$

c) $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \frac{\sum fd}{\sum f}}$ d) $\sigma = \sqrt{\left(\frac{\sum fd}{\sum f}\right)^2 - \frac{\sum fd^2}{\sum f}}$

v) If V is the variance and σ is the standard deviation, then

a) $V=\sigma^2$ b) $V = \frac{1}{\sigma^2}$ c) $V^2=\sigma$ d) $V=\frac{1}{\sigma}$

vi) If n=10, $\bar{X} = 12$ and $\sum x_i^2 = 1530$, then the co-efficient of variation is

a) 36% b) 41% c) 25% d) None of these

vii) The mean deviation of the series $a, a+d, a+2d, ..., a+2nd$ from its mean is

a) $\frac{(n+1)d}{2n+1}$ b) $\frac{nd}{2n+1}$ c) $\frac{n(n+1)d}{2n+1}$ d) None of these

viii) The sum of the squares deviations for 10 observations taken from their mean 50 is 250. The co-efficient of variation is

a) 10% b) 40% c) 50% d) None of these.

ix) The sum of 10 items is 12 and the sum of their squares is 18. The standard deviation is

a) $\frac{2}{5}$ b) $\frac{1}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

x) The algebraic sum of the deviation of 20 observations measured from 30 is 2. So, the mean of observations is

a) 30 b) 30.1 c) 30.2 d) 30.3

xi) The cofficient of variation is computed by –

a) $\frac{S.D}{Mean}\times 100$ b) $\frac{S.D}{Mean}$ c) $\frac{Mean}{S.D}\times 100$ d) $\frac{Mean}{S.D}$

xii) If mode of a series exceeds its mean by 12, then mode exceeds the median by

a) 4 b) 8 c) 6 d) 12

xiii) The median and SD of a distribution are 20 and 4 respectively. If each item is increased by 2, the new median and SD are

a) 20, 4 b) 22, 6 c) 22, 4 d) 20, 6

xiv) Range of the data 13, 17, 21, 37, 8, 10 13, 26 is

a) 19 b) 39 c) 29 d) None of these.

xv) If Mean = Median = Mode, then it is

a) Symmetric distribution

b) Asymmetric distribution

c) Both symmetric and asymmetric distribution

d) none of these

xvi) If the mean of first n natural numbers is $\frac{5n}{9}$, then n =

a) 5 b) 4 c) 9 d) 10

xvii) Which one is measure of dispersion method ?

a) Range b) Quartile deviation c) Mean deviation d) all of the above

xviii) Variance is independent of change of

a) Origin only b) Scale only c) Origin & scale both d) none of these.

xix) Let $x_1, x_2, \ldots\ldots\ldots, x_n$ be values taken by a variable X and $y_1, y_2, \ldots\ldots\ldots y_n$ be the values taken by a variable Y such that $y_i = ax_i + b_i$, $i = 1, 2, \ldots\ldots., n$. Then,

a) Var(Y) = a^2Var(X) b) Var(X) = a^2Var(Y)

c) Var(X) = Var(X)+b d) None of these.

xx) If the standard deviation of a variable x is σ, then the standard deviation of variable $\dfrac{ax+b}{c}$ is

a) $a\sigma$ b) $\dfrac{a}{c}\sigma$ c) $\left|\dfrac{a}{c}\right|\sigma$ d) $\dfrac{a\sigma+b}{c}$

xxi) If two variates X and Y are connected by the relation $Y = \dfrac{aX+b}{c}$, where a, b, c are constants such that $ac<0$, then

a) $\sigma_Y = \dfrac{a}{c}\sigma_X$ b) $\sigma_Y = -\dfrac{a}{c}\sigma_X$ c) $\sigma_Y = \dfrac{a}{c}\sigma_X + b$ d) None of these.

xxii) The standard deviation of first ten natural numbers is

a) 5.5 b) 3.87 c) 2.97 d) 2.87

xxiii) The following information relates to a sample of size 60, $\sum x^2 = 18000$, and $\sum x = 960$. Then, the variance is

a) 6.63 b) 16 c) 22 d) 44

xxiv) If the co-efficient of variation of two distribution are 50, 60 and their arithmetic means are 30 and 25 respectively, then the difference of their standard deviation is

a) 0 b) 1 c) 1.5 d) 2.5

xxv) The standard deviation of some temperature data in ^{0}C is 5. If the data were converted into ^{0}F, then the variance would be

a) 81 b) 57 c) 36 d) 25

xxvi) Consider the first 10 positive integers. If we multiply each number by –1 and, then add 1 to each number, the variance of the numbers, so obtain is

a) 8.25 b) 6.5 c) 3.87 d) 2.87

2. Very short answer type questions :

i) Define statistics.

ii) Find the mean of the cubes of first n natural numbers.

iii) The mean of the set of numbers is $\bar{x}$. If each item is decreased by 4, then find the mean of new observations set of observations.

iv) If $x_1, x_2, \ldots\ldots\ldots, x_n$ is the set of n observations whose mean is $\bar{X}$ then find $\sum_{i=1}^{n}(x_i - \bar{x})$.

v) If a variate assumes the values 0,1,2,.......,n wth frequencies ${}^nc_0, {}^nc_1, \ldots\ldots\ldots\ldots {}^nc_n$ then, what is the mean square deviation about the value x=0 ?

vi) The mean of observations $x_1, x_2, \ldots\ldots, x_{10}$ is $\overline{x}$, then find the value of

$$\left[\left(x_1-\bar{x}\right)+\left(x_2-\bar{x}\right)+\left(x_3-\bar{x}\right)+\cdots\cdots\cdots+\left(x_{10}-\bar{x}\right)\right]^2$$

vii) Find the mean of the co-efficient in the expansion of $(1+x)^{30}$.

viii) The mean marks secured by 25 students of a section A of class XI is 47 that of 35 students of section B is 51 and that of 30 students of section C is 53. Find the mean marks of all student of three sections.

ix) Find S.D. of the first $(n+1)$ natural number.

x) If x_1, x_2, x_3, x_4 and x_5 be the observations with mean m and standard deviation s then find the standard deviation of the observations Kx_1, Kx_2, Kx_3, Kx_4 and Kx_5.

Group - B

3. Short answer type questions : (3 marks each)

i) The mean of 200 items is 48 and their standard deviation is 3. Find the sum of items and sum of squares of all items.

ii) The mean and standard deviation of 100 observations were calculated as 40 and 5.1 respectively by a student who took by mistake 50 instead of 40 for one observation. What are the correct mean and standard deviation.

iii) Find the mean deviation about mean for the following data :

6.5, 5, 5.25, 5.25, 5.5, 4.75, 4.5, 6.25, 3, 5, 4, 9, 3, 3.75, 4, 3, 4, 5

iv) Find the mean deviation from the median for the data :

34, 66, 30, 38, 44, 50, 40, 60, 42, 51

v) Find the co-efficient of variation for the following data :

12, 6, 7, 3, 15, 10, 18, 5

vi) The variance of 10 observations is 4. If each observation is multiplied by 3, find the variance of the new data.

vii) For a frequency distribution we have C.V=26.73%, A.M = 39.5, find σ.

viii) For the distribution $\sum(x-5)=3$ and $\sum(x-5)^2=43$, whose total no. of items is 18. Find the mean and the standard deviation.

ix) The variance of n-observations is σ^2. Show that if each observation is multiplied by a, then the variance of the new set of observations is $a^2\sigma^2$.

x) Two sets each of 20 observations, have the same standard deviation 5. The first set has a mean 17 and the second has a mean 22. Determine the standard deviation of the set obtained by combining the given two sets.

Group - C

4. Long answer type questions : [4 or 6 marks each]

i) Find the mean deviation about the mean of the distribution

size	20	21	22	23	24
frequency	6	4	5	1	4

ii) Calculate the mean deviation about the mean of first n natural numbers

iii) Calculate the mean deviation from the median of the following data

Class interval	0-6	6-12	12-18	18-24	24-30
frequency	4	5	3	6	2

iv) The frequency distribution

x	A	2A	3A	4A	5A	6A
f	2	1	1	1	1	1

where, A is a positive integer, has a variance of 160. Determine the value of A.

v) For the frequency distribution

x	2	3	4	5	6	7
f	4	9	16	14	11	6

Find the standard deviation.

vi) Find the mean and variance of the frequency distribution given below

x	$1 \leq x < 3$	$3 \leq x < 5$	$5 \leq x < 7$	$7 \leq x < 10$
f	6	4	5	1

vii) The weights of cofee in 70 jars is shown in the following table

Weight(in/g)	200-201	201-202	202-203	203-204	204-205	205-206
frequency	13	27	18	10	1	1

Determine variance and S.D. of the above distribution.

viii) Determine mean and S.D. of first n terms of an AP whose first term is a and common difference is d.

ix) Following are the marks obtained, out of 100, by two students Ravi and Hashina in 20 tests.

Ravi	Hashina
25	10
50	70
45	50
30	20
70	95
42	55
36	42
48	60
35	48
60	80

Who is more intelligent and who is more consistent ?

x) The mean and S.D. of a group of 100 observations were found to be 20 and 3 respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and S.D. if the incorrect observations were omitted.

xi) For a group of 200 candidates the mean and S.D. were found to be 40 and 15 respectively. Later on it was found that the score 43 was misread as 34. Find the correct mean and correct S.D.

ANSWERS

Group - A

1.	i) b	ii) c	iii) b	iv) a	v) a	vi) c
	vii) c	viii) a	ix) c	x) b	xi) a	xii) b
	xiii) c		xiv) c	xv) a	xvi) c	xvii) d xviii) a
	xix) a	xx) c	xxi) b	xxii) d	xxiii) d	xxiv) a
	xxv) a		xxvi) a			

2. i) Statistics can be defined as the collection, presentation, classification, analysis and interpretation of quantitative data.

ii) $\frac{n(n+1)^2}{4}$ iii) $\bar{x}-4$ iv) 0 v) $\frac{n(n+1)}{4}$ vi) 0 vii) $\frac{2^{30}}{31}$

viii) 153.4 ix) $\sqrt{\frac{n(n+2)}{12}}$ x) Ks

Group - B

3. i) 9600; 4,62,600 ii) Mean = 39.9, σ=5 ii) 1.041 iv) 8.7 v) 51.26%

vi) 36 vii) 10.56 viii) Mean = 5.16, σ= 1.53 x) 5.59

Group - C

i) –1.15 ii) $\frac{n}{4}$ ii) 7 iv) 7 v) 1.38 vi) 4.043

vii) 1.1655, 1.08 viii) Mean=$a+\frac{(n-1)}{2}d$, S.D=$d\sqrt{\frac{n^2-1}{12}}$

ix) Hashina is more consistent and intelligent x) Mean=39.9, S.D.=5

xi) 40.045, 14.995

Chapter - 16

Probability

Important points and Results :

- **Experiment :**

  An investigation which can produce some well defined outcomes, is known as experiment. There are two types of experiments. These are

  i) Deterministic experiment and

  ii) Random experiment.

- **Random experiment :**

  An experiment conducted repeatedly under the identical conditions does not give necessarily the same result everytime, then the experiment is called random experiment.

  e.g. tossing a coin, rolling a die, drawing a card from a well shuffled pack of cards etc.

- **Outcomes :**

  A possible result of a random experiment is called its outcome.

- **Sample space :**

  The set of all outcomes in a random experiment is called sample space and is denoted by S. Each element of a sample space is called a sample point or an event point.

  e.g. When we roll a die, the possible outcomes of this experiment are 1, 2, 3, 4, 5 or 6.

  $\therefore$ The sample space, $S = \{1, 2, 3, 4, 5, 6\}$

- **Event :**

  A subset of the sample space associated with a random experiment is called an event, generally denoted by 'E'.

  e.g. Suppose a die is thrown, then we have the sample space $S = \{1, 2, 3, 4, 5, 6\}$.

  Then, $E = \{2, 3, 5\}$ is an event.

- **Types of events :**

  On the basis of the element in an event, events are classified into the following types.

  i) **Simple event :**

  If an event has only one sample point or an element of the sample space, it is called a simple event.

e.g. Let two coins are tossed, then sample space, $S = \{HH, HT, TH, TT\}$. Then, $A=\{HH\}$, $B=\{TT\}$ are simple events.

ii) **Compound event :**

If an event has more than one sample point of the sample space, then it is called compound event.

e.g. On rolling a die, we have the sample space, $S=\{1, 2, 3, 4, 5, 6\}$. Then, $E=\{2, 4, 6\}$, $F=\{1, 3, 5\}$ are compound events.

iii) **Sure event :**

The event which is certain to occur is said to be the sure event. The whole sample space 'S' is a sure or certain event, since it is a subset of itself.

e.g. On throwing a die, event of getting a natural number less than 7 is a sure event.

iv) **Impossible event :**

The event which has no element is called an impossible event or null event. The empty set 'ϕ' is an impossible event.

e.g. On throwing a die, event of getting a number less than 1, is an impossible event.

v) **Equally likely events :**

Events are said to be equally likely if they have equal chance of happening i.e., we don't have such that one will occur more frequently than others.

vi) **Mutually exclusive events :**

Two events are said to be mutually exclusive, if the occurance of any one of them excludes the occurance of the other event i.e., they cannot occur simultaneously.

Thus, two events E_1 and E_2 are said to be mutually exclusive, if $E_1 \cap E_2 = \phi$.

e.g. In throwing a die, we have the sample space $S = \{1, 2, 3, 4, 5, 6\}$.

Let, $E_1 = \{2, 4, 6\}$, $E_2 = \{1, 3, 5\}$.

Then $E_1 \cap E_2 = \phi$.

So, E_1 and E_2 are mutually exclusive events.

In general, events $E_1, E_2, ..., E_n$ are said to be mutually exclusive, if they are pairwise disjoint, i.e., If $E_i \cap E_j = \phi$, $\forall$ $i \neq j$.

vii) **Exhaustive events :**

Two or more events associated with a random experiment are exhaustive if their union is the sample space i.e., events $E_1, E_2, ..., E_n$ associated with a random experiment with sample space S are exhaustive if $E_1 \cup E_2 \cup .... \cup E_n = S$.

e.g. In a single throw of a die, let us consider the following events.

$S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1, 2\},\ E_2 = \{1, 3, 5\},\ E_3 = \{4, 5, 6\}$

Thus, $E_1 \cup E_2 \cup E_3 = S$. Hence, E_1, E_2, E_3 are exhaustive events.

viii) Mutually exclusive and exhaustive system of events :

Let S be the sample space associated with a random experiment. A set of events E_1, E_2,......,E_n is said to form a set of mutually exclusive and exhaustive system of events if

a) $E_1 \cup E_2 \cup .... \cup E_n = S$

b) $E_i \cap E_j = \phi$ for $i \neq j$.

e.g. Let us consider an experiment of drawing a card from a well-shuffled deck of 52 playing cards.

Let us consider the following events :

E_1 = Card drawn is spade.

E_2 = Card drawn is club.

E_3 = Card drawn is heart.

E_4 = Card drawn is diamond.

Here, E_1, E_2, E_3 and E_4 form a mutually exclusive and exhustive system of events.

ix) **Favourable elementary events** :

Let S be the sample space associated with a random experiment and E be an event associated with the experiment. Then, elementary events belonging to E are knwon as favourable elementary events to the event E.

Thus, an elementary event A is favourable to an event E if the occurance of A ensures the happening or occurance of event E.

- **Algebra of events :**

Let E and F be two events associated with a sample space S, then –

i) **Complementary event :**

For every erent E, there corresponds another event E' or $\bar{E}$ called the complementary event of E, which consists of those outcomes that do not correspond to the occurance of E. E' or $\bar{E}$ is called the complementary event of E.

e.g. In tossing three coins, the sample space is

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

Let, $E = \{THT, TTH, HTT\}$ = the event of getting only one head.

Then, $E' = \{HHT, HTH, THH, TTT, HHH\} = S - E$

ii) **The event E or F :**

The event ' E or F' is same as the event E⋃F and it contains all those element which are either in event E or in F or in both. Thus,

$E \text{ or } F = E \cup F = \{x : x \in E \text{ or } X \in F\}$

iii) **The event E and F :**

The event E and F is same as the event E⋂F and it contains all those elements which are both in E and F. Thus,

$E \text{ and } F = E \cap F = \{x : x \in E \text{ and } x \notin F\}$

iv) **The event E but not F :**

The event E but not F is same as the event E–F or (E ⋂ F') and it contains all those elements which are in E but not in F.

Thus, $E \text{ but not in } F = E - F = \{x : x \in E \text{ and } x \notin F\}$

- **Some events and their corresponding equivalent sets :**

	Events	Equivalent Sets
i)	Not E	$\bar{E}$
ii)	Neither E nor F	$\bar{E} \cap \bar{F}$
iii)	Exactly one of E and F	$(E \cap \bar{F}) \cup (\bar{E} \cap F)$
iv)	At least one of E, F or G	$E \cup F \cup G$
v)	All three of E, F and G	$E \cap F \cap G$
vi)	Exactly two of E, F and G	$(E \cap F \cap \bar{G}) \cup (E \cap \bar{F} \cap G) \cup (\bar{E} \cap F \cap G)$

Note :

i) The complementary events A and $\bar{A}$ are always mutually exclusive.

ii) A sample space is called a discrete sample space, if S is a finite set.

iii) If n(S)=p, then there are 2^p events of a sample space S, where p is the number of elements in S.

iv) Elementary events associated with a random experiments are also known as in decomposable events.

v) All events other than elementary events and impossible events associated with a random experiment are called compound events.

vi) For any event E, associated with a sample space S, E' = not E = S–E = $\{\omega : \omega \in S \text{ and } \omega \notin E\}$

vii) Simple events of a sample sapce are always mutally exclusive.

- **Probability of occurance of an event :**

A numerical value that conveys the chance of occurance of an event, when we perform an experiment, is called the probability of that event. The different approaches of probability are –

i) **Statistical approach to probability :**

In statistical approach, probability of an event 'E' is the ratio of observed frequency to the total frequency.

i.e. $P(E) = \dfrac{\text{Number of observed frequencies}}{\text{Total frequency}}$

ii) **Classical approach to probability :**

According to classical theory of probability

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{n(E)}{n(S)}$$

iii) **Axiomatic approach to probability :**

Let 'S' be a sample space. The probability P can be treated as a real valued function whose domain is the power set of S and range is the interval [0, 1] satisfying the following axioms.

A_1 : For any event E, $P(E) \geq 0$

A_2 : $P(S) = 1$

A_3 : If E & F are mutually exclusive, then $P(E \cup F) = P(E) + P(F)$

Let 'S' be a sample space containing the events $E_1, E_2, \ldots\ldots, E_n$. Then, from the axiomatic approach to probability, we have

i) $0 \leq P(E_i) \leq 1, \quad E_i \subseteq S$

ii) $P(E_1) + P(E_2) + \ldots\ldots + P(E_n) = 1$, if $E_i \cap E_j = \phi$, $i \neq j$ and $\bigcup_{i=1}^{n} E_i = S$

- **Probability of equally likely outcomes :**

The outcomes of a random experiment are said to be equally likely, if the chance of occurance of each outcome is same.

Let $S = \{s_1, s_2, \ldots\ldots\ldots, s_n\}$. Also, let all the outcomes are equally likely, i.e. $P(s_i) = p$, $\forall s_i \in S$, $0 \leq p \leq 1$.

By axiomatic approach to probability,

$$\sum_{i=1}^{n} P(s_i) = 1$$

$$\Rightarrow \quad \frac{\text{P} + \text{P} + \dots\dots\dots + \text{P}}{\text{n-times}} = 1$$

$\Rightarrow$ np = 1

$$\Rightarrow \quad p = \frac{1}{n}$$

$$\therefore \quad P(s_i) = \frac{1}{n}, \text{ i=1, 2,.........,n}$$

- **Addition rule of probability :**

If A and B are two events associated with a random experiment, then–

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Or $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

When A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

When A and B are mutually exclusive and exhaustive events, then $P(A \cup B) = P(A) + P(B) = 1$

For three events A, B and C

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

- **Probability of complementary event :**

Let E be an event and $\bar{E}$ be its complementary event. Then, $P(\bar{E}) = 1 - P(E) \Rightarrow P(E) + P(\bar{E}) = 1$

- **Some results on probability of events :**

i) For any two events A and B, $A \subseteq B \Rightarrow P(A) \leq P(B)$

ii) For an event A, $0 \leq P(A) \leq 1$

iii) If A and B are any two events, then

$P(A \cap B) \leq P(A), \ P(B) \leq P(A \cup B) \leq P(A) + P(B), \ P(A \cap B) \leq P(B)$

iv) If P(A)>P(B), then event A has a higher chance of occuring than event B.

v) If P(A) = P(B), the events A and B are equally likely to occur.

vi) For any two events A and B,

$P(A - B) = P(A) - P(A \cap B)$

$\Rightarrow \quad P(A \cap B') = P(A) - P(A \cap B)$

vii) Probability of an impossible or null event is zero and the probability of a sure or certain event is 1.

viii) In case of equally likely outcomes, axiomatic approach coincide with the classical approach of probability.

ix) If A, B, and C are mutually exclusive events, i.e., $A \cap B = B \cap C = C \cap A = A \cap B \cap C = \phi$, then $P(A \cup B \cup C) = P(A)+P(B)+P(C)$

x) For any two events A and B, P(neither A nor B) = $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

xi) If A and B be two events associated to a random experiment. Then

$$P\left[(A \cap \bar{B}) \cup (\bar{A} \cap B)\right] = P(A)+(B)-2P(A \cap B)$$

xii) For any two events A and B, then $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$

xiii) For any two events A and B, then $P(A) + P(B) - 2P(A \cap B) = P(A \cup B) - P(A \cap B)$

xiv) If odds in favour of A are p:q, then $P(A) = \frac{p}{p+q}$ and $P(\bar{A}) = \frac{q}{p+q}$

xv) If odds against A are p:q, then $P(A) = \frac{q}{p+q}$ and $P(\bar{A}) = \frac{p}{p+q}$

Exercise – 16

Group - A

Objective Type Questions : [1 or 2 marks each]

1. Multiple choice type questions :

i) In a non-leap year, the probability of having 53 Sunday or 53 Monday is

a) $\frac{1}{7}$ b) $\frac{2}{7}$ c) $\frac{3}{7}$ d) none of these

ii) One card is drawn from a pack of 52 cards. The probability that it is the card of a King or Spade is

a) $\frac{1}{26}$ b) $\frac{3}{26}$ c) $\frac{4}{13}$ d) $\frac{3}{13}$

iii) If M and N are any two events, the probability that atleast one of them occurs is

a) $P(M) + P(N) - 2P(M \cap N)$ b) $P(M) + P(N) - P(M \cap N)$

b) $P(M) + P(N) + 2P(M \cap N)$ d) $P(M) + P(N) + P(M \cap N)$

iv) The probability that atleast one of the events A and B occurs is 0.6. If A and B occur simultaneously with probabiliby 0.2, then $P(\bar{A})+P(\bar{B})$ is equal to

a) 0.4 b) 0.8 c) 1.2 d) 1.6

v) Three numbers are choosen from 1 to 20. The probability that they are not consecutive is

a) $\frac{186}{190}$ b) $\frac{187}{190}$ c) $\frac{188}{190}$ d) $\frac{18}{^{20}C_3}$

vi) While shuffling a pack of 52 playing cards, 2 are accidentally dropped. The probability that the missing cards to be of different colours is

a) $\frac{29}{52}$ b) $\frac{1}{2}$ c) $\frac{26}{51}$ d) $\frac{27}{51}$

vii) If seven persons are to be seated in a row. Then, the probability that two particular persons sit next to each other is

a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{2}{7}$ d) $\frac{1}{2}$

viii) If without repetition of the numbers, four-digit numbers are formed with the numbers 0, 2, 3 and 5, then the probability of such a number divisible by 5 is

a) $\frac{1}{5}$ b) $\frac{4}{5}$ c) $\frac{1}{30}$ d) $\frac{5}{9}$

ix) If A and B are mutually exclusive events, then

a) $P(A) \le P(\bar{B})$ b) $P(A) \ge P(\bar{B})$ c) $P(A) < P(\bar{B})$ d) none of these

x) If $P(A \cup B) = P(A \cap B)$ for any two events A and B, then

a) $P(A)=P(B)$ b) $P(A)>P(B)$ c) $P(A)<P(B)$ d) none of these

xi) If 6 boys and 6 girls sit in a row at random, then the probability that all the girls sit together is

a) $\frac{1}{432}$ b) $\frac{12}{431}$ c) $\frac{1}{132}$ d) none of these

xii) If a single letter is selected at random from the word 'PROBABILITY', then the probability that it is a vowel is

a) $\frac{1}{3}$ b) $\frac{4}{11}$ c) $\frac{2}{11}$ d) $\frac{3}{11}$

xiii) If the probabilities for A to fail in an examination is 0.2 and that for B is 0·3, then the probability that either A or B fails is

a) >0.5 b) 0.5 c) ≤0.5 d) 0

xiv) If S is the sample space in which A and B two mutually exclusive events and $P(A)=\frac{1}{3}P(B)$ and S=A∪B, then P(A) is

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) $\frac{3}{8}$

xv) An urn contains 9 balls two of which are red, three blue and four yellow. Three balls are drawn at random. The probability that they are of the same colour is

a) $\frac{5}{84}$ b) $\frac{3}{9}$ c) $\frac{3}{7}$ d) $\frac{7}{17}$

xvi) Out of 30 consecutive integers, 2 are choosen at random. The probability that their sum is odd

a) $\frac{14}{29}$ b) $\frac{16}{29}$ c) $\frac{15}{29}$ d) $\frac{10}{29}$

xvii) A person write 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is

a) $\frac{1}{4}$ b) $\frac{11}{24}$ c) $\frac{15}{24}$ d) $\frac{23}{24}$

xviii) One of the two events must occur. If the chance of one is $\frac{2}{3}$ of the other, then odds in favour of the other are –

a) 1:3 b) 3:1 c) 2:3 d) 3:2

xix) If three dice are thrown simultaneously, then the probability of getting a score of 5 is

a) $5/216$ b) $1/6$ c) $1/36$ d) none of these

xx) A pack of cards contains 4 aces, 4 kings, 4 queens and 4 Jacks. Two cards ard drawn at random. The probability that at least one of them is an ace is

a) $\frac{1}{5}$ b) $\frac{3}{16}$ c) $\frac{9}{20}$ d) $\frac{1}{9}$

xxi) If $\frac{(1-3p)}{2}$, $\frac{(1+4p)}{3}$, $\frac{(1+p)}{6}$ are the probabilities of three mutually exclusive and exhaustive events, then the set of all values of p is

a) (0, 1) b) $\left[-\frac{1}{4},\frac{1}{3}\right]$ c) $\left(0,\frac{1}{3}\right)$ d) $(0, \infty)$

xxii) The probabilities of three mutually exclusive events A, B and C are given by $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ respectively. The statement –

a) is true b) is false c) nothing can be said d) could be either

xxiii) Two dice are thrown together. The probability that neither they show equal digits nor the sum of their digit is 9 will be

a) $\frac{13}{15}$ b) $\frac{13}{18}$ c) $\frac{1}{9}$ d) $\frac{8}{9}$

xxiv) Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there are exactly 2 children in the selection is

a) $\frac{11}{21}$ b) $\frac{9}{21}$ c) $\frac{10}{21}$ d) none of these

xxv) Two dice are thrown together. The probability that at least one will show its digit greater than 3 is

a) $\frac{1}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{1}{8}$

xxvi) Out of (2n+1) tickets consecutively numbered, three are drawn at random, the probability that the numbers on them are in A.P.

a) $\frac{3n}{4n^2-1}$ b) $\frac{2n}{4n^2-1}$ c) $\frac{n}{4n^2-1}$ d) none of these

xxvii) From a set of 40 cards numbered 1 to 40, 5 cards are drawn at random and arranged in ascending order of magnitude $x_1<x_2<x_3<x_4<x_5$. The probability of $x_3=24$, is

a) $\frac{{}^{16}c_2}{{}^{40}c_5}$ b) $\frac{{}^{23}c_2}{{}^{40}c_5}$ c) $\frac{{}^{16}c_2 \times {}^{23}c_2}{{}^{40}c_5}$ d) none of these

xxviii) The probability that in a random arrangement of the letter of the word "FAVOURABLE" the two 'A' do not come together is

a) $\frac{1}{5}$ b) $\frac{1}{10}$ c) $\frac{9}{10}$ d) $\frac{4}{5}$

xxix) A die is thrown three times, the probability of getting a larger number than the previous number each time is

a) $\frac{15}{216}$ b) $\frac{5}{108}$ c) $\frac{13}{216}$ d) none of these

xxx) Two squares are choosen from a chess board. the probability that they are of different colour is

a) $\frac{63}{64}$ b) $\frac{32}{63}$ c) $\frac{23}{64}$ d) none of these

2. Very short answer type questions :

i) A die has two faces each with number '1' three faces each with number '2' and one face with number '3'. If die is rolled once, determine,

a) P (1 or 3) b) P (not 3)

ii) A card is drawn from a pack of 52 cards. Calculate the probability that card is an ace of spade.

iii) Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that both are face cards.

iv) What is the probability that a leap year has 53 Wednesday ?

v) A coin is tossed twice, then find the probability of getting atleast one head.

vi) In a single throw of two dice, what is the probability of getting a total of 8 on the faces of the dice ?

vii) What is the probability that a letter choosen at random from word 'EQUATIONS' is a consonant ?

viii) A card is draw in from a pack of 52 cards. What is the probability that it is a King or Queen?

ix) If E and F are events such that P(E)=$\frac{1}{4}$, P(F)=$\frac{1}{2}$ and P(E and F)=$\frac{1}{8}$, find P(E or F).

x) A and B are two events such that P(A)=$\frac{1}{4}$, P(B)=$\frac{2}{5}$ and P(A$\cup$B)=$\frac{1}{2}$. Find the values of P(A$\cap$B) and P(A$\cap$$B^C$).

xi) A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A), if P(B)=$\frac{3}{2}$ P(A) and 2P(C)=P(B).

xii) A and B are two exhaustive events of an experiment. If $P(\bar{A})=0.25$, P(A $\cap$ B)=0.65 and P(B)=x, find x.

xiii) The odds in favour of occurance of an event are 5:13. Find the probability that it will occur.

xiv) What is the probability that 2 letters chosen at random from the word VALUES are vowels?

xiv) A coin is tossed repeatedly until a head comes for the first time. Describe the sample space.

xvi) A coin is tossed twice. If the second draw results in a head, a die is rolled. Write the sample space for this experiment.

Group - B

3. Short answer type questions : (3 marks each)

i) If a card is drawn from a deck of 52 cards, then find the probability of getting a King or a heart or a red card.

ii) Find the probability that when 7 cards are drawn from the well-shuffled deck of 52 cards, it contains

a) all Kings b) 3 Kings.

iii) A coin whose faces are marked by 3, 4 is tossed 5 times. Determine the probability of getting a total of 24.

iv) A natural number x is chosen at random from 150 natural numbers. Find the probability that $\frac{x^2 - 70x + 1200}{x - 35} < 0$.

v) Three of six vertices of a regular hexagon are chosen at random. What is the probability that the triangle formed by these vertices is equilateral ?

vi) In a bag there are 21 balls marked by number 1, 2, 3,........, 21. Two balls are drawn one by one with replacement. What is the probability that from drawn balls first shows odd number and second shows even number ?

vii) 7 boys and 3 girls are seated in a row randomly. Find the probability that no boy sit between two girls.

viii) If for two events A and B, $P(A \cap B) = \frac{1}{2}$ and $P(A \cup B) = \frac{2}{5}$, then find $P\left(A^C\right) + P\left(B^C\right)$.

ix) Events A, B and C are exhustive events. The odds against A are 8:3 and in favour of B are 2:5. Find the odds in favour of C.

x) An integer is chosen at random from the first 200 positive integers. Find the probability that integer is divisible by 6 or 8.

Group - C

4. Long answer type questions : [4 or 6 marks each]

i) Two students Ankur and Advik appeared in an examination. The probability that Ankur will qualify the examination is 0.05 and that Advik will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that :

a) Both Ankur and Advik will not qualify the examination.

b) At least one of them will not qualify the examination.

c) Only one of them will qualify the examination.

ii) Four persons are chosen at random from a group consisting of 3 men, 2 women and 3 children. Find the probability that out of 4 chosen persons, exactly 2 are children.

iii) Two dice are thrown. The events A, B, C, D, E, F are as follows :

A : getting an even number on the first dice

B : getting an odd number on the first dice

C : getting sum of numbers on two dice ≤ 5

D : getting the sum of numbers on two dice less than 10 and greater than 5.

E : getting the sum of numbers on two dice ≤ 10

F : getting an odd number on one of the dice.

Describe the events –

a) b) A and B c) B or C d) A and E

e) A or F f) A and F

iv) The probabilities that a student will receive A, B, C or D grade are 0.40; 0.35; 0.15 and 0.10 respectively. Find the probability that a student will receive

a) not an A grade b) B or C grade c) atmost C grade

v) A five digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number formed is divisible by 4.

vi) A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that

a) all the three balls are white

b) all the three balls are red

c) one ball is red and two balls are white

vii) Four candidates A, B, C and D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, then what are the probabilities that

a) C will be selected ?

b) A will not be selected ?

viii) A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, complex, routine, simple or very simple are respectively, 0.15, 0.20, 0.31, 0.26 and 0.80. Find the probabilities that a particular surgery will be rated

a) complex or very complex b) neither very complex nor very simple.

c) routine or complex d) routine or simple

ix) If an integer from 1 through 1000 is chosen at random, then find the probability that the integer is a multiple of 2 or a multiple of 9.

x) If the letters of the word 'ALGORITHM' are arranged at random in a row then what is the probability that the letter 'GOR' must remain together as a unit ?

xi) If the letters of the word 'ASSASSINATION' are arranged at random. Find the probability that

a) Four S's come consecutively in the word. b) Two I's and two N's come together.

c) All A's are not coming together. d) No two A's are coming together.

xii) An experiment consists of rolling a die until a 2 appears.

a) How many elements of the sample space correspond to the event that the 2 appears on the K^{th} roll of the die ?

b) How many elements of the sample space correspond to the event that the 2 appears not later than the K^{th} roll of the die ?

ANSWERS

Group - A

1].	i) b	ii) c	iii) b	iv) c	v) b	vi) c
	vii) c	viii) d	ix) a	x) a	xi) c	xii) b
	xiii) c		xiv) a	xv) a	xvi) c	xvii) d xviii) d
	xix) c	xx) c	xxi) b	xxii) b	xxiii) b	xxiv) c
	xxv) b	xxvi) a	xxvii) c	xxviii) d	xxix) d	xxx) b

2]. i) a) $\frac{1}{2}$, b) $\frac{5}{6}$ ii) $\frac{1}{52}$ iii) $\frac{11}{221}$ iv) $\frac{2}{7}$ v) $\frac{3}{4}$ vi) $\frac{5}{36}$

vii) $\frac{4}{9}$ viii) $\frac{2}{13}$ ix) $\frac{5}{8}$ x) $\frac{3}{20}, \frac{1}{10}$ xi) $\frac{4}{13}$ xii) 0.9

xiii) $\frac{5}{18}$ xiv) $\frac{1}{5}$ xv) S = {H, TH, TTH, TTTH, ...}

xvi) {TT, HT, (TH,1), (TH,2), (TH,3), (TH,4), (TH,5), (TH,6), (HH,1), (HH,2), (HH,3), (HH,4) (HH,5), (HH,6)}

Group - B

3]. i) $\frac{7}{13}$ ii) a) $\frac{1}{7735}$ b) $\frac{9}{1547}$ iii) 0 iv) $\frac{11}{50}$ v) $\frac{1}{10}$

vi) $\frac{110}{441}$ vii) $\frac{1}{15}$ viii) $\frac{11}{10}$ ix) 34:43 x) $\frac{1}{4}$

Group - C

4. i) a) 0.87 b) 0.98 c) 0.11 ii) $\frac{3}{7}$

ii) a) Getting odd number on the first dice.

b) Null set

c) {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5, 5), (5,6), (2,1), (2,2), (2,3), (4,1)}

d) {(1,2), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2) (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4)}

e) {(45) (4, 6) (61) (62) (63) (64) (65) (66) (1, 1) (12) (13) (14) (15) (1, 6) (31) (32) (33) (34) (35) (3, 6) (5, 1) (5, 2) (53) (54) (55) (56)}

f) {(21) (2, 3) (25) (41) (4, 3) (45) (61) (6, 3) (6, 5)}

iii) a) 0.6 b) 0.5 c) 0.25

iv) $\frac{1}{5}$

v) a) $\frac{5}{143}$ b) $\frac{28}{143}$ c) $\frac{40}{143}$

vi) a) $\frac{2}{9}$ b) $\frac{5}{9}$

vii) a) 0.35 b) 0.77 c) 0.51 d) 0.57

viii) 0.556

ix) $\frac{1}{72}$

x) a) $\frac{2}{143}$ b) $\frac{2}{143}$ c) $\frac{25}{26}$ d) $\frac{11!}{3!\ 8!}$

xi) a) 5^{k-1} b) $\frac{5^k - 1}{4}$

NOTE

NOTE