ECONOMICS WORK BOOK

CLASS-XI

Azadi Ka Amrit Mahotsav

Ek Tripura Shrestha Tripura

State Council of Educational Research and Training
Govt. of Tripura

Economics Work Book
Class - XI

First Edition
September, 2021

Cover Design
Asoke Deb, Teacher

রতন লাল নাথ
মন্ত্রী
শিক্ষা দপ্তর
ত্রিপুরা সরকার

বার্তা

শিক্ষার প্রকৃত বিকাশের জন্য, শিক্ষাকে যুগোপযোগী করে তোলার জন্য প্রয়োজন শিক্ষাসংক্রান্ত নিরন্তর গবেষণা। প্রয়োজন শিক্ষা সংশ্লিষ্ট সকলকে সময়ের সঙ্গে সঙ্গে প্রশিক্ষিত করা এবং প্রয়োজনীয় শিখন সামগ্রী, পাঠ্যক্রম ও পাঠ্যপুস্তকের বিকাশ সাধন করা। এস সি ই আর টি ত্রিপুরা রাজ্যের শিক্ষার বিকাশে এসব কাজ সুনামের সঙ্গে করে আসছে। শিক্ষার্থীর মানসিক, বৌদ্ধিক ও সামাজিক বিকাশের জন্য এস সি ই আর টি পাঠ্যক্রমকে আরো বিজ্ঞানসম্মত, নান্দনিক এবং কার্যকর করবার কাজ করে চলেছে। করা হচ্ছে সুনির্দিষ্ট পরিকল্পনার অধীনে।

এই পরিকল্পনার আওতায় পাঠ্যক্রম ও পাঠ্যপুস্তকের পাশাপাশি শিশুদের শিখন সক্ষমতা বৃদ্ধির জন্য তৈরি করা হয়েছে ওয়ার্ক বুক বা অনুশীলন পুস্তক। প্রসঙ্গত উল্লেখ্য, ছাত্র-ছাত্রীদের সমস্যার সমাধানকে সহজতর করার লক্ষ্যে এবং তাদের শিখনকে আরো সহজ ও সাবলীল করার জন্য রাজ্য সরকার একটি উদ্যোগ গ্রহণ করেছে, যার নাম 'প্রয়াস'। এই প্রকল্পের অধীনে এস সি ই আর টি এবং জেলা শিক্ষা আধিকারিকরা বিশিষ্ট শিক্ষকদের সহায়তা গ্রহণের মাধ্যমে প্রথম থেকে দ্বাদশ শ্রেণির ছাত্র-ছাত্রীদের জন্য ওয়ার্ক বুকগুলো সুচারুভাবে তৈরি করেছেন। ষষ্ঠ থেকে অষ্টম শ্রেণি পর্যন্ত বিজ্ঞান, গণিত, ইংরেজি, বাংলা ও সমাজবিদ্যার ওয়ার্ক বুক তৈরি হয়েছে। নবম দশম শ্রেণির জন্য হয়েছে গণিত, বিজ্ঞান, সমাজবিদ্যা, ইংরেজি ও বাংলা। একাদশ দ্বাদশ শ্রেণির ছাত্র-ছাত্রীদের জন্য ইংরেজি, বাংলা, হিসাবশাস্ত্র, পদার্থবিদ্যা, রসায়নবিদ্যা, অর্থনীতি এবং গণিত ইত্যাদি বিষয়ের জন্য তৈরি হয়েছে ওয়ার্ক বুক। এইসব ওয়ার্ক বুকের সাহায্যে ছাত্র-ছাত্রীরা জ্ঞানমূলক বিভিন্ন কার্য সম্পাদন করতে পারবে এবং তাদের চিন্তা প্রক্রিয়ার যে স্বাভাবিক ছন্দ রয়েছে, তাকে ব্যবহার করে বিভিন্ন সমস্যার সমাধান করতে পারবে। বাংলা ও ইংরেজি উভয় ভাষায় লিখিত এইসব অনুশীলন পুস্তক ছাত্র-ছাত্রীদের মধ্যে বিনামূল্যে বিতরণ করা হবে।

এই উদ্যোগে সকল শিক্ষার্থী অতিশয় উপকৃত হবে। আমার বিশ্বাস, আমাদের সকলের সক্রিয় এবং নিরলস অংশগ্রহণের মাধ্যমে ত্রিপুরার শিক্ষাজগতে একটি নতুন দিগন্তের উন্মেষ ঘটবে। ব্যক্তিগত ভাবে আমি চাই যথাযথ জ্ঞানের সঙ্গে সঙ্গে শিক্ষার্থীর সামগ্রিক বিকাশ ঘটুক এবং তার আলো রাজ্যের প্রতিটি কোণে ছড়িয়ে পড়ুক।

(রতন লাল নাথ)

Class-XI

ECONOMICS WORK BOOK

CONTRIBUTORS & PROOF CHECKING

01. Chandan Debnath, Teacher
02. Rakesh Ghosh, Teacher
03. Sukanta Saha, Teacher
04. Rajesh Dutta, Teacher
05. Abhijit Saha, Teacher

Class-XI

CONTENT

Part-A : Macro Economics

Part-B : Statistics for Economics

Introduction

You must observed various activities happening around you in your daily life, like- factories, shops, markets, offices, roads etc. All these institutions and organisations may be collectively called an economy. These institutions enable us to earn income and at the same time, help to produce goods and services for the requirement of people.

So an economy is a system which provides people, the means to work & earn a living. To fulfil that object, every economy should undertake the three basis economic activities-

a) Production

b) Consumption

c) Investment or capital formation.

Performance of these activities are based on resources and we know resources are scarce according to its requirement. Our wants or requirements are unlimited & it cannot be fulfilled with the limited resources– it creates economic problem.

1.1 Economic problem:

Economic problem is a problem of choice involving satisfaction of unlimited wants out of limited resources having alternative uses. The main reasons behind economic problems are-

a) **Limited resources:** Resources are limited according to its requirement. So all the wants of every individuals cannot be satisfied at a given point of time. It creates economic problem .

b) **Unlimited wants:** Our wants are not only unlimited but also recurring in nature. With the development of education, technology etc. our wants also increasing and changing day by day. So by using limited resources we cannot satisfy all our wants.

c) **Alternative uses:** Resources also having various alternative uses. For example :- Bamboo can be used to make paper, food items <bamboo shoots>, construction work, for fuel, craft items etc. As a result economy has to make choice between the alternative uses of the given resources, otherwise it will create economic problem.

1.2 Economics-meaning:

Economics is all about making choices in the presence of scarcity. It studies human behaviour as a relationship between resources & human wants.

Economics is the branch of social science which deals with the way a society chooses to use its limited resources, which have alternative uses, to produce goods & services and to distribute them among different groups of people.

1.2.1 Positive and normative economics:

Positive economics Deals with what is or how the economic problems are actually solved. It can be verified with actual data, i.e. based upon facts. It does not give any value judgements. Example: Inflation is rising in India, there is inequalities in distribution of income & wealth in India.

Normative economics deals with what ought to be or how the economic problem should be solved. It cannot be verified with actual data, i.e. not based upon facts and so suggestive in nature. Example: Rising inflation should be controlled in India, Inequalities in income &Wealth distribution should be reduced in India.

1.2.2 Micro & Macro Economics:

The subject matter of economics has been studied under the following branches-

a) **Micro economics:** It is the part of economic theory which studies the behaviour of individual units of an economy . The basic objective of such study is to determine 'price' and the instruments which are used to determine price are 'demand' & 'supply'. It is also called 'price theory'. It deals with partial equilibrium analysis.

   Example : Demand of a consumer, production of a firm.

b) **Macro economics:** It is the part of economic theory which studies the behaviour of aggregates of the economy as a whole. The basic objective of such study is to determine 'Income & employment' level of the economy and the instruments which are used to determine are 'Aggregate demand' & 'Aggregate supply'. It is also called "Income & employment theory". It deals with Aggregate equilibrium analysis.

   Example: National income, money, Banking etc.

1.3 Central problem of an economy:

There are three fundamental and interdependent problems in an economic organisation- 'What', 'How' & 'Whom'- Which are grouped as allocation of resources.

a) **WHAT to produce**: The problem of 'WHAT' is related with what possible commodities to produce & how much to produce by using available resources and production technique.The economy has to decide which consumer good (Rice, cloths etc) and which capital goods (machine etc) are to be produced and in what quantity.Here the decision has to take in such way that provides maximum aggregate satisfaction.

b) **'HOW' to produce:** The problem of "HOW" is related with the choice of production technique to produce required goods and services.Generally techniques are classified as–

1. Labour intensive technique, where more labour & less capital is used.

2. Capital intensive technique,where more capital and less labour is used.

   In the country like India labour intensive method are used due to presence of abundant labour, but country like USA where shortage of labour is their and so capital based techniques are used.

c) **'For WHOM' to produce:**

The problem of "FOR WHOM" is related with distribution because the citizen who are the consumer of the country maybe economically sound or not. The problem can be classified as–

i) Personal distribution:

It means how national income of an economy is distributed among the different groups of people.

ii) Functional distribution:

It involves deciding the share of different factors of production in the total national product of the country.

1.4 Production possibility curve <PPC> :

Production possibility curve (PPC) or production possibility frontier (PPF) is the curve which represents the various production possibilities of two goods that can be produced in the economy with given resources and production techniques.

Assumption of PPC: PPC is based on the following assumption-

a) The economy is producing only two good (good x & y say)

b) The amount of resources are fixed & it can be shifted from one production to another.

c) Resources are fully & efficiently utilised.

d) The level of technology is fixed.

Suppose following are the various production possibilities of two good X & Y :

Production Possibility	good Y (units)	good X (units)
A	7	0
B	6	1
C	4	2
D	0	3

Now if we plot all the production possibilities on a graph paper, taking good x on the X-axis and good y on the Y-axis then a concave shape of curve will be obtained. This curve is called PPC.

good x
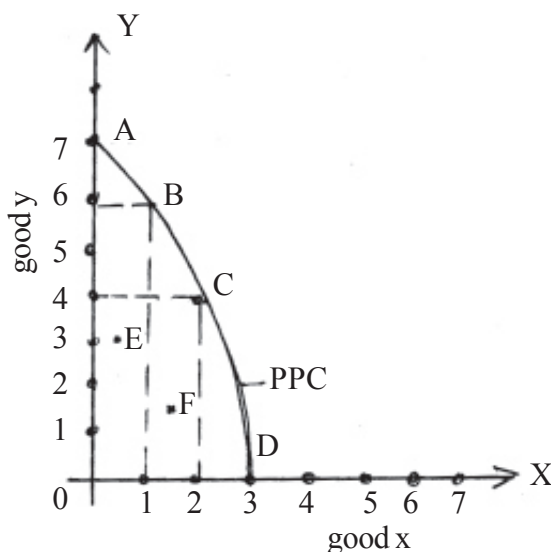
1. The PPC indicates the following points-

a) Points on the PPC <A,B,C& D> means resources are fully utilised.

b) Points inside the PPC <E,F> means resources are not fully utilised or misused or under utilised.

c) Production of both the good cannot be simultaneously increased since resources are fixed.

d) If production of one good is increased the production of another has to be reduced.

1.4.1 Properties of PPC:

a)   PPC is downward sloping from left to right.
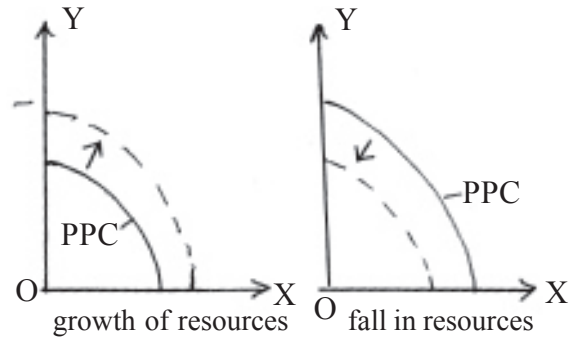
b)   PPC is concave to origin in general <may be convex or straight line>

1.4.2 Shifting of PPC:

PPC shifts due to change in resources. It shifts rightward when resources increase or upgradation of Technology and shifts backward due to fall is resources or degradation of technology.

1.4.3 Rotation of PPC:

PPC rotates when there is a change in productive capacity with respect to one good only.

a)   When there is technology upgradation of good x, then PPC will rotate rightward along X-axis. But when there is degradation of technology or fall in resources of good x the PPC will rotate backward along X-axis.

b)   When there is growth of resources of good y or technological upgradation of good y then PPC will rotate upward along y-axis. But in case of fall in resources or degradation of technology good y then PPC will rotate downward along y-axis.
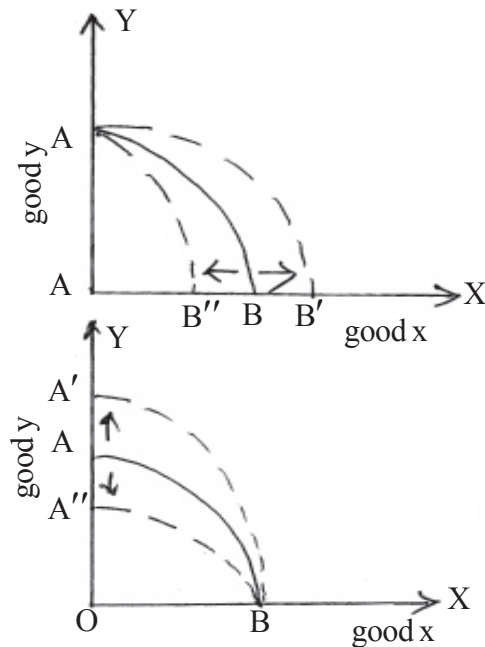
1.5   Opportunity cost:

Opportunity cost means opportunity lost i.e the cost of next best alternative foregone.

For example: Suppose a person received 2 job offers: one for ₹10,000 from a bank and another for R8000 from an insurance company. If he accepts the job of bank of ₹10,000 salary then his opportunity cost is ₹8000 that is offered from insurance company which he sacrificed.

1.6   Marginal opportunity cost (MOC) or Marginal Rate of Transformation (MRT):

MOC or MRT is the rate of sacrifice of a good to gain extra unit of another good

$$MOC/MRT = \frac{\Delta \text{ Unit sacrificed}}{\Delta \text{ Unit gained}}$$

For example : Suppose to increase the production of Rice from 100 to 120kg, a farmer has to reduce his production of wheat from 500 to 400 kg then-

$$\text{MOC} = \frac{\Delta \text{ Loss in production of Wheat}}{\Delta \text{ Gain in production of Rice}} = \frac{500-400}{120-100} = \frac{100}{20} = 5$$

1.6.1 The shape of PPC and MOC or MRT:

a)   Concave PPC: When MOC rises PPC becomes concave. MOC rises because when production of a good is increased continuously the factors producing it becomes less & less productive. So more and more unit of another good has to sacrifice to increase the production of formar good by an extra unit. So, MOC rises & PPC becomes concave.

good y (in uints)	good x (in units)	ΔLoss	Δgain	MOC $\left(\frac{\Delta L}{\Delta G}\right)$
20	1	-	-	-
18	2	2	1	2
12	3	6	1	6
0	4	12	1	12

'Concave PPC'
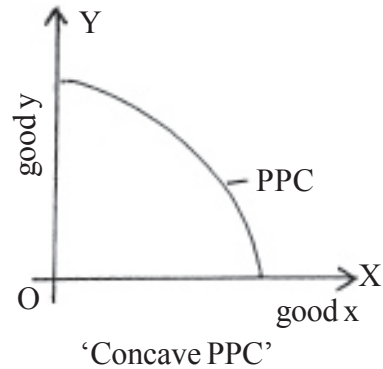
b)   Convex PPC : When MOC falls, PPC becomes Convex

good y (in uints)	good x (in units)	ΔLoss	Δgain	MOC $\left(\frac{\Delta L}{\Delta G}\right)$
20	1	-	-	-
10	2	10	1	10
4	3	6	1	6
0	4	4	1	4

'Concave PPC'   good x
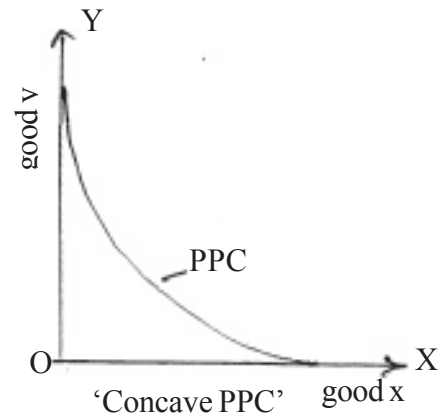
c)   Straight line PPC : When MOC remains constant the PPC becomes straight line

good y (in uints)	good x (in units)	ΔLoss	Δgain	MOC $\left(\frac{\Delta L}{\Delta G}\right)$
20	1	-	-	-
15	2	5	1	5
10	3	5	1	5
5	4	5	1	5

'Straight PPC'

Exercise

True/False type of question:

1. Price theory deals with factor pricing & product pricing.
2. Economic problem arises because resources are unlimited.
3. Shape of PPC is downward sloping.
4. If earthquake takes place, the PPC will shift inward.
5. If PPC shifts toward right, it means advancement of technology.
6. Theory of distribution studies the problem of 'How to produce'.
7. Shape of PPC may be concave or convex.
8. PPC shifts backward if there is large inflow of foreign capital.

Fill in the blanks:

1. _____ is the basic reason for economic problem.
2. PPC Shifts towards right due to _____ of resources.
3. Micro economics studies the economic behaviour of _____ units.
4. All the points on the PPC indicate _____ utilisation of resources.
5. 'How ' to produce deals with choice of _____.

Multiple choice question:

1. PPC can be straight when-
   a) MOC rises b) MOC falls c) MOC constant d) None.
2. The word 'economics' is most closely with the word-
   a) Free b) Scarcity c) unlimited d) Restricted
3. Which of the following will not lead to shift of PPC?
   a) Upgradation of technology b) exploration of new oil reserves c) massive unemployment
   d) Destruction of resources
4. Which of the following is a cause of economic problem?
   a) Scarcity of resources b) unlimited wants c) Alternative uses of resources d) all of the above.
5. Which of the following is related with micro economics?
   a) Inflation b) unemployment problem c) National income d) price of a good
6. PPC is-
   a) Downward sloping b) concave to origin c) both (a) & (b) d) either (a) & (b)

7. Macro economics focuses on all of the following except-

a) Unemployment problem b) Aggregate demand c) inflation in the economy

d) price of a firm product.

8. If PPC shifts to the left, it means-

a) Resources are destroyed b) more unemployment c) uses of outdated technology d) all the above.

9. If production of good x rises by 1 unit & that of good y falls from 10 to 8 units. Then, marginal opportunity cost of x is –

a) 2   b) 10   c) 8   d) 18.

10. Which one of the following is not an assumption of PPC?

a) Resources are fixed   b) production technique is fixed

c) There are more than 2 goods   d) None.

Very short answer type question (1 mark)

1. Define scarcity.

2. What is economising of resources?

3. What do you mean by economic problem?

4. What is economics all about?

5. Define opportunity cost.

6. Give examples when PPC shift towards back.

7. Give examples when PPC shift towards right.

8. Give an example of normative statement.

9. Give an example of positive statement.

10. What does concavity of PPC indicate?

11. What do you mean by alternate use of resources.

12. What does a rightward shift of PPC indicate?

13. What does a leftward shift of PPC indicate?

14. What is the opportunity cost of an input which has not alternative use?

15. Name the two main branches of Economics.

Short answer type question (3/4 marks)

1. Explain the reasons behind economic problem.

2. Write the difference between micro & macro economics.

3. Write the difference between positive & normative economics.

4. Explain the problem of 'WHAT' with examples.

5. Explain the problem of 'How' with examples.

6. Explain the problem 'For WHOM' with examples.

7. Write a short note on production possibility curve.

8. Explain marginal opportunity cost with an example.

9. Why does PPC slope concave to origine?

10. Why does PPP slope downward?

11. Draw a PPC and show the situation of fuller use of resources, under utilisation of resources.

12. Give the various reasons for shifting of PPC towards right and left <with figure>

13. Give the various reasons for rotation of PPC <with figure>.

14. Explain marginal opportunity cost with the help of PPC.

15. Explain the problem 'What' with the help of PPC.

16. Explain the problem 'How' with the help of PPC.

Numerical questions:- (3/4 marks)

1. Calculate Marginal Opportunity Cost (MOC) for good A from the given combination :

good A	0	1	2	3	4	5
good B	15	14	12	9	5	0

2. Find MOC

good A	0	10	20	30	40
good B	200	180	140	80	0

3. Find MOC & based on that explain the shape of PPC

good X	0	1	2	3	4
good Y	10	9	7	4	0

4. Comment on the shape of PPC based on the following schedule:

good X	0	1	2	3	4
good Y	16	12	8	4	0

Answer Key

True/False

      1. True   2. False   3. True   4. True   5. True   6. False   7. True   8. False.

Fill in the blanks:

      1. Scarcity 2. Growth 3. Individual 4. Fuller/efficient 5. Technique.

Multiple choice question :

    1)   c MOC constat   2)   b. scarcity   3) c. massive unemployment

    4)   d. all of the above   5)   d. Price of a good.   6) c. both (a) & (b)

    7)   d. Price of a firm product   8)   d. all of the above   9) a. 2

    10) c. There are more than 2 good.

Short answer type question:

1. Scarcity refers to limitation of supply in relation to the demand of a good or resources.

2. Economising of resources means making the best use of available resources.

3. Economic problem is the problem of choice involving satisfaction of unlimited wants out of limited resources having alternative uses.

4. Economics is all about making choices in the presence of scarcity.

5. Opportunity cost is the cost of next best alternatives.

6. Loss of resources , loss of lives due to earthquake.

7. Technological development, rise in literacy rate.

8. Birth rate should be controlled in India, Government should not provide subsidy to the former.

9. India having 2nd largest population in the world, PCI is low in India as compared to USA.

10. Rising MOC.

11. Resources can be put to more than one use.

12. Growth of resources.

13. Decrease in resources.

14. Zero.

15. Micro & macro economics.

Numerical problems <solution>

1.

good A	good B	ΔLoss	Δ Gain	MOC $\left(\frac{\Delta L}{\Delta G}\right)$
0	15	-	-	-
1	14	1	1	1
2	12	2	1	2
3	9	3	1	3
4	5	4	1	4
5	0	5	1	5

2.

good A	good B	ΔLoss	Δ Gain	MOC $\left(\frac{\Delta L}{\Delta G}\right)$
0	200	-	-	-
10	180	20	10	2
20	140	40	10	4
30	80	60	10	6
40	0	80	10	8

3.

good x	good y	ΔLoss	Δ Gain	MOC $\left(\frac{\Delta L}{\Delta G}\right)$
0	10	-	-	-
1	9	1	1	1
2	7	2	1	2
3	4	3	1	3
4	0	4	1	4

Here the shape of PPC is concave, Since MOC rises.

4.

good x	good y	ΔLoss	Δ Gain	MOC $\left(\frac{\Delta L}{\Delta G}\right)$
0	16	-	-	-
1	12	4	1	4
2	8	4	1	4
3	4	4	1	4
4	0	4	1	4

Here since MOC is constant, So, PPC will be straight line.

Consumer Behaviour & Demand

A consumer is one who consumes/buys goods and services to satisfy his/her wants. He takes decisions with regard to the kind of goods to be purchased in order to satisfy his wants. The main objective of the consumer is to maximise his satisfaction through spending his income on different goods and services. Since consumers income or resources are limited and he has to satisfy his unlimited wants with that limited resources, so to reach his maximum satisfaction level some principles or laws are followed. The two main approaches to study consumer's behaviour and consumer's equilibrium are:

a)   Cardinal approach or utility approach < Marshall analysis >

b)   Ordinal approach or indifference curve approach < Hicksian analysis >.

2.1 Cardinal approach:

In this approach, consumers satisfaction or utility is measured in terms of numerial values by giving numbers.

2.1.1 Utility :

Utility refers to the psychological satisfaction which is obtained from the consumption of a commodity.

The sum total of satisfaction which is obtained from the consumption of all the units of a commodity is called total utility (TU). $TU = \Sigma MU$.   [MU = Marginal Utility]

The extra satisfaction which is obtained from the consumption of an additional unit of a commodity is called marginal utility (MU). $MU = TU_n - TU_{n-1}$

2.1.2 Relation between TU & MU.

Schedule

Qn. of good X (units)	TU	MU
1	10	10
2	17	7
3	19	2
4	19	0
5	15	-4
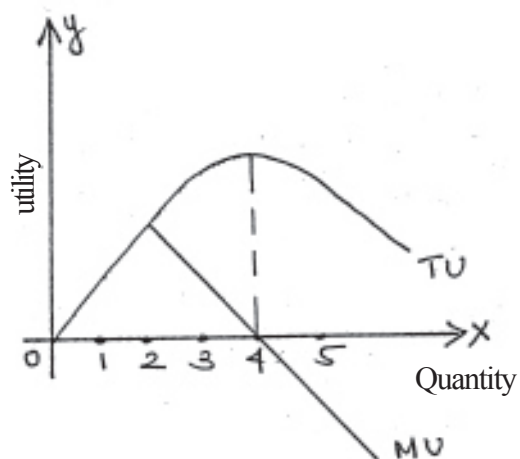
a)   MU is calculated from TU as MU is the change in TU. $MU = TU_n - TU_{n-1}$.

b)   With the increase in the level of consumption, TU Rises in the beginning & MU decreases continuously but remains positive.

c)   When TU is maximum, MU becomes O<This level of Consumption is called saturation point>

d)   When TU starts falling, MU becomes negative.

2.1.3 Law of Diminishing Marginal utility (DMU)

The law of DMU states that with the increase in the continuous consumption of a commodity, MU Obtains from every successive units which tends to diminish.

Assumption of law of DMU-

a)   The consumer is rational.

b)   Consumers income & price of the commodity is fixed.

c)   Utility can be measured cardinally & also in terms of money.

d)   Consumption of every successive units should be in continuous manner.

e)   Every unit of the commodity should be a standard unit and same in quality.

Exception of law of DMU-

a)   The law of DMU Does not apply in case of habits, hobbies etc. of the consumer. Here satisfaction level may increase with every successive unit of consumption like smoking, collection of stamp, gardening etc.

b)   The law of DMU Does not hold in case of miser people. Here MU of such people rises with every successive unit of savings.

2.1.4 Consumer's equilibrium:

Consumer's equilibrium refers to the situation where he can maximise his satisfaction with his given income & the given price of the good & having tendency to change his way of existing expenditure.

Consumer's equilibrium through utility/cardinal approach:

Case-I : single good

In case of consumption of single good, the consumer's equilibrium is based on the law of DMU. The condition of equilibrium in case of single good is-

MUx = Px   where- MUx = MU of good x

$\qquad\qquad$ Px = Price of good x

It means that the consumer reaches in equilibrium where his MU Obtains from the good is equal to the price of the good. It is explained in the following schedule and figure-

Qn. of good X	Px (₹/u)	MUx
1	10	17
2	10	15
3	10	10
4	10	7
5	10	2
6	10	-3
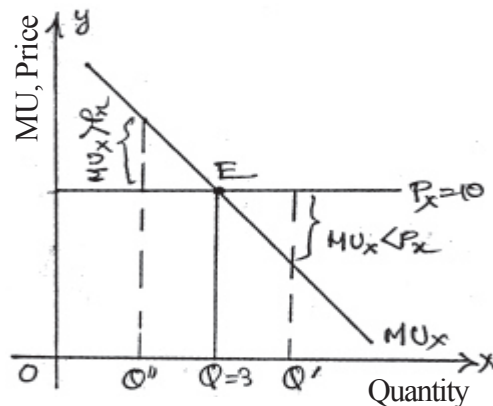
The above schedule & figure shows, E is the equilibrium point where the MUx & Px line intersect each other, i.e. at E, MUx = Px. So he reaches in equilibrium through consuming 3 unit of good x.

If he consumes more than the equilibrium level <OQ′ level say>then MUx<Px. It means he has to increase his MU through reducing the consumption of good x. This process will continue until he reaches in equilibrium again where MUx = Px.

If he consumes less than the equilibrium level (OQ″ level say) then MUx>Px. Which means he has to reduce his MU through increasing the consumption of good x. This process will continue until he reaches in equilibrium again where MUx = Px.

Case-2: Double good:

In case of consumption of two goods, the consumers equilibrium is based on the law of Equi-Marginal utility <EMU>.

The law of EMU states that-

A consumer spends his income on two goods in such a way that the MU Obtains from both the goods should be equal to the price ratio of the two good.

The following conditions are to be satisfied to reach in equilibrium in case to consumption of two goods–

a) $\dfrac{MUx}{Px} = \dfrac{MUy}{Py}$ =MUm – **it is called Necessary condition**

b) X.Px + Y.Py = M – It is called sufficient condition.

→ *good x* →

←*good y* ←

6 5 4 3 2 1

Where–

X=Quantity of good *x*
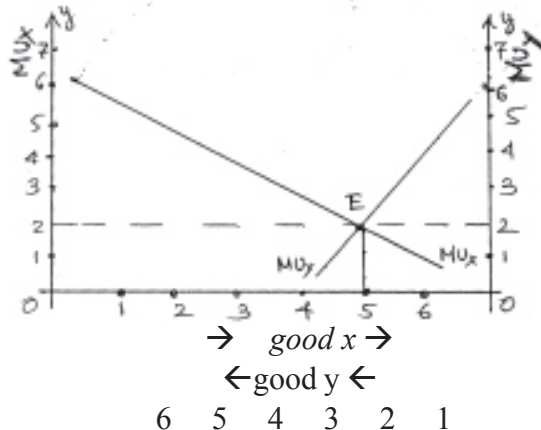
Y=Quantity of good *y*

Px=Price of good *x*

Py=Price of good *y*

MUm=MU of Money

M=consumer's budget

The above figure shows E is the equm point where, the MUx & MUy line intersect each other, i.e. MUx = MUy =2. So the consumer will be in equilibrium through consuming 4 unit of good *x* & 2 unit of good *y*.

Now if $\dfrac{MU_x}{P_x} > \dfrac{MU_y}{P_y}$, then he obtains more satisfactions in good x. So to reach back in equilibrium again, he has to reduce his MUx through consuming more and more units of x. This process will continue until he reaches in equilibrium again where $\dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y}$

Now if, $\dfrac{MU_x}{P_x} < \dfrac{MU_y}{P_y}$, Then he obtains more satisfaction in good y, So to reach back in equilibrium has to reduce MUy through consuming more and more unit of y. This process will continue until he reaches in equilibrium again where $\dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y}$

2.2 Ordinal approach or indifference curve approach:

In this approach utility or satisfaction of consumer is measured by giving "Rank" not by giving numbers.For example if the consumer likes mango more than apple, then he will give 1st rank to mango and 2nd rank to apple.So ordinal utility is the utility expressed in ranks.

2.2.1 Indifference set-

Indifference set refers to the various combination of two commodities among which the consumer is indifferent.

Combination	good 1	good 2
A	1	20
B	2	18
C	3	14
D	4	8

Indifference curve
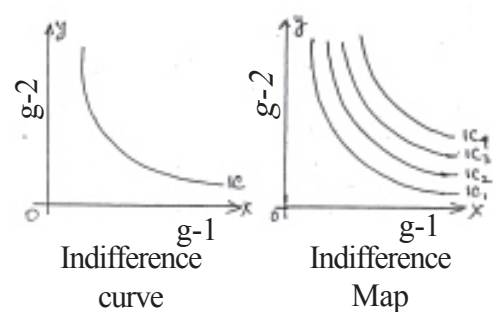
Indifference Map

The graphical presentation of indifference set is called indifference curve. Which shows various combination of two goods providing same level of satisfaction to the consumer.

Properties of indifference curve (IC)-

a)   ICs are downward/negatively sloped.

b)   ICs are convex to origin.

c)   Every point on an IC shows equal level of satisfaction.

d)   Higher IC shows higher level of satisfaction.

e)   Two different IC never touch or intersect each other.

2.2.2 Budget set:

It refers to the different Combination of two goods which can be purchased within the given budget and the given price of the two goods.

Suppose, consumer's budget is 100 and price of 2 goods are $Px = ₹10/Unit$ $\&Py = ₹20ý/Unit$ respectively. Then the budget set is as follows-

Combintion	Qn.of X	Qn.of Y
A	0	5
B	2	4
C	4	3
D	6	2
E	8	1
F	10	0
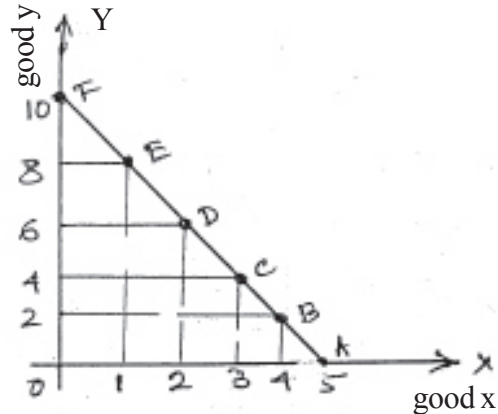
Fig : Budget line

If we plot all the point of budget set on the graph paper, then a downward sloping straight line curve will be obtained which is called budget line.

Properties of budget line:-

a) It is downward/negatively sloped straight line.

b) slope of budget line is the price ratio of two good $\left(\dfrac{Px}{Py}\right)$

c) The equation of budget line- $X.Px + Y.Py \leq M$

d) It shifts due to change in budget or the price of both the good.

e) It rotates due to change in price of a good.

2.2.3 Shifting of budget line-

a) Budget line shifts due to change in total budget or change in price of both the good simultaneously.

b) Budget line shifts towards right due to rise in budget or fall in the price of both the good simultaneously.

c) Budget line shifts towards left due to fall in budget or rise in the price of both the good simultaneously.
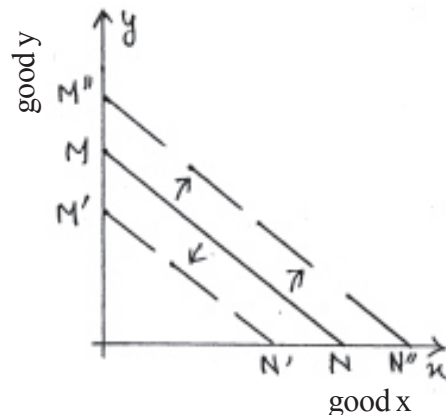
good x

2.2.4 Rotation of budget line-

good x
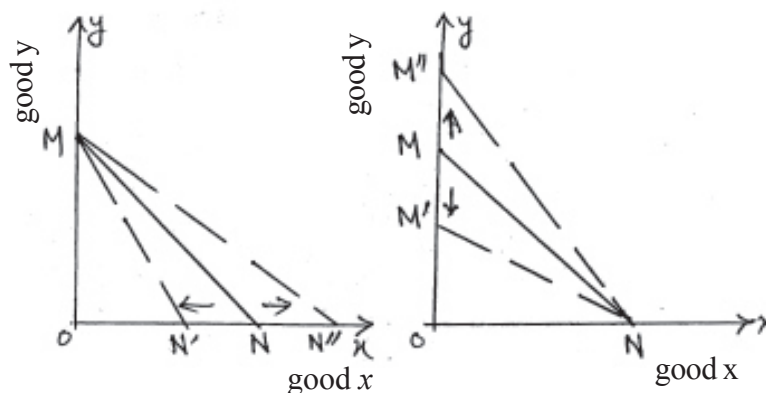
Budget line rotates upward (MN to M″N) or rightward (MN to MN″) due to fall in price of good y or good x respectively.

Budget line rotates downward (MN to M′N) or leftward (MN d to MN′) due to rise in price of good y or good x respectively.

2.2.5 Consumers equilibrium through IC approach or ordinal approach-

According to IC/ Ordinal approach the consumer reaches in equilibrium where his budget line becomes tangent with the IC.

The following conditions are required to satisfy in this method-

a) At the point of equilibrium the slope of IC must be equal to the slope of budget line.

b) at the point of equilibrium IC must be convex to the origin & the budget line becomes tangent with IC.
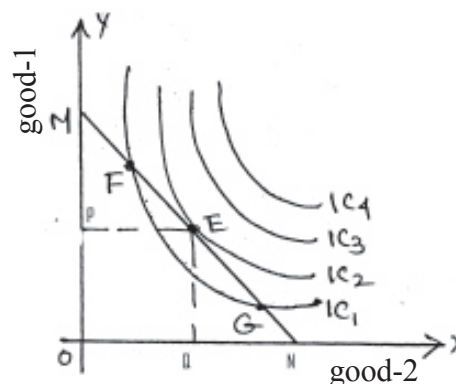
The above figure shows, E is the equilibrium Point where the budget line (MN) becomes tangent with the indifference curve IC_2. Although he can attain any other combination of good 1 & good 2 Which is available on the budget line (Say point E & G). But except E the other points on the budget line lies on lower IC, i.e. lower level of satisfaction. So he will attain equilibrium only at E, Where he consumes OP & OQ quantity of good-1 & good-2 respectively.

- **Why ICs are downward/-very sloped?**

Due to monotonic preference of the consumer, ICs are negatively sloped.

Monotonic preference means a rational consumer always prefers more of at least one good without sacrificing another or prefers more of both the good.

Accordingly to maintain same level of satisfaction, if he want to increase the consumption of a good he has to reduce or sacrifice another good. So ICs are downward or negatively sloping.

- **Why ICs are convex to origin?**

ICs are convex to origin due to diminishing tendency of marginal rate of substitution (MRS). MRS refers to the rate of sacrifice of a good to increase the consumption of another good by an extra unit.
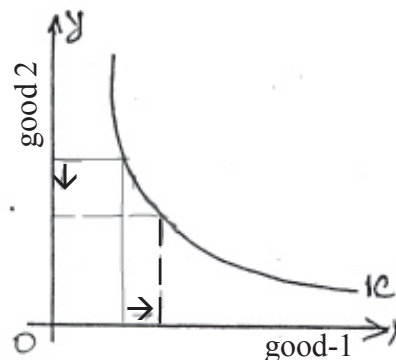
$$MRS = \frac{\Delta \text{ Loss in consumption of good 1}}{\Delta \text{ Gain in consumption of good 2}}$$

Now, MRS diminishes because with the increase in the consumption of a good the consumer is prepared to sacrifice less and less amount of another good. So the ratio between the rate of sacrifice and rate of gain i.e. MRS decreases continuously with the rise in the level of consumption. So, ICs are convex to origin.

- **Show that : two different IC Never touch or intersect**.

Suppose two indifference curve IC_1 & IC_2 Intersect each other at point A.

We know that all the points on an IC shows equal level of satisfaction. So point A & B on IC_1 shows equal satisfaction. So A=B. Similarly point A & C on IC_2 shows equal satisfaction level. So A=C.

So we can write, B = C. But B & C lies on different IC & B<C as C lies on higher IC.
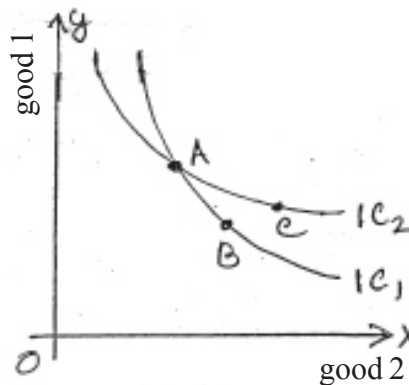
It proves that our assumption was wrong. So we can write two IC never intersect each other.

Difference:-

Cardinal/Utility approach	Ordinal indifference curve approach
a) utility that can be measured numerically by giving numbers is called cardinal utility.	a) utility that cannot be measured numerically by giving numbers but measured in terms of 'Rank' is called ordinal utility.
b) It is measured interms of UTILES.	b) It is measured in terms of RANKS.
c) It follows MU analysis.	c) It follows indifference curve (IC) analysis.
d) It is quantitative measure of utility.	d) It is qualitative measure of utility.

Solved numerical examples:

1. Find marginal utility from the following table:

Quantity of Good x (Unit) :	1	2	3	4	5	6
Total Utility :	20	36	46	50	50	44

Solution :

Quantity of x	Total utility (TU)	Marginal Utility (MU)
1	20	20
2	36	16
3	46	10
4	50	4
5	50	0
6	44	−6

2. Find TU From the following table:

Quontity of x (Units):	1	2	3	4	5	6
MU of x :	10	7	5	2	0	−3

Solution :

Quantity of x	MUx	TUx [= Σ MU]
1	10	10
2	7	17
3	5	22
4	2	24
5	0	24
6	−3	21

3. Find consumers equilibrium level of consumption from the following table, [if price of x = ₹6/unit and utility is expressed in terms of utils & 1 util = ₹1]

Quantity of x :	1	2	3	4	5	6
TUx :	10	18	25	31	34	34

Solution :

Quantity of x	TUx	MUx
1	10	10
2	18	8
3	25	7
4	31	6
5	34	3
6	34	0

Here, at 4 unit of consumption of x, his MUx = Price of x per unit (Px) = 6

So, he will be at equilibrium through consuming 6 unit of x.

4. If price of two good x & y both are ₹1/unit and consumer's budget is ₹8, find his equilibrium level from the following table.

Quantity :	1	2	3	4	5	6	7	8
MUx	11	10	9	8	7	6	5	4
MUy	19	17	15	13	12	10	8	6

Solution :

Quantity	MUx	MUy	$\dfrac{MUx}{Px}$	$\dfrac{MUy}{Py}$
1	11	19	11	19
2	10	17	10	17
3	9	15	9	15
4	8	13	7	13
5	7	12	6	12
6	6	10	6	10
7	5	8	5	8
8	4	6	4	6

Now, if he consumes 2unit of x & 6unit of y then

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y} = 10$$

and X.Px + Y.Py = 2×1 + 6×1 = ₹ 8 = Budget

i.e. both the condition of equilibrium are satisfied at the level and so at equilibrium he will buy 2 unit of x & 6 unit of y.

2.3 Demand:-

Dimand refers to the various quantity of a good which a consumer is willing to consume at different possible prices of that good during a given period of time and at a particular place.

2.3.1 Individual demand:

The demand of a good by an individual consumer at a given price & at a given period of time, is called Individual demand.

The tabular presentation of the various demand of a good by an individual consumer at different possible prices is called individual demand schedule. The graphical presentation of individual demand schedule is called individual demand curve.

Individual demand schedule :

Price of X (₹/U)	Qn. demanded of X (units)
10	50
12	30
14	20
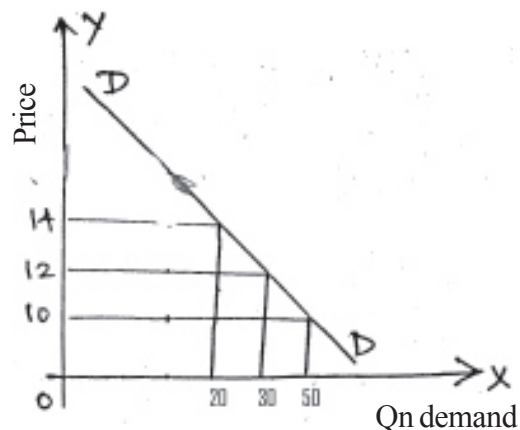
2.3.2 Market demand:

The total quantity demanded of a good which all the consumer of the market are willing and able to consume at different possible prices of that good during a given period of time is called market demand .

The tabular presentation of different quantity demanded of a good by all the consumer in the market at different prices is called market demand schedule. The graphical presentation of market demand schedule is called market demand curve.

Market demand Schedule

Price of X (₹/U)	Qn d^{d}ed by A (units)	Qn d^{d}ed by A (Units)	Market d^d (Units)
10	100	40	40
12	70	30	100
14	50	10	60

A's demand   B's demand   Market demand curve
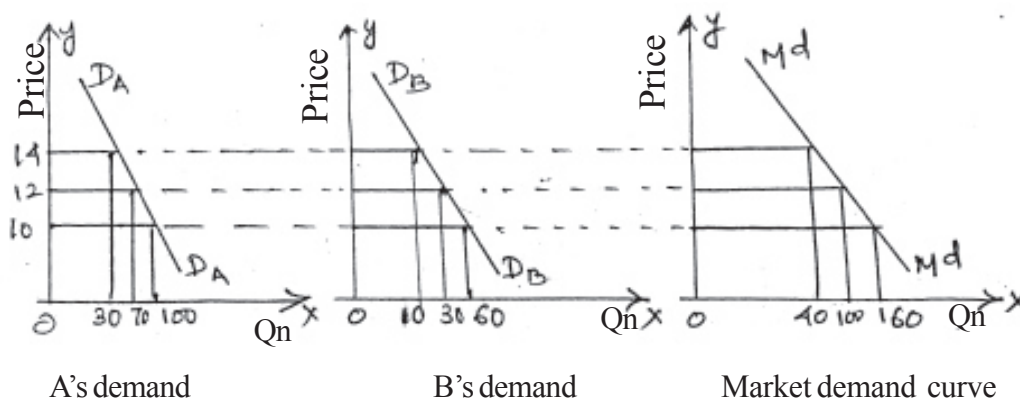
2.4. Demand function:

The functional relationship between the demand of a good and its different factors or determinants are called demand function. Symbolically-

$Dx = f(Px, Pr, Pe, Y, T, Dn, N)$

Where Dx = demand for good x

Px = price of x

Pr = Price of related good of x

Pe = Expectation of price change in near future.

Y = Consumers income

T = Taste, choice etc. of consumer

Dn = Distribution of National Income

N = No. of consumer.

2.5 Different types of commodities-

a) Normal good- Demand for such good rises with the fall in price.

b) Inferior good- Demand for such good falls with the rise in income. Ex:- Coarse grains like Jower, maize, Bazra.

c) Giffen good- Are those inferior good where demand falls with the fall in price of such good. Example rotten fruits, vegetables etc.

d) Luxurious good- Demand for such good rises with the rise in income. Ex: Car, TV etc.

e) Vablen good- Are those luxurious goods where demand rises with the rise in price of such good. Ex: Diamond, costly jewellery.

f) Essential good- Demand for such good does not affect with the change in price and income. Ex: salt, medicine.

g) Related good- If two or more good are related with each other, i.e. Demand for a good is affected with changing price of another.-It is of the following two type-

i) Complimentary good- When two or more good are demanded simultaneously. Ex: Car & petrol.

ii) Substitute good- When one good can be consumed instead of another. Ex: Tea & Coffee.

2.6 Law of demand:

The law of demand states that when the other factors remaining the same the quantity demanded of a good increases with the fall in its price and the quantity demanded decreases with the rise in its price.

Assumption:-

a) Consumers income remains fixed

b) Price of related good remains fixed

c) Consumers taste, choice etc. remains fixed

d) There is no expectation of price change in near future

e) No. of consumer remains same

Exception:

a) The law of demand does not hold in case of Giffen in good < rotten fruits, vegetable etc >. As demand for such good decreases with the fall in price.

b) The law of demand is not applicable in case of Vablen good < like diamond etc>, where demand for such good rises with the rise in price.

c) The law of demand does not hold in case of essential good, as demand for such good cannot be avoided with the change in price.

2.7 Factors affecting the demand for a good-

A) Price of the good:

In general demand for a good is inversely related with its price. i.e. with the rise in the price of a good, it's demand falls and vice versa. But in case of Giffen good <Rotten food items>demand falls with the price fall & in case of veblen good<Diamond>, demand rises with the price rise of that good. Where as in case of essential goods <medicine>demand does not change with the price change.

Normal good   Vablen good   Giffen good
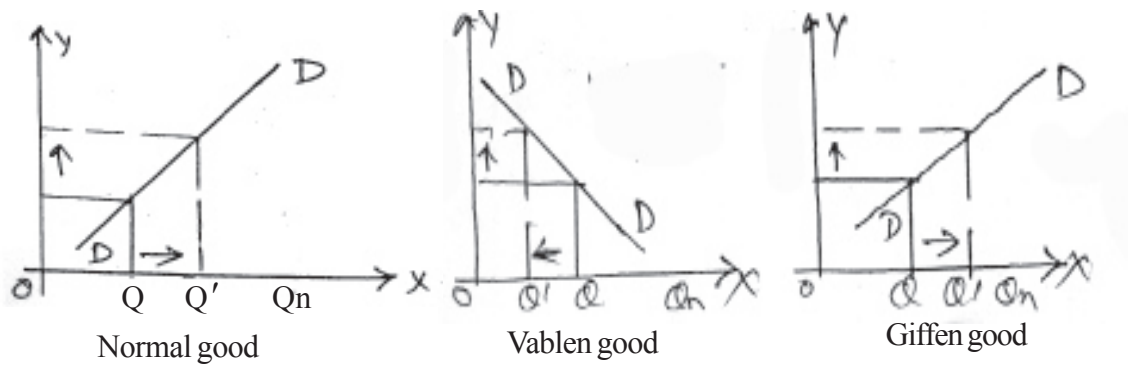
B) Price of other related good:-

Related goods are generally two types-

- In case of complimentary goods with the rise in the price of a good, the demand for its complimentary good falls and vice versa. Ex: Car & Petrol.

- In case of substitute good, with the rise in the price of a good, demand for its substitute goods also rises and vice versa. Ex: Tea & Coffee.
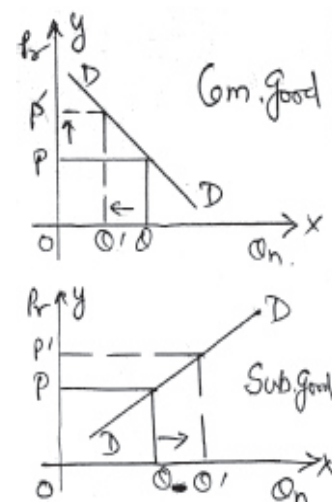
C) Expectation of price change:

The demand for a good is affected directly with the expectation of change in price of a good in near future.If there is expectation of price rise in near future then the present demand rises and vice versa.

D) Consumers income:

Demand for a normal good is directly related with the income of the consumer in general. But demand for inferior good <Jowar, Bazra> falls with the rise in income and demand for luxurious good rises with the rise in income <car>, where as income does not affect the demand for essential good <medicine> as its demand cannot be avoided in case of emergencey, and requirement
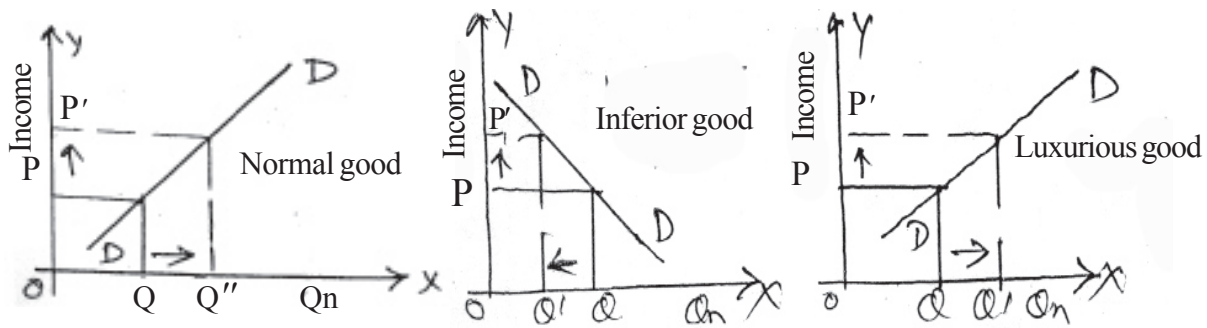
E) Consumer's habit, taste etc:-

The demand for a good is affected by the taste, choice, habit, preferences, likes & dislikes of a consumer. Example- if a consumer prefers a particular band most or if he is habituated with a particular product , then his demand may not change even with the change in price or income and demand may increase even with the price rise.

F) Number of consumer:-

The demand for good is directly related with the number of consumer in the market. The demand will increase with the rise in number of consumer & vice versa. Demand also depends upon composition of Population i.e. Male-female ratio, children senior citizen ratio etc. If there is more of children then demand for child related product will rise.
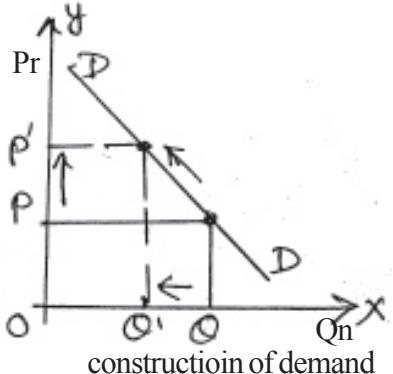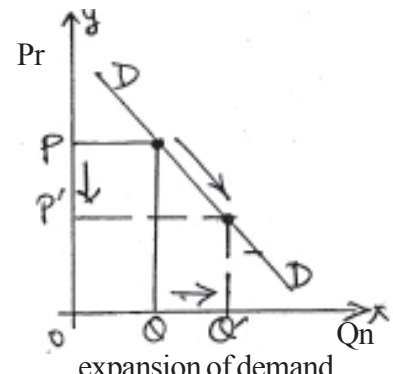
G) Distribution of National Income:

The market demand of a good is affected by the distribution of National Income. If the distribution of National Income is even then the gap between rich & poor will fall and then the market demand will rise. But in case of uneven distribution the number of poor will rise & market demand will fall.

2.8 Change in quantity demanded-
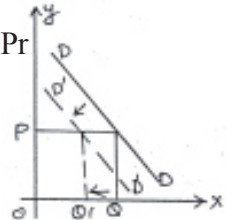
If the demand of a good change due to change in its price,other factors remaining the same, it is called change in quantity demanded.

Then there is movement along the demand curve- upward or downward.

Upward movement along the demand curve (contraction of demand)	Downward movement along the demand curve (Expansion of demand)
1) If the demand for good falls due to rise in its price, other factors remaining the same, it is called contraction of demand. 2) Then there is upward movement along the demand curve 3) The reason behind it is price rise.	1) If the demand for a good rises due to fall in its price, other factors remaining the same, it is called expansion of demand. 2) Then there is downward movement along the demand curve. 3) The reason behind it is price fall.
 constructioin of demand	 expansion of demand

2.9 Change in demand-

When the demand of a good changes due to change in its factors other than price, it is called change in demand. Then there is shifting of demand curve- towards right or left.

Rightword shifting of demand curve (increase in demand)	Leftward shifting of demand curve (decreas in demand)
a) When the demand of a good rises due to change in its factors other than price, it is called increase in demand. b) Then the demand curve shifts rightward	a) When the demand of a good falls due to change in its factors other than price it is called decrease in demand. b) Then the demand curve shifts leftward.

Price (R/U)	Qn. Units
10	100
10	130

increase in demand

Price (R/U)	Qn. Units
10	100
10	70

decrease in demand

c) The reasons behind it are-	c) The reasons behind it are-
i) Expectation of price rise	i) Expectation of price fall
ii) Rise in income	ii) Fall in income
iii) Rise in price of substitute good & fall in the price of complementary good.	iii) Fall in price of substitute good & rise in price of complementary good.
iv) Change in consumer's choice, habit etc in favour of the good.	iv) Change in consumer's choice, habit etc. against the good.
v) Rise in number of consumer in the market.	v) Fall in number of consumer in the market.

2.10. Reasons behind downward or, negatively sloped demand curve-

A) The law of DMU:-

According to DMU, in case of Single good consumption, the consumers equilibrium condition is $MUx = Px$. <wherePx = Price of goodx, MUx = MU obtains from goodx>. Now if Price (Px) falls then.

$MUx > Px$, It means the consumer has to reduce his MUx to reach back in equilibrium and it can be possible when he increases the consumption of good x. So with the fall in price demand rises and demand curve is downward sloping.

B) Income effect:

When the price of a good falls, the real income or purchasing power of the consumer rises. So, With the same monetary income he can buy more quantity of that good.Or, after consuming same quantity as earlier at less price, he can save some part of his income and can purchase some extra units through using that saved income.So, demand rises with the fall in price & demand curve is negatively sloped.

C) Substitution effect:

When the price of a good falls, it becomes cheaper compared to its other substitute goods.So the earlier consumers of that good can buy more of it and consumers of other substitute goods also may shift themself towards that cheaper good, So demand of that good rises with the fall in its price.

2.11 Price Elasticity of Demand (ED):

It is the responsiveness or reaction of the demand of a good with the change in its price.In other words we can say, price elasticity of demand is the ratio between the percentage or proportionate change in quantity demanded with the percentage or proportionate change in price.

It can be measured in the following two methods-

a) Percentage method:

In this method elasticity is measured from the ratio between percentage change in quantity demanded with percentage change in price.

$$\text{Price elasticity of demand} = \frac{\text{Percentage change in Quantity demanded}}{\text{Percentage change in Price}}$$

$$= \frac{\dfrac{\Delta Q}{Q} \times 100\%}{\dfrac{\Delta P}{P} \times 100\%}$$

Where, P=initial Price
Q=initial Quantity demanded
ΔP=change in Price
ΔQ= change in Quantity demanded

b) Proportionate method:

In this method elasticity is measured from the ratio between the proportionate change in quantity demanded with the proportionate change in price.

$$\text{Price elasticity of demand} = \frac{\text{Proportionate change in quantity demanded}}{\text{Proportionate change in price}}$$

$$= \frac{\dfrac{\Delta Q}{Q}}{\dfrac{\Delta P}{P}} = \frac{\Delta Q}{Q} \times \frac{P}{\Delta P} = \frac{\Delta Q}{Q} \times \frac{P}{\Delta P}$$

2.12 Degrees of price electricity of demand:

The value of price elasticity demand lies between zero (0) & Infinite (α). Based on that range, there are five degrees of price electricity of demand–

a) Perfectly inelastic demand ($e^d=0$) :

If the responsiveness of the demand of a good becomes zero with the price change it is called perfectly inelastic demand. Then the demand curve becomes parallel to Y-axis. Ex: salt, medicine etc.

Price (₹/U)	Qn. demanded (Units)
20% $\begin{cases} 10 \\ 12 \end{cases}$	$\begin{rcases} 100 \\ 130 \end{rcases}$ 0%
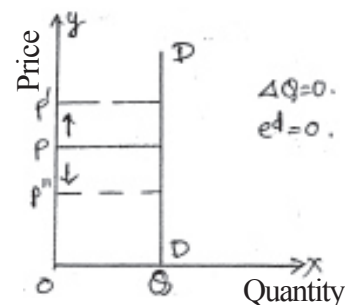
b) Inelastic demand ($e^d > 0$):

If the responsiveness of the demand of a good becomes less than the change in its price, it is called inelastic demand. Then the slope of demand curve lies between 45° to 90°.

Price (₹/U)	Qn. demanded (Units)
50% $\begin{cases} 10 \\ 15 \end{cases}$	$\begin{rcases} 100 \\ 90 \end{rcases}$ 10%
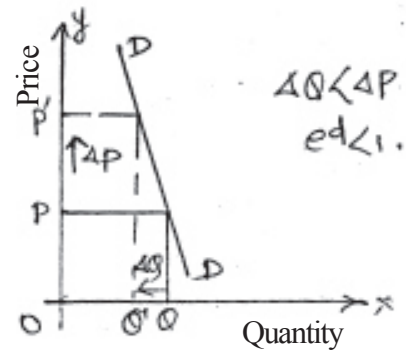
c) Unit elastic demand ($e^d = 1$):

If the responsiveness of the demand of a good becomes same as the change in its price, it is called unit elasticity of demand. Then the demand curve's slope becomes 45° with x-axis.

Price (₹/U)	Qn. demanded (Units)
50% $\begin{cases} 10 \\ 15 \end{cases}$	$\begin{rcases} 100 \\ 50 \end{rcases}$ 50%
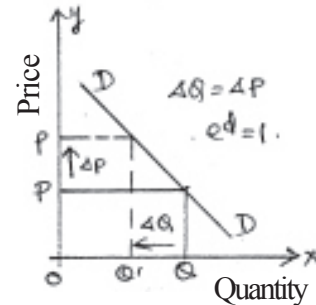
d) Elastic demand (ed>1):

If the responsiveness of the demand of a good becomes more than the change in its price, it is called elastic demand. Then the demand curve's slope becomes 0° to 45° with x-axis. Example: Luxurious goods like freeze, TV etc.

Price (R/U)	Qn. demanded (Units)
20% $\begin{cases} 10 \\ 12 \end{cases}$	$\begin{rcases} 100 \\ 60 \end{rcases}$ 40%
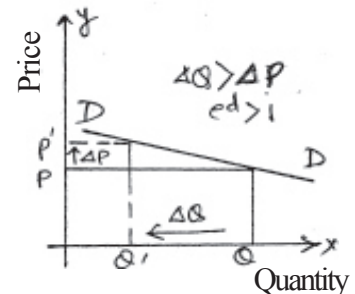
e) Perfectly elastic demand (ed = α):

If the demand of a good changes irrespective of change in its price it is called perfectly elastic demand. Then the demand curve becomes parallel to x-axis. It is an imaginary concept. Slope of demand curve becomes O^o with x-axis.

Price (₹/U)	Qn. demanded (Units)
0% {10 / 10	100 / 80 } 20%
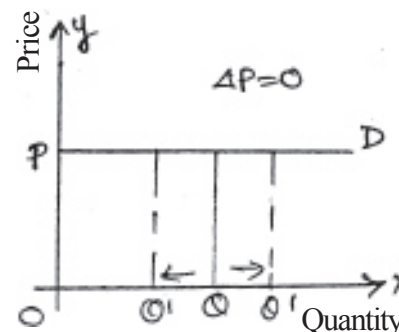
2.13 Factors affecting price elasticity of demand:

Following are the various factors which affect the price electricity of demand for a good.

a) Availability of substitutes:

More is the availability of substitutes for a good, more is the price elasticity of demand and vice- versa. Because when more substitutes are available then consumers can easily react with the price change. Example: Demand for sugar is more elastic than salt as salt having no substitute.

b) Variety of uses:

More is the variety of uses of a good, more is the price electricity of demand and vice-versa. Example: demand for electricity is more elastic than newspaper, as newspaper having very limited uses.

c) Time period:

Longer the time period available with the consumer, more is the price elasticity of demand and vice-versa. Because during long period, consumer can easily adjust his demand with the change in price, which is not possible in case of short period.

d) Amount of income spent:

More is the amount of income spent on a good, more is the price elasticity of demand and vice-versa. Example: Demand for a car is more elastic than demand for a pen, because more amount is spent on purchase of car and so price reaction i.e. elasticity is also high.

e) Nature of the good:

Generally demand for luxuries is more elastic than necessities. Become demand for necessities (medicine etc) can't be avoided even at higher price, but demand for luxuries (car etc) can be avoided with high prices.

Solved numerical questions on price elasticity of demand (e^d)

1. If due to 10% rise in the price of a good it's demand falls by 20% , find price e^d.

Solution : Price e^d (by percentage Method) $= \dfrac{\text{Percentage change in demand}}{\text{Percentage change in price}}$

$$= \frac{20\%}{10\%} = 2 \text{ [elastic demand, } \because e^d>1]$$

2. The demand of a good rises from from 100 to 120 units due to fall in its price from ₹10/unit to ₹8/unit. Find price e^d.

Solution: Here Initial Price(P) = ₹10/U. Change in Price (ΔP)=10–8=2

Initial Quantity (Q)=100U. Change in Quantity (ΔQ)=100–120=(–) 20.

$$\therefore \text{Price } e^d= \frac{\Delta Q}{\Delta P}.\frac{P}{Q} = \frac{-20}{2}\times\frac{10}{100} = (-)\ 1 \text{ [Unit } e^d, \text{ as } e^d=1]$$

3. Due to 40% fall in price of a good its demand rises from 500 to 600 units . Find price e^d.

Solution: Here, Q=500 & ΔQ=500–600= –100

$$\therefore \text{Percentage change in Quantity} = \frac{\Delta Q}{Q}\times100\% = \frac{(-)100}{500}\times100\% = (-)20\%$$

$$\therefore \text{Price } e^d = \frac{\text{Percentage change in Quantity}}{\text{Percentage change in Price}} = \frac{(-)20\%}{40\%} = (-)\frac{1}{2}$$

$$=(-)\ 0.5 \text{ [Inelastic demand, } \therefore\ e^d<1]$$

4. Find the demand for a good if its price rises from ₹10/unit to ₹12/unit and initial demand was 400 unit. [given price e^d = (-) 2]

Solution: here P=10 $\therefore \Delta$P=10–12=(–)2

Given, e^d=(–)2

or, $\dfrac{\Delta Q}{\Delta P}.\dfrac{P}{Q} = (-)2$

or, $\dfrac{\Delta Q}{(-)2}\times\dfrac{10}{400} = (-)2 \therefore \Delta Q=160$

$\therefore$ New demand=400–160=240 units. (Here demand falls due to price rise]

5. The demand for a good rises from 1000 to 1200 units due to fall in price by ₹10/unit. If the initial price was ₹40/unit, find price e^d.

Solution : Here, P=40, ΔP=10

Q=1000, ΔQ=1000–1200=(–)200

$$\therefore e^d = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \frac{(-)200}{10} \times \frac{40}{1000} = (-)0.8 \quad \text{(Inelastic demand)}$$

EXERCISE

True/False type question:

1. Different points on an indifference curve show different level of satisfaction.
2. Marginal rate of substitution (MRS) indicates the slope of budget line.
3. MRS remains same along the indifference curve.
4. Slope of indifference curve is different at different points on the curve.
5. Only one indifference curve will pass through a given point on an indifference map.
6. When total utility is maximum, marginal utility $= 0$
7. If $\dfrac{MU_x}{P_x} > \dfrac{MU_y}{P_y}$, then the consumer will buy more unit of Y and less unit of X to reach in equilibrium.
8. Marginal utility may be zero or negative.
9. In case of two good, MU of a good must fall to attain consumers equilibrium.
10. Market demand curve is flatter than individual demand curve.
11. Expansion of demand leads to an upward movement along the same demand curve.
12. In case of Giffen good, demand curve slopes upwards.
13. Size & composition of population affect the demand of an individual.
14. Demand of a good always rises with the rise in price of other good.
15. Demand of all types of good rises with the rise in income of consumer.
16. Goods having more close substitute show high elasticity of demand.
17. Goods having diverse uses show less elasticity of demand.
18. Demand is more elastic in long period than in the short.
19. Price e^d is infinite in case of horizontal demand curve.
20. If 20% rise in price of a good leads to 20% fall in its demand then price $e^d = 0$.

Multiple choice question:

1. Indifference curves are convex to origin due to-
   a) Rising MRS b) diminishing MRS c) Law of diminishing marginal utility d) none.
2. In case of 2 good, the necessary condition for consumers equilibrium-

   a) $\dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y}$ b) $MRSxy = \dfrac{Px}{Py}$ c) $MUx = Px$ d) None

3. Indifference map refers to-
   a) Highest indifference curve b) lowest indifference curve c) Family of indifference curves
   d) None of these

4. Condition of consumer's equilibrium by indifference curve analysis-

   a) $MUx = Px$ b) $\dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y}$ c) $MRSxy = \dfrac{Px}{Py}$ d) $MUx = MUy$

5. In case of single good consumers equilibrium is achieved when-
   a) $MUx > Px$ b) $MUx < Px$ c) $MUx \neq Px$ d) $MUx = Px$

6. Utility differs from-
   a) Person to person   b) Time to time   c) Product to product   d) All of the above

7. At the point of satiety-
   a) MU is negative   b) MU is Zero   c) MU is rising   d) none.

8. With the rise in income of consumer, demand for normal good-
   a) Rises   b) Falls   c) Remains same   d) none.

9. Law of demand states the _____ relation between the price of a good and its demand-
   a) Inverse   b) positive   c) proportionate   d) none of these.

10. If more demand at same price, this fact of demand is called-
    a) Extension of demand b) Increase in demand c) Contraction of demand
    d) Decrease in demand.

11. If price of A affects the demand of B, than A & B are –
    a) Complimentary good b) Substitute good c) both (a) & (b) d) either (a) or (b).

12. The slope of demand curve is generally-
    a) Negative b) positive c) constant d) either (a) or (b).

13. Which of the following is the factor of market demand-
    a) Income of buyer b) Consumers habit c) Price of related good d) None

14. If the percentage change in demand is less than percentage change in price, then-
    a) $e^d > 1$ b) $e^d = 1$ c) $e^d < 1$ d) $e^d = 0$

15. Which of the following influence the price e^d-
    a) Nature of the good b) Variety of uses of the good c) Availability of substitutes
    d) All of the above

16. The demand for a good made for rich consumers is generally-
    a) Less elastic b) Highly elastic c) Unitary elastic d) Perfectly elastic.

17. Which of the following will have elastic demand?
    a) Match box b) Text books c) Medicines d) car.
18. Shape of perfectly inelastic demand curve is-
    a) Parallel to x-axis b) parallel to y-axis c) 45° with origin d) none.
19. If demand of good does not change with the rise in price then its demand is-
    a) Perfectly elastic b) Perfectly inelastic c) Less elastic d) More realistic.
20. Slope of demand curve is _____ with x-axis when $e^d = \alpha$.
    a) 60° b) 45° c) 90° d) 0°

Very short answer type question   **(1 mark):**
1. Define utility.
2. Define MRS.
3. Define an indifference map.
4. What is budget set?
5. What is budget line?
6. What is cardinal Utility?
7. What is ordinal utility?
8. Write a reason for rightward shifting of demand curve.
9. What is purchasing power of money?
10. When is a good considered as normal good?
11. Why is price e^d generally negative?
12. Under what condition will the demand curve be parallel to y-axis?
13. Why the demand for water inelastic?
14. If two demand curve intersect, which one has the higher price e^d?
15. Demand for a good x is perfectly elastic. If price of x rises by 10% how the demand for x will change?

Short answer type question   **(3/4 marks):**
1. Write the relationship between total utility & marginal utility through using a schedule & figure.
2. What is the law of "Diminishing marginal utility". Give one exception of it.
3. What is budget line? Why is it downward sloping?
4. What is indifference curve? Write the properties of indifference curve.
5. Why indifference curve is downward sloping?
6. why indifference curve is convex to origin?
7. Explain consumers equilibrium condition under indifference curve approach.

8. What is law of demand? Write the assumptions of law of demand.

9. Write the difference between contraction & decrease in demand.

10. Write the difference between expansion & increase in demand.

11. How price of related good affects the demand for a good.

12. How income of the buyer affects the demand for a good.

13. Write the reasons behind leftward shifting of demand curve or decrease in demand.

14. Write the reasons behind rightward shifting of demand curve.

15. Explain any four factors affecting the price elasticity of demand.

16. What is price elasticity of demand? Explain the proportionate method of measuring price e^d.

17. Write the difference between cardinal & ordinal utility.

Long answer type question   **(6 marks)**

1. Explain consumer's equilibrium through Utility approach when the consumer is consuming only one good.

2. Explain consumer's equilibrium through indifference curve approach or ordinal approach.

3. Explain the factors affecting the market demand of a good.

4. Why is the demand curve downward sloping?Or why there is inverse relation between the demand for a good and & its price?

5. Distinguish between an inferior and normal good. Explain the effect of change in income on each good <give suitable examples>

Numerical Quesyions (3/4 marks):

1. Find marginal utility (MU) from the following table:

Units	0	1	2	3	4	5
Total Utility	0	12	21	29	32	30

[Ans : MU :–, 12, 9, 8, 3, –2]

2. Calculate total utility (TU):

Units	1	2	3	4	5	6
MU	14	10	8	6	0	–2

[Ans : TU : 14, 24, 32, 38, 38, 36]

3. Calculate the missing figures:

Units	1	2	3	4	5
TU	5	9	–	14	–
MU	–	–	3	–	1

[Ans : TU : 5, 9, 12, 14, 15

MU : 5, 4, 3, 2, 1]

4. Following are the utility schedule of a consumer:

Units	1	2	3	4	5
MU	40	32	15	10	7

If the good is sold at ₹ 5 and MU of one rupee is 2 utiles, Find the equilibrium level of consumption of the consumer.   [Ans: 4 unit]

5. MU schedule of two good A & B given below. If the price of both the good is ₹1 & consumer's Budget is ₹ 8, find the equilibrium level of consumption of good A & B. Also find the TU at that level of equilibrium.

Quantity :	1	2	3	4	5	6
MU of A :	26	21	17	13	8	3
MU of B :	11	9	8	6	4	2

6. Find price e^d from the table:

Price (₹)	10	20
Demand (Unirts)	20	15

[Ans : $e^d = (-)\,0.25$]

7. Due to fall in the price of a good from ₹10 to ₹8, its demand rises from 80 to 100 units. Find price e^d.

[Ans : $e^d = (-)\,1.25$]

8. At a price of ₹ 10/unit, The demand for a good was 40 units. Find new demand if the price falls to ₹ 5/unit & price $e^d = 0.2$ given.

[Ans : 44 units]

9. The demand for a good was 100 units at ₹ 10/unit. When the price changes, demand falls to 50 units. Find the new price, if price $e^d = (-)\,2$.

[Ans : R 12.50]

10. When the price of a good falls by ₹ 1/unit, its demand rises by 3 units. If price $e^d = (-)2$, find the original demand if the price before change was ₹10/unit.   [Ans : 15 units]

11. Find price e^d, if due to 10% rise in the price of a good its demand falls by 5%.

[Ans : $e^d = 0.5$]

12. As a result of 10% fall in the price the demand for a good rises from 40 to 50 units. Find price e^d.

[Ans : (–) 2.5]

13. At a price of ₹ 20% unit, The demand of a good is 300 units. If price falls by 10%, and the demand rises by 60 units find price e^d.   [Ans : $e^d = (-)2$]

Answer key

True/False type question:-

1. False 2. False 3. False 4. True 5. True 6. True 7. False 8. True 9. True 10. True

11. False 12. True 13. False 14. False 15. False 16. True 17. False 18. True 19. True 20. False.

Multiple choice type question:

1. b) Diminishing MU 2. a) $\dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y} =$ 3. c) Family of indifference curve

4. c) $MRSxy = \dfrac{Px}{Py}$ 5. d) MUx = Px 6. d) All of the above

7. b) MU = 0 8. a) Rises 9. a) Inverse 10. b) Increase in demand 11. d) either (a) or (b)

12. a) Negative 13. c) Price of related good 14. c) $e^d<1$ 15. d) All of the above

16. a) Less elastic 17. d) car 18. b) Parallel to y-axis 19. b) Perfectly inelastic 20. b) 45°.

Very short question- answer:-

1. Want satisfying power of a good is called utility.

2. Marginal rate of substitution (MRS) refers to rate at which a good is substituted with another good so that the total satisfaction of the consumer remains the same.

3. A group of indifference curve is called indifference map which represent consumer Preferences over all the bundles of the two good.

4. Budget set is the set of all possible combinations of two goods which a consumer can afford, given his income and price in the market. < Equation of budget set : $X.P_x + Y.P_2 = M$ >.

5. The graphical presentation of budget set is called budget line which represents the various combination of two goods which consumer can afford, given his income and price of the two good. <Equation of budget line :

$$X. P_x + Y. P_Y \leq M>$$

6. Utility that can be expressed in numbers is called cardinal utility.

7. Utility that can expressed in Ranks is called ordinal utility.

8. Rise in income of the consumer.

9. Purchasing power of money means the power of money to consume other commodities in its exchange.

10. Normal goods are those where demand for such good rises with the rise in income of buyer.

11. Due to opposite/inverse relation between price of a good & its demand.

12. When the demand for a good is perfectly elastic.

13. Because water is a necessity and so its demand is inelastic.

14. If two demand curves intersect, then the flatter curve is more elastic.

15. There will be no change in demand for x.

Producers Behaviour and Supply

In every economy consumers and producers are integral part. Both are required for smooth functioning of an economy. Producers provide goods and services to the consumers to fulfill their requirements and for that they conduct production of goods & services.

3.1 Production:

Production is the process of converting the physical input into the physical output.

Production function is the functional relationship between the production of a good & its various factors.

Symbolically- $Y = f(X_1, X_2, X_3 \dots\dots Xn)$

Where, $Y = $ prod/yield & $X_1, X_2, X_3 \dots\dots$ are the

Various, factors required in prod^n

3.2 Inputs or factors:

Input refers to various ingradients which are required in the production process of a good. It is of the following 2 types-

Fixed factors	Variable factors
a) It refers to those factors which cannot be changed during short run. Ex: Land, building, machine technology etc.	a) It refers to those factors which can be changed during short run. Ex: fuel, raw materials, transportation, casual workers etc.
a) It remains same with the change in production level during short period.	b) It changes with the change in production during short.
c) It remains same even if the production is in loss or there is no production.	c) It does not require when the production is in loss or there is no production.

3.3 Time period:

The functional relationship between change in output, due to change in inputs is studied in two phases:

a) Short period/short run

b) Long period/long run.

Short period:	Long period:
a) It refers to that period of time when output can be changed by changing only one variable factor (say labour).	a) It refers to that period of time when output can be changed by changing all the factor inputs.

b) During this period Some of the factors are fixed & others are variable.	b) Here all the factors are variable, nothing is fixed.
c) Then demand is more active to determine price, as supply cannot be increased with the rising demand.	c) Then both demand and supply play equal role to determine price, as both can be increased.

3.4 Concept of Total Product (TP) or Total Physical Product (TPP)

TP is the sum total of production of a good by a producer through using various fixed and variable factors.

$TP = AP \times Q_L$ <Q_L = Quantity of variable factor Say, Labour> (AP- Average Product

$TP = \sum MP$ MP = Maginal Product)

3.4.1 Average Product(AP)/ Average Physical Product(APP) is the total product per unit of variable factor <Say Labour>.

$$AP = \frac{TP}{O_L} \quad [TP = \text{Total Product}, \ Q_L = \text{Quantity of Labour})$$

3.4.2 Marginal Product(MP)/ Marginal Physical Product(MPP) is the change in TP due to change in variable factor(Lab)by an extra unit. i.e. ratio between the change in TP & change in variable factor.

$MP = TP_n - TP_{n-1}$

or, $MP = \Delta TP / \Delta Q_L$

3.4.3 Relationship between TP, AP & MP.

Schedule:

Qn. of Labour	TP	AP	MP
1	10	10	10
2	28	14	18
3	38	12.6	10
4	38	9.5	0
5	30	6	−8
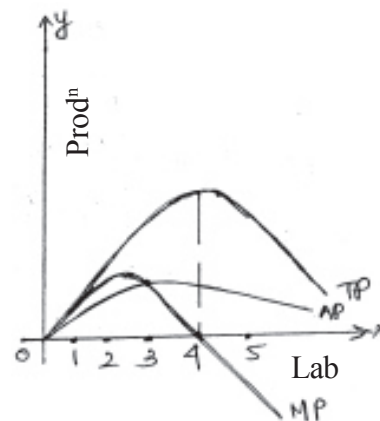
Relation:

a) Both the AP & MP is calculated from TP. [$AP = TP/Q_L$, $MP = TP_n - TP_{n-1}$]

b) When TP rises at faster rate, both AP & MP tends to rise.

c) When TP rises at slower rate, AP & MP tends to fall but remains positive.

d) When TP is at maximum, MP = 0.

e) When TP falls, MP becomes negative & AP also falls continuously but remains positive.

f) When AP rises, MP>AP

g) When AP is maximum, MP=AP

h) When AP falls, MP<AP.

3.5 Short run production function:

Law of variable proportion/ Law of returns to a variable factor:

The law states that- during short run when one of the variable factor (Say Labour) is increased continuously along with the other fixed factors then,TP rises at a faster rate in the begining, then after rises at a slower rate and finally starts falling.

Assumption:-

a) It is a short run concept

b) There is only one variable factor(Lab) & the rest are fixed.

c) The state of technology is fixed.

d) All the unit of variable factors are homogeneous & equally efficient.

e) Factors become imperfect substitute of each other beyond a certain level of production.

The law having the following three stages-

Stage 1: Stage of increasing returns to a variable factor. In this 1st stage-

    a) TP rises at a faster rate.

    b) AP also rises &

    c) MP rises to its maximum.

Reasons:

a) Due to under utilisation of fixed factors compared to variable factors, it helps greater utilisation of fixed factor and it results in increasing return.

b) Due to increase in variable factor, specialisation and division of Labours took place, which leads to increasing return.

c) So long fixed factors are under utilised, additional application of variable factor improves co-ordination between fixed & variable factor.

INPUTS	TP	AP	MP	
1L2K3S	5	5	5	} Stage 1
2L2K3S	15	7.5	10	
3L2K3S	19	6.3	4	} Stage 2
4L2K3S	19	4.7	0	
5L2K3S	17	3.2	-2	} Stage 3

[Where,
L = Labour
K = Capital
S = Land]
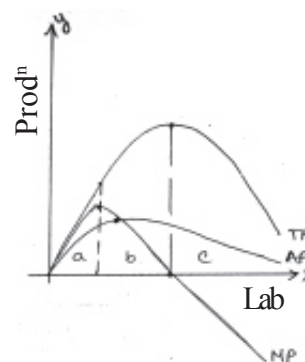
Stage 2: Stage of diminishing returns to a variable factor. In this second stage-

a) TP rises at a slower rate & reaches to maximum.

b) AP also rises to maximum & then starts falling

c) MP decreases to zero.

Reasons:-

    a) Due to optimum use of fixed factor compared to variable factor employed on it, returns starts diminishing.

b) Factors of production are imperfect substitute to each other and more & more labour cannot be used in place of capital, so returns starts diminishing.

Stage 3: Stage of negative returns to a variable factor. In this final stage–

    a) TP starts falling

    b) AP also decreases continuously but remains positive.

    c) MP becomes negative.

Reasons:

    a) Over utilisation of fixed factor.

    b) Lack of proper co-ordination between the factors.

    c) Due to more and more increase in variable factors beyond certain limit, benefit of specialisation & division of labour starts extinguishing, resulting fall in TP.

3.6 Long run production function: Law of Returns To SCALE:

The law state that, during long-run when all the factor inputs are increased in same proportion, then the rate of increase of output may be more or constant or less compared to the rate of increase in input.

Following are the stages in this law-

Stage 1: Law of Increasing Returns to Scale<IRS> : The stage of IRS occurs when the rate of increase in output becomes more than the rate of increase in input.

INPUTS	Output
$100\%\begin{cases} 5L3K3S \\ 10L4K6S \end{cases}$	$\begin{matrix} 100 \\ 300 \end{matrix} \Big\} 200\ \%$
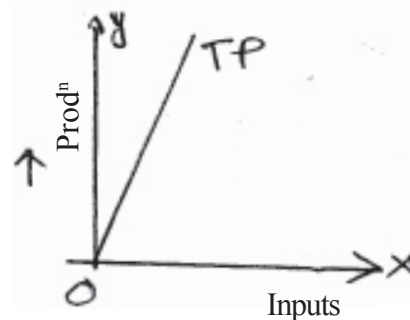
Stage 2: Law of Constant Returns to Scale<CRS> : The stage of CRS occurs when the rate of increase in output becomes equal to the rate of increase in input.

INPUTS	Output
$100\%\begin{cases} 10L4K6S \\ 10L8K12S \end{cases}$	$\begin{matrix} 300 \\ 600 \end{matrix} \Big\} 100\ \%$
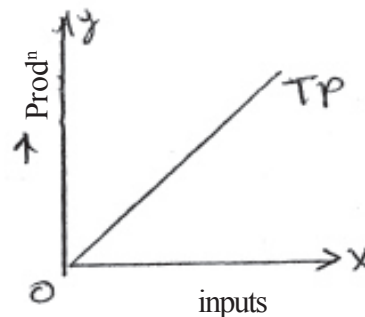
Stage 3 : Law of Diminishing Returns to Scale(DRS) : The stage of DRS occurs when the rate of increase in output becomes less than the rate of increase in input.

INPUTS	Output
100% { 20L8K12S 40L16K24S	600 1000 } 60 %
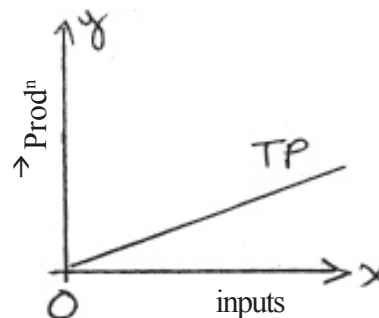
3.7 Difference:

Returns to Variable Proportion	Returns to Scale
a) It occurs during short run	a) It occurs during long run
b) There is only one variable factor & the rest are fixed.	b) All the factors are variable & nothing is fixed
c) Factor ratio changes	c) Factors ratio remains fixed
d) Scale of production does not change.	d) Scale of production changes
e) Its various stages are- Increasing returns, diminishing returns & negaive returns.	e) Its various stages are- Increasing returns, constant returns & diminishing returns.

Solved Numerical Question :

1. Find AP & MP from the following table.

Labour	0	1	2	3	4	5
Tp	0	8	20	28	28	25

Solution:

Labour	TP	AP	MP
0	0	–	8
1	8	8	8
2	20	10	12
3	28	9.33	0
4	28	7	0
5	25	5	–3

2. Find TP & MP from the following table.

Labour	1	2	3	4	5
AP	8	10	8	6	4

Solution:

Labour	AP	TP	MP
1	8	8	8
2	10	20	12
3	8	24	4
4	6	24	0
5	4	20	-4

3. Find the various stages of law of variable proprotion from the following Table:

Labour	1	2	3	4	5
TP	4	9	13	15	12

Solution:

Labour	TP	MP	
1	4	4 }	Stage 1
2	9	5 }	
3	13	4 }	Stage 2
4	15	2 }	
5	12	-3 }	Stage 3

4. Suppose the production function is - $Q = 5\,L^{\frac{1}{2}}K^{\frac{1}{2}}$

Find maximum level of output by using 100 units of L & 100 units of K.

Solution :

$$Q = 5\,L^{\frac{1}{2}}K^{\frac{1}{2}}$$

$$= 5\,100^{\frac{1}{2}} \times 100^{\frac{1}{2}}$$

$$= 5 \times \sqrt{100} \times \sqrt{100}$$

$$= 500 \text{ units}$$

5. Find AP from the following table.

Labour	1	2	3	4	5
MP	8	12	10	6	4

Solution:

Labour	MP	TP	AP
1	8	8	8
2	12	20	10
3	10	30	10
4	6	36	9
5	4	40	10

3.8 Cost : Cost refers to the expenditure incurred by the producer on various factors of production required in the production process of a good.

Total Cost(TC)- It refers to sum total of expenditure made by the producer on various fixed and variable factors required in the production process of good.

TC = FC + VC

Fixed Cost/Total Fixed Cost(TFC or FC) :

FC refers to the expenditure on fixed factors of production required in the production process of a good.It is also called supplementary cost/indirect cost/overhead cost. Ex: Cost of machine, land, technology, salary to the permanent staff, bank interest, insurance premium, licence fee, permit fee, minimum telephone bill etc.

Variable Cost (V/C) or Total Variable Cost (TVC):

It refere to the the expenditure on variable factors of production required in the production process of a good. It is also called prime cost/ direct cost.

Example : cost of raw material , Transportation, wages to casual workers, advertisement, telephone bill beyond the minimum etc.
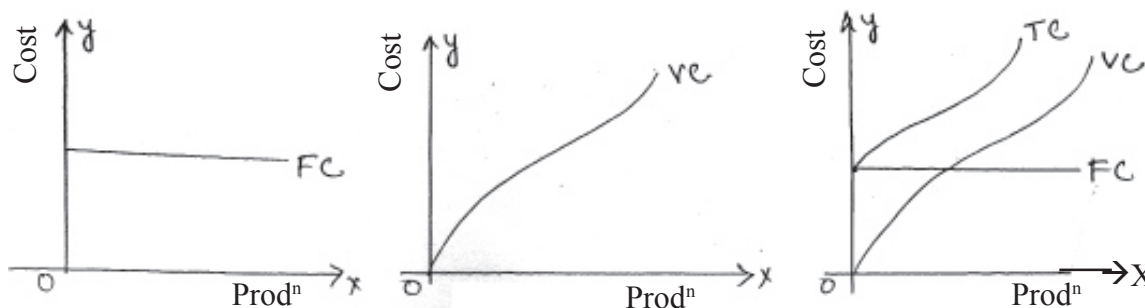
3.9 Difference between fixed &variable cost:

Fixed cost	Variable cost
a) It is the cost of fixed factors of production	a) It is the cost of variable factors of production
b) It cannot be changed during short run	b) It can be changed during short run
c) It remains same with the change in production level	c) It changes with the change in production level
d) It never be zero even when production is in loss or there is no production.	d) It becomes zero when production is in loss or production becomes zero.
e) FC curve is parallel to x-axis.	e) VC curve is upward sloping starting from origin.

3.10 Average cost: Average Cost(AC) or Average total cost (ATC) is the total cost per unit of output.i.e,Ratio between Total cost and quantity of output. AC is also the sum of Average Fixed Cost (AFC) and Average Variable Cost (AVC).

$$AC = \frac{TC}{Q} \text{ (Q = Qn. of output)}$$

AC = AFC + AVC.

■ Average Fixed Cost(AFC) is the fixed cost per unit of production, i.e. Ratio between Fixed Cost & output.

$$AFC = {FC}/{Q} \text{ or, } {TFC}/{Q} \text{ (Q= Quantity of output)}$$

■ Average Variable Cost(AVC) is the variable cost per unit of output. i.e. Ratio between variable Cost & output.

$$AVC = {VC}/{Q} \text{ or, } {TVC}/{Q} \text{ (Q= Quantity of output)}$$
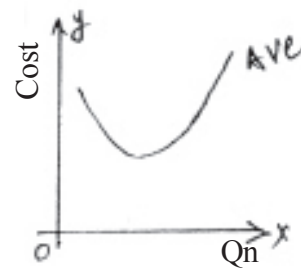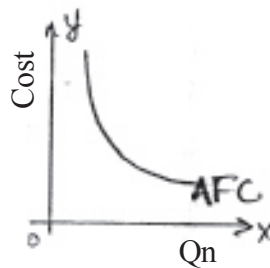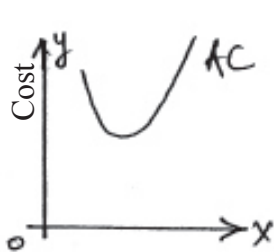
3.11 Marginal cost (MC) :

It is the ratio between the change in VC & change in output. i.e. MC is the extra VC which is required to increase the output by an extra unit.

$$MC = VC_n - VC_{n-1} \text{ or } MC = \frac{\Delta VC}{\Delta Q}$$
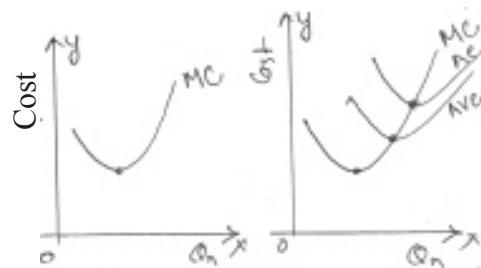
3.12 Relationship between the different types of cost

3.12.1 MC & AC :

a) Both the AC & MC are calculated from TC.

$$AC = {TC}/{Q},$$

$MC = TC_n - TC_{n-1}$ (Q= Quantity of output)
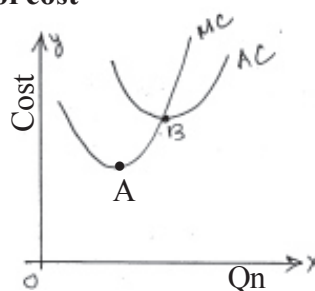
Qn	TC	AC	MC
1	10	10	10
2	16	8	6
3	24	8	8
4	39	9.7	15

b) Slope of AC & MC curve is 'U' shaped due to the law of variable proportion.

c) When AC falls, MC<AC

d) When AC is minimum, MC = AC, i.e. MC passes through the minimum point of AC <at point B>.

e) When AC rises, MC>AC

f) MC reaches to its minimum point (A) earlier than AC.

3.12.2 MC & AVC :

a) Both AVC & MC is calculated from VC
   $MC = VC_n - VC_{n-1}$;
   $AVC = VC/Q$ (Q= Quantity of output)
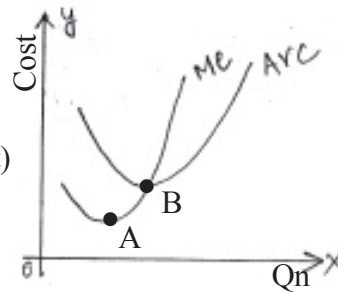
b) Slope of both the AVC & MC curve is 'U' shaped due to law of variable proportion.

Qn	TC	AC	MC
1	10	10	10
2	16	8	6
3	24	8	8
4	36	9	12

c) When AVC falls, MC<AVC

d) When AVC is minimum <at B>, MC = AVC

e) When AVC is rising, MC>AVC

f) MC reaches to its minimum (A) earlier than AC (B)

3.12.3 AC & AVC

a) AVC is the part of AC.
   AVC = AC-AFC

b) Both the AC & AVC curve is 'U' shaped due to the law of variable proportion.

c) The gap between AC & AVC falls with the rise in production level, because AC = AFC +AVC & AFC falls continuously with the rise in production. So AVC becomes closer to AC with the rise in production level.
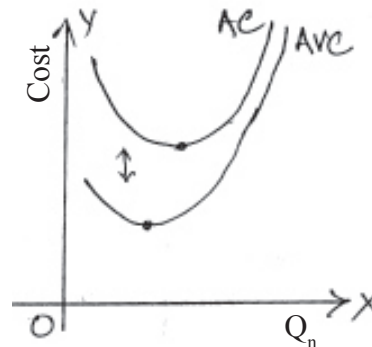
Qn.	AC	AVC
1	10	3
2	7	4
3	15	12
4	25	24

d) AC never be equal to AVC i.e. AC≠AVC i.e. AVC curve never intersect AC curve, because AFC never be zero as FC never be zero at any level of production.

3.12.4 AC & AFC

a) AFC is a part of AC
   AFC = AC-AVC.

Qn.	AC	AFC
1	10	8
2	7	5
3	12	3
4	20	1

B) AC curve is 'U' shaped but AFC curve is downward sloping as FC remains fixed with the rise in production level.

C) The gap between AC & AFC rises with the rise in production level, as AC rises with production & AFC falls with the rise in production level.

3.12.5 TC & MC

a) MC is the rate of change of TC

$MC = TC_n - TC_{n-1}$

b) When TC rises at a diminishing rate (upto A), MC falls,

c) When TC rises at a constant rate, MC becomes minimum.

d) When TC rises at an increasing rate, MC also rises.

(upto A)
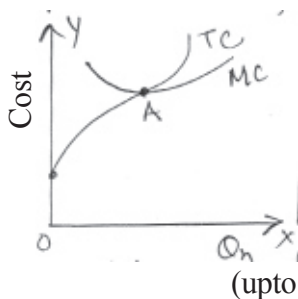
Qn.	TC	MC
1	10	10
2	17	7
3	29	12
4	49	20

<after A>

3.12.6 VC & MC

a) MC is the rate of change of VC

$MC = VC_n - VC_{n-1}$

b) When VC rises at a diminishing rate (upto point A), MC falls

c) When VC rises at constant rate (at A), MC becomes minimum.

d) When VC rises at an increasing rate (after point A), MC also rises.
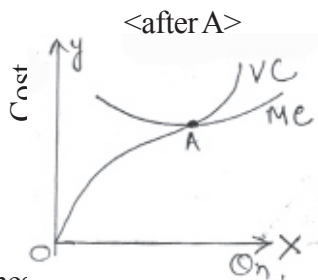
Qn.	VC	MC
1	10	10
2	17	7
3	29	12
4	49	20

 # Why AC curve is U-shaped?

AC curve is 'U' shaped due to "law of variable proportion".

In the beginning of production, the stage of increasing returns operates and so firm gets economies in production. So per unit of production cost i.e AC falls. But after the optimum combination, economies turned in diseconomies & so per unit of production cost ie AC rises. It is the shage of diminishing returns.

So AC first falls & then after rises and the shape of AC curve becomes U-shaped.

 # Why AFC curve is negatively or downward sloping?

AFC is the ratio between FC & quantity of production. But FC remains same at any level of production. So the ratio between FC & output falls with the rise in the production level.

But AFC never be zero or -ve. Because FC never be zero or -ve. So AFC curve never touch or intersect the x-axis
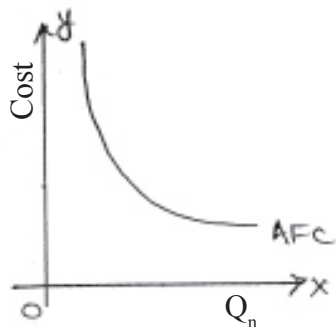
Qn.	AC	AFC
1	60	60
2	60	30
3	60	20
4	60	15

Solved Numerical Question on Cost :

1. Find FC, VC, AC, AVC, AFC, & MC from the following table:

Output (Units)	0	1	2	3	4
TC	50	65	90	120	160

Solution:

Output	TC	FC	VC	AVC	AFC	AC	MC
0	50	50	0	-	-	-	-
1	65	50	15	15	50	65	15
2	90	50	40	20	25	45	25
3	120	50	70	12.33	16.33	40	30
4	160	50	110	27.25	12.25	40	40

[At Zero level of production TC = FC, ∴ VC = 0]

2. Find AVC from the following table:

Output (Units)	1	2	3	4
MC	40	30	35	39

Solution:

Output (Q)	MC	VC (ΣMC)	AVC ($^{VC}/_Q$)
1	40	40	40
2	30	70	35
3	35	105	35
4	39	144	36

3. Calculate ATC & AVC if FC = Rs. 60 given

Output (Units)	1	2	3	4
MC	30	26	28	32

Solution:

Output	MC	VC	FC	TC (FC+VC)	ATC	AVC
1	30	30	60	90	90	30
2	26	56	60	116	58	28
3	28	84	60	144	48	28
4	32	116	60	176	44	29

$$ATC = \frac{TC}{Q}, \quad AVC = \frac{VC}{Q}$$

3.13 Concept of revenue :

Revenue refers to the money received by a firm from the sale of its product.

The sum total of money received by a firm from the sale of all its quantity is called Total Revenue (TR).

$$TR = P \times Q \qquad P = \text{Price/ Unit Sold, } Q = \text{Qn. Sold}$$

$$TR = AR \times Q \text{ or } TR = \sum MP < MR = \text{Marginal Revenue}>$$

♦ Average revenue (AR) is the total revenue per unit of quantity sold i,e ratio demand between TR & quantity sold

$$AR = \frac{TR}{Q}$$

$$\boxed{AR = \frac{TR}{Q} = \frac{P \times Q}{Q} = P}$$

i,e AR is nothing but price / Unit

♦ Marginal revenue (MR) is the rate of change in TR due to change in quantity sold by an extra unit. i.e. MR is the ratio between change in TR & change in quantity Sold.

$$MR = TR_n - TR_{n-1} \text{ or, } \Delta TR / \Delta Q$$

3.14 Relationship between TR, AR & MR:

a) Both the AR & MR is calculated from TR

$$AR = \frac{TR}{Q}$$

$$MR = TR_n - TR_{n-1}$$
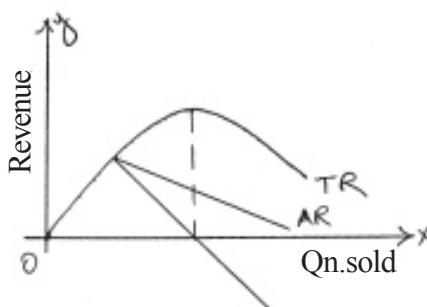
Qn.	TR	AR	MR
1	10	10	10
2	17	8.5	7
3	19	6.3	2
4	19	4.35	0
5	17	3.4	-2

b) When TR rises both AR & MR falls continuously, but remains positive.

c) When TR is maximum.

$$MR = O$$

d) When TR falls, MR becomes negagive AR also falls but remains positive

---

Solved Numericals on Revenue:

1. Find TR, AR & MR from the following table:

Price	12	10	8	6
Output (Units)	1	2	3	4

Solution:

Price (Rs.)	Output (Q)	TR (P × Q)	AR ($^{TR}/_Q$)	MR ($TR_n - TR_{n-1}$)
12	1	12	12	12
10	2	20	10	8
8	3	24	8	4
6	4	24	6	0

2. Find TR, AR from the following table:

Quentity Sold (Units)	12	10	8	6
MR (Rs.)	1	2	3	4

Solution:

Quantity (Q)	MR	TR (ΣMR)	AR (TR/$_Q$)
1	14	14	14
2	10	24	12
3	7	31	10.33
4	5	36	9

3.15 Supply:

Supply refers to the quantity of a commodity which a producer or firm is willing to sale at different price of that commodity during a given period of time.

Quantity supplied refers to the specific quantity of a commodity which a producer is willing to sale at a given price and at a given point of time.

Stock refers to the total quantity of a commodity which is available with the producer to supply, i.e. Supply is a part of stock.

3.16 Supply function:

Supply function refers to the functional relationship between the supply of a good & its various factors or determinants. Symbolically-

$S_x = f$ (Px, Pr, Pe, Pt, T, Gp, Go, N)

where, S_x = Supply(S^y) of good X

Px = Price of goodx.

Pr = Price of related good of x.

Pe = Expectation of price change.

Pf = Price of factors of production of goodx

T = State of Technology used in production of X.

Gp = Govt. policy.

Go = Goal of the producer.

N = No. of Producer/ firm.

3.17 Determinants or factors of S^y of a good:

1) **Price of the good:** Generally the supply of a good is directly related with the price of that good. i.e supply rises with the rise in price and vice versa.It is because with the rise in price, the profit of the producer rises which encourage them to increase production and supply also rises.

2) **Price of related good:** Supply of a good is affected with the change in the price of its related good, specially substitute good (rice & wheat).Here with the rise is price of a good, the S^y of its substitute good falls & vice-versa.

3) **Price of factors of production:** Supply(S^y) of good is inversely related with the price of its factors of production. If the factor price rises, then production cost will rise & profit margin of the producer fall.It will discourage them to increase production. So S^y will fall. On the other hand S^y rises with the fall in factor price.

4) **Production Technique :** If modern & advance technique is used in the production process then maximum production is possible with minimum resource & expenditure. So S^y will rise. But if primitive techniques are used then production falls and S^y also falls.

5) **Govt. Policy:** The tax and subsidy policy of govt. effects the supply of a good.If more taxes are imposed on the production of a good then it will discourage the producer to produce more & so S^y will fall.On the other hand if subsidies are provided then it will encourage the producer to produce more and so S^y rises.

6) **Goal of the producer:** Generally producer's goals are either profit maximisation or sales maximisation. If the goal is profit maximisation, i.e. maximum profit & minimum sales then the S^y will fall. But if the goal is sale or welfare or production or employment maximisation then more will be supplied even at same price or less price so S^y will rise

7) **Number of producer:** Supply of a good is directly related with the number of producer or seller of that good. If the number of producer of good rises its supply also will rise &vice-versa.

3.18 Law of supply:

The law states that- when other factors remain same, quantity supply of commodity rises with the rise in its price, and quantity supply falls with the fall in its price.

Assumption of law of supply:

   a) Price of related good remains fixed
   b) Price of factors of production remains fixed
   c) State of technology remains fixed
   d) Govt. Policy remains fixed
   e) Goal of the producer remains fixed
   f) No. of producer remains fixed.

Exception of law of supply:

a) The law of S^y doesn't hold in case of agriculture product. Because product of such goods depends on monsoon.and so are not affected by market price.

b) The law does not hold in case of perishable good like fruits, vegetables etc. Because such goods cannot be stored for long period. so producers are bound to sale at any market price.

c) The law does not hold in case of auction and rare goods. Because S^y of such good is so limited and so S^y cannot be increased with the rise in price of such good. Ex: writings of Tagore, Kohinoor Diamond etc.

3.19 Individual supply:

The supply of a good by an individual producer/firm at different price of that good in the market is called individual supply.

When that individual supply is presented in a tabular form it is called individual supply schedule.

The graphical presentation of individual supply schedule is called individual supply curve- and through that we can get the idea that the various supply of a good by an individual producer in the market at different prices of that good.

Indivisual Supply Schedule :

Price of good x ₹/ Units	Quantity Supplied of x (Units)
10	70
12	90
4	100
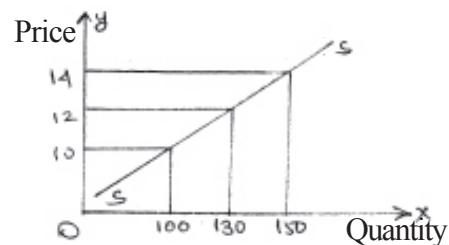
Fig - Individual supply curve

3.20 Market supply:

The sum total of quantity supplied of a good by all the producers in a market at a given price is called market supply. When the market supply is represented in a tabular form it is called market supply schedule.

The graphical presentation of market supply schedule is called market supply curve through which we can get the idea that what are the various supply of a good by all the producers in the market at different prices of that good.

Market supply schedule :

Price of good x (₹/ Units)	Supply of firm A (Units)	Supply of firm B (Units)	Market Supply (Units)
10	70	50	120
12	90	60	150
14	100	90	190
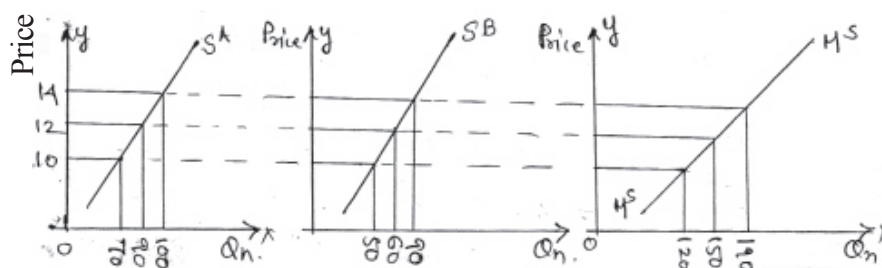
Fig : Market S^y curve

3.21 Change in quantity supplied:

When the supply of a good changes due to change in its price, other factors remaining the same, it is called change in quantity supplied.

Then there is movement along the supply curve upward & downward. So two types of change in quantity supplied- expansion and contraction of supply.

Upward movement along the supply curve (Expansion of supply)

It happens when the supply of a good rise is due to rise in its price, other factors remaining the same. Then there is upward movement along the supply curve.

Price of x (₹/ U)	Quantity Supplied (Units)
10	10
12	12

Expansion of supply
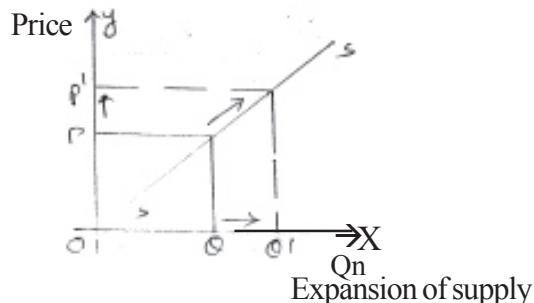
The reason behind expansion of supply is price rise.

Downward movement along the supply curve (contraction of supply)

It happens when the supply of a good falls due to fall in its price, other factors remaining the same.Then there is downward movement along the supply curve.

Price of x (Rs./ U)	Quantity Supplied (Units)
10	100
8	80

Construction of Supply
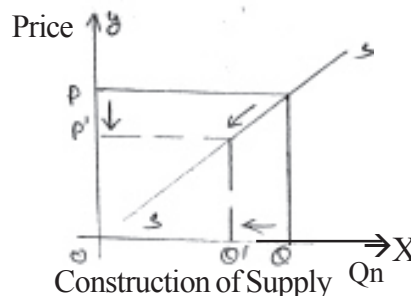
The reason behind contraction of supply is price fall.

3.22 Change in supply:

When supply of a good changes due to change in its factors, other than price, it is called change in supply.Then the supply curve shifts towards right or left. So two types of change in supply- increase in supply and decrease in supply.

Rightward shifting of supply curve (Increase in supply)

It happens when the supply of a good rises due to change in its various factors other than price. Then the supply curve shifts rightward or forward.

Price of good x (₹/ U)	Supply of X (Units)
10	100
10	120

<Increase in sy>
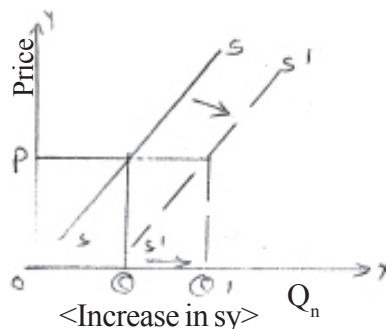
The reasons behind increase in supply are-

a) Fall in factor price
b) Tax reduction & subsidy expansion by government.
c) Use of advance technology
d) Sales maximisation target of firm
e) Rise in number of firm
f) Expectation of price fall in near future
g) Fall in the price of substitute good.
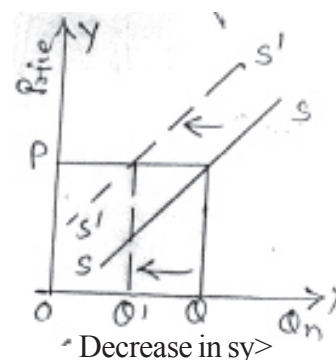
← Decrease in sy>

Leftward shifting of supply curve (Decrease in supply)

It happens when the supply of a good falls due to change in its various factors other than price. Then the supply curve shifts leftward or backward.
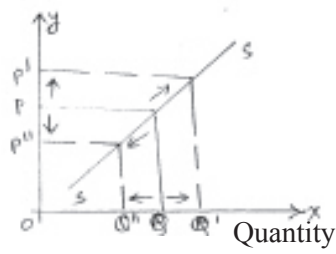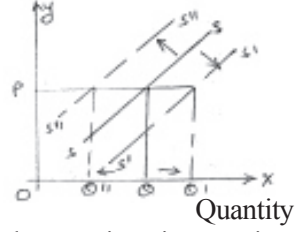
Price of good x (₹/ U)	Supply of X (Units)
10	100
10	80

The reasons behind decrease in supply are-

a) Rise in factor price
b) Tax expansion and subsidy reduction by government
c) Use of primitive technology
d) Profit maximisation target of firm
e) Fall in number of firm.
f) Expectation of price rise in near future.
g) Rise in the price of substitute good.

Difference:

Change in quantity supplied	Change in supply
a) It occurs when the supply of a good changes due to change in its price, other factors remaining the same.	a) It occurs when the supply of a good changes due to change in its factors, other than price.
b) Then there is movement along the supply curve- upward and downward	b) Then there is shifting of supply curve- rightward and leftward.

Change in quantity supplied	Change in supply
c) It is of two types- contraction & expansion of supply Construction of supply expansion of supply d) The reason behind it is change in price.	c) It is of two types- increase and decrease in supply. decrease in supply increase in supply d) The reasons behind it are change in factor price, Govt. policy, goal of the producers, technology, price of related good etc.

3.23 Price elasticity of supply:

It is the responsiveness of the supply of a good with the change in its price. in other words, price elasticity of supply is the rate of change in quantity supplied of a good with the change in its price. Numerically it can be expressed in two methods-

a) **Proportionate method :**

Price elasticity of supply (E^s) $= \dfrac{\text{Proportionate change in Quantity Supplied}}{\text{Proportionate change in Price}}$

$$= \dfrac{\dfrac{\Delta Q}{Q}}{\dfrac{\Delta P}{P}} = \dfrac{\Delta Q}{Q} \times \dfrac{P}{\Delta P} = \dfrac{\Delta Q}{\Delta P} \cdot \dfrac{P}{Q}$$

Where, P= Initial Price

Q = Initial Quantity Supplied

ΔP = Change in Price

ΔQ = Change in Quantity Supplied

b) **Percentage Method:**

Price elasticity of supply (E^s) $= \dfrac{\text{Percentage change in Quantity Supplied}}{\text{Percentage change in Price}}$

$$= \dfrac{\dfrac{\Delta Q}{Q} \times 100\%}{\dfrac{\Delta P}{P} \times 100\%}$$

3.24 Factors affecting the price Demand elasticity of supply:

Following are the factors affecting the price elasticity of supply-

a) Nature of the good:

Supply of perishable goods(fruits, milk etc) are less elastic compared to durable good (freeze, fan etc).Because perishable goods cannot be stored for long period, but it is possible for durable good and so price reaction is more in durable good.

b) Time period:

Longer the time period required in the production process of a good, more is the price elasticity of supply & vice-versa. Because during longer period, production can be easily changed with the change in price and so price reaction is high.

c) Production technique:

If more simple technique is used in the production process of a good, more is the price elasticity of supply and vice-versa. In case of simple techniques of production, production can be easily changed or closed with the change in price. But it is not possible in complex technique of production and so price reaction i.e. elasticity is also low.

d) Limited supply:

In case of limited supply of a good or, where reproduction of a good is not possible or goods in auction etc., price elasticity of supply is perfectly inelastic.Because supply cannot be changed with the change of price of such goods.

e) Availability of factors :

More is the availability of factors of production of a good, more is the price elasticity of supply vice-versa. Because when factors are easily available the production can be easily increased with the price rise. So price reaction i.e. elasticity is also high.

3.25 Degrees of price elasticity of Supply:

The range of the value of price elasticity of supply is zero to infinity. $o \leq e^s \leq \alpha$

In between there are five degrees of price elasticity of supply

a) Perfectly inelastic supply ($e^s = 0$) : It occurs when the responsiveness of supply of a good become zero with the change in its price. Then the supply curve becomes vertical i.e. parallel to y-axis.

Example : rare goods, goods in a auction etc. Here slope of supply curve is 90° with x axis.
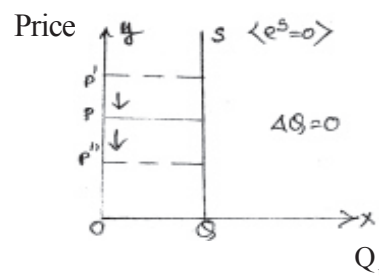
b) Inelastic supply ($e^s < 1$): It occurs when the responsiveness of the supply of a good is less than the change in its price. Then the slope of supply curve remains in between 45° to 90° with x-axis.

Example : Agricultural Product.
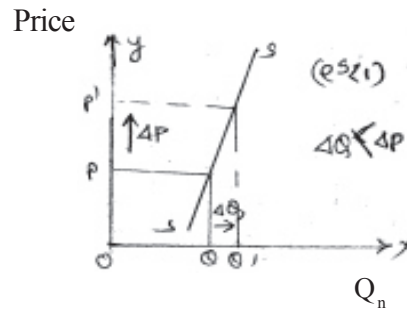
c) Unit elasticity of supply ($e^s = \alpha$): It occurs when the responsiveness of the supply of a good is same as the change in its price. Then the supply curve's slope becomes 45° with origin.
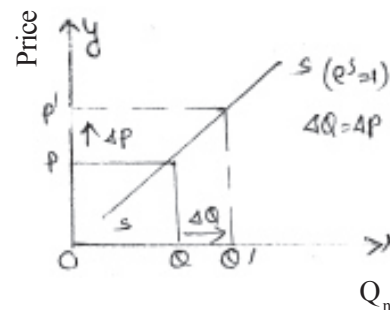
d) Elastic Supply ($e^s > 1$): It occurs when the responsiveness of the supply of a good is more than the change in its price. The slope of supply curve remains in between 0° to 45°.

Example: Durable goods.
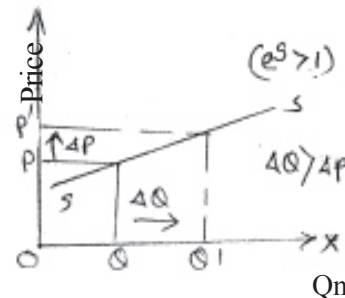
e) Perfectly elastic supply ($e^s = \alpha$): It occurs when the supply of a goods changes irrespective of the change in its price. Then the supply curve becomes horizontal i.e. paralled to x-axis.
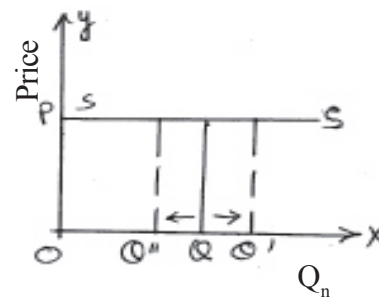
Solved Numerical on price elasticity of supply (e^s):

1. The supply of a good is 50 units at Rs. 10/ units. Its supply rises to 100 units at Rs.15/units. Find price e^s.

Solution: Initiat Price (P) = 10   Change in Price ΔP = 15-10 = 5

Initiat Quantity (Q) = 50   Change in Quantity ΔQ = 150-100 = 50

$$\therefore \text{Pricee's} = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} = \frac{50}{5} \times \frac{10}{50} = 2 \qquad \text{(elastic supply)}$$

2. Price of a good falls by Rs. 5/unit and due to that its supply decreases from 100 to 50 units. Find e^s if original price was Rs. 15/unit.

Solution : $P = 15$ $\Delta P = 5$

                    $Q = 100$ $\Delta Q = 100 - 50 = 50$

$$\therefore e^s = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \frac{50}{5} \times \frac{10}{100} = 1 \cdot 5 \text{ (elastic supply)}$$

3. If the supply of a good rises by 10% due to rise in its price by 5% then find e^s.

Solution :   price $e^s = \dfrac{\text{Percentage change in Quantity}}{\text{Percentage change in Price}} = \dfrac{10\%}{5\%} = 2$

4. If due to rise in price by 10% the supply of a good rises from 100 to 120 units. Find e^s.

Solution : $Q = 100$ $\Delta Q = 120 - 100 = 20$

$$\therefore \text{Percentage change in quantity} = \frac{\Delta Q}{Q} \times 100\% = \frac{20}{100} \times 100\% = 20\%$$

$$\therefore e^s = \frac{\% \text{ change } \Delta Q}{\% \text{ change } \Delta P} = \frac{20\%}{10\%} = 2$$

5. Price e^s of a good is 2 when its price falls from Rs. 10 to 8/ unit, its supply falls by 500 units. Find Quantity supplied at reduced price.

Solution : $P = 10$ $\Delta P = 10 - 8 = 2$

            $\therefore e^s = 2 \text{ (given)}$ $\Delta Q = 500$

or, $\dfrac{\Delta Q}{\Delta P} \cdot \dfrac{P}{Q} = 2$ or, $\dfrac{50}{2} \cdot \dfrac{10}{Q} = 2$ $\therefore Q = 1250$

    $\therefore$ Quantity supplied at reduced price $= 1250 - 500 = 750$ units

Formula at a glance:

→ **Production**

a) Total Production (TP) = Average Production (AP) × Quantity

b) $AP = \dfrac{TP}{Q}$

c) $MP = TP_n - TP_{n-1}$ (MP = Marginal Product)

→ **Cost**

a) $TC = FC + VC$ TC = Total Cost

b) $AC = \dfrac{TC}{Q}$ FC = Fixed Cost

c) $AFC = \dfrac{FC}{Q}$ VC = Variable Cost

d) $AVC = \dfrac{VC}{Q}$ AC = Average Cost

e) $MC = VC_n - VC_{n-1}$ Q = Quantity of Production

f) $AC = AFC + AVC$ AVC = Average Fixed Cost

g) $VC = \Sigma MC$ AVC = Average Variable Cost

MC = Marginal Cost

→ **Revenue**

$TR = P \times Q$ TR = Total Revence

$TR = \Sigma MR$ P = Price/ Units

$AR = \dfrac{TR}{Q}$ Q = Quantity Sold

$AR = P$ AR = Average Revence

$MR = TR_n - TR_{n-1}$ MR = Marginal Revence

→ **Elasticily of Supply (Eˢ)**

1. $\therefore E^s = \dfrac{\Delta Q}{\Delta P} \cdot \dfrac{P}{Q}$ P = Initial Price

$\therefore E^s = \dfrac{\dfrac{\Delta Q}{Q} \times 100\%}{\dfrac{\Delta P}{P} \times 100\%}$

Q = Initial Quantity

ΔP = Change in Price

ΔQ = Change in Quantity

Exercise

State True / False :

1. When MP falls, AP also will fall.

2. Both AP & MP can be negative.

3. During long run, all factors of production are variable.

4. Under "Law of Variable Proportions", factor ratio keeps on changing.

5. MP cuts AP from its top.

6. "Law of Variable Proportions", operater in long period.

7. When TP is constaut, AP will fall.

8. Production function does not depend on the state of technology.

9. The gap between total cost & total veriable cost rises with the rise in production.

10. Fixed cost remains same even if output is zero.

11. Total fixed cost is more than total veriable cost at zero level of output.

12. Total cost of production is the sum of fixed & variable cost.

13. MC is not affected by total fixed cost.

14. Both the TC & TFC curve start from the same point.

15. When MC rises, AC also rises.

16. AFC curve is U-shaped.

17. AR and price are one and the same thing.

18. MR can never be negative.

19. TR can be obtained by adding MR.

20. When TR is maximum, MR is also maximum.

21. When TR is constant, AR falls.

22. MR can never be zero.

23. When MR is falling but positive, TR will also be falling.

24. An increase in number of firms, leads to 'Expantion in Supply'

25. A cost saving technology shifts the supply curve of good towards left.

26. Contraction of supply occurs when there is change in factors other than price of the given good.

27. Rise in price of Limca, may reduce the supply of coke.

28. When the supply curve forms an angle of 45° with origin then the price elasticity of supply is equal to one.

29. In case of perfectly elastic supply, the supply curve becomes vertical.

Multiple Choice Question:

1. When MP is zero, TP

    a) increasing   b) maximum

    c) falling   d) None of this

2. When AP rises, MP is

    a) less then AP   b) equal to AP

    c) more than AP   d) none

3. What happens to AP, when MP < AP ?

    a) AP rises   b) AP falls

    c) AP constant   d) none

4. When AP is maximum, MP is equal to-

    a) AP   b) TP

    c) zero   d) one

5. Both AP & MP curve is generally-

    a) 'U' shaped   b) inversely 'U' shaped.

    c) rising   d) falling

6) When MP rise TP-

    a) falls   b) rises

    c) can rise/ fall   d) constant

7. Identify the two curves which start from the same point of Y-axis

    a) TVC, TFC   b) TFC, AVC

    c) TFC, TC   d) TFC, AFC

8) which one is not affected by FC

    a) MC   b) TC

    c) AC   d) AFC

9) MC can be directly derived from

    a) TFC   b) TVC

    c) AC   d) AFC

10. AC, AVC & MC curve is 'U'shaped due to the law of

    a) diminishing marginal utility   b) diminishing ruturns

    c) variable proportions   d) none

11. When AC is rising, MC is

    a) equal to AC   b) more than AC

    c) less than AC   d) constant

12. AFC curve

    a) Touch x-axis   b) Touch y-axis

    c) both 'a' & 'b'   d) neither 'a' nor 'b'

13. Which cost rises continueously with the rise in production ?

    a) Average cost   b) Marginal cost

    c) Variable cost   d) fixed cost

14. Which of the following statement is true ?

    a) ATC = AFC-AVC   b) AVC = AFC+ATC

    c) AFC = ATC+AVC   d) AFC = ATC-AVC

15. Out of the following, which one can never be zero

    a) AFC   b) AVC

    c) TVC   d) none

16. When MR is positive, TR

    a) rises   b) falls

    c) maximum   d) none

17. If TR is R 1,00,000 when 20,000 units are sold, then AR is

    a) 10,000   b) 20,000

    c) 5   d) 1,20,000

18. Average revenue equals -

    a) Total revenue divided by quantity sold   b) price

    c) both 'a' & 'b'   d) none

19. The market supply is affected by

    a) state of technology   b) number of firms

    c) Govt. policy   d) all of the above

20. Supply curve shifts rightwards due to

    a) rise in price   b) fall in price

    c) fall in factor price   d) none

21. Upward movement along the supply curve is due to

    a) rise in price   b) fall in price

    c) rise in factor price   d) none

22. Change in quantity supplied occurs due to

    a) change in price   b) change in other factors except price

    c) both 'a' & 'b'   d) none

23. In case of perfectly elastic supply, the supply curve is

    a) parallel to x axis   b) parallel to y axis

    c) 45° with origin   d) none

24. In case of perfectly inelastic supply, slope of supply curve with x axis is

    a) 90°   b) 45°

    c) 0°   d) 80°

25. 'Decrease in supply' of a good due to

    a) fall in price   b) rise in factor price

    c) rise in price   d) none

Very Short Answer Type Question: (Mark- 1)

1. Can AP be zero or negative ?

2. Can MP be zero or negative ?

3. What is the general shape of TP, AP & MP curves ?

4. Define short run production function.

5. Difine long run Production function.

6. Define explicit cost ?

7. Define implicit cost ?

8. How is MC related to FC ?

9. What is the behaviour of AFC when output rises ?

10. Can MR be zero or negative ?

11. How does TR reacts when MR = 0 ?

12. What is the shape of supply curve ?

13. In which situation does the supply fall at the same price.

14. Give one reason for decrease in supply of a good.

15. When the supply of a good is said to be elastic ?

Short Answer type Question: (Mark- 3/4)

1. Write the relationship between TP, AP & MP.

2. Write the difference between fixed & variable factors of production.

3. Write the difference between returns to a factor & returns to scale.

4. Discuss the concept of short & long run production function.

5. Write the differance between fixed & variable cost.

6. Write the difference between explicit & implicite cost.

7. Why does AFC falls with the rise in output? explain with a schedule & figure.

8. Why does the gap between AC & AVC fall with the rise in output.

9. State the relation between MC & AC.

10. State the relation between MC & AVC.

11. State the relation between TR, AR & MR.

12. Why is AR always equal to price ?

13. Distinguish between stock & supply.

14. Explain any two factor of supply of a good ?

15. Distinguish between contraction & decrease in supply.

16. Distinguish between expansion & increase in supply.

17. Write the reasons behind change in supply.

18. Write the reasons behind decrease in supply.

19. How does Govt. policy affect the supply of a good.

20. Explain any four factors affecting the price elasticity of supply.

21. What is the law of supply ? Give 2 exceptions of Law of supply.

22. What is elasticity of suppply? Explain the proportionate method of measuring elasticity of supply.

Long Answer type Question: (Mark- 6)

1. Explain the law of variable proportion with the help of TP, AP & MP curves.

2. Draw AVC, ATC & MC curves in a single diagram. State the relation between MC, AVC and ATC .

3. Discuss in brief the various kind of price elasticity of supply.

4. Explain the factors of supply of good.

5. What is change in supply ? Explain its various kinds with numerical example & figure.

6. What is change in quantity supply ? Explain its various kinds with numerical example & figure.

Numerical Question: (Mark- 3/4)

PRODUCTION

Calculate AP & MP,

Variable factors	1	2	3	4	5	6
TP (units)	8	16	24	29	29	25

2. Find TP & AP from the following table:

Labour (Units)	1	2	3	4	5
MP (units)	10	11	10	9	8

3. Find TP & MP from the following table:

Labour (Units)	1	2	3	4	5
AP (Units)	50	45	40	35	30

4. ldentify the various stages of law of veriable proportion from the following table:

a)

Labour (Units)	0	1	2	3	4	5	6
TP (Units)	0	10	28	45	52	52	48

b)

Labour Input (Units)	1	2	3	4	5
TP (Units)	50	45	40	35	30

COST

5. Find FC & VC from the following table:

Output (Units)	0	1	2	3	4	5
TC (R)	40	100	120	130	150	190

6. Find TC, (FC = R 60 at 0 level of output)

Output	0	1	2	3	4	5
TVC (R)	0	16	22	29	42	48

7. Find AVC & MC

Output (Units)	0	1	2	3	4
TC (R)	100	160	212	280	356

8.   Find TFC, TVC, AFC, AVC, MC

Output (Units)	0	1	2	3	4	5	6
TC (R)	120	150	170	186	200	220	270

9.   Find TVC & TC [if TC = R 20 given]

Output (Units)	1	2	3
MC (R)	10	15	25

10. Find MC

Output (Units)	1	2	3	4	5	6
AC (R)	60	40	30	26.25	28	35

11. Find TC, VC, AFC, AVC & AC, If FC = R 400 given.

Output (Units)	1	2	3	4	5	6
MC (R)	150	110	130	150	210	310

12. Find MC.

Output (Units)	1	2	3	4	5	6
AVC (R)	13	11	10	10	11	12

REVENUE

13. Find TR, AR & MR

Unit Sold	1	2	3	4
Price (R)	5	4	3	2

14. Estimate TR & MR

Unit Sold	1	2	3	4	5	6
AR	25	23	21	19	18	15

15. Calculate TR, AR

Output Sold	0	1	2	3	4	5	6	7
MR	-	14	10	7	5	0	-3	-5

16. Complete the following table:

Output	1	2	3	4	5
AR (R)	10	-	8	-	-
MR (R)	10	8	-	0	-
TR (R)	10	-	-	-	20

17. Complete the following table:

Price (R)	7	-	-	1
Output (U)	-	2	3	-
TR (R)	7	10	-	-
MR (R)	-	-	-1	-5

PRICE ELASTICITY OF SUPPLY

18. Due to rise in the price of potato from ₹15 to ₹18/ unit, its supply rises from 100 kg to 150 kg. Find elasticity of supply. (Ans : 2.5)

19. The supply of a good prices by 60% due to rise in its price by 30% calculate price e^s. (Ans : 2)

20. Price e^s of good is 2. If its price falls from ₹ 10 to ₹ 8/u its supply falls by 500 units. Find supply at reduced price. Ans : 2)

21. The supply of a good rises by 25% due to rise in its price from ₹ 4/Unit to ₹ 5/ Unit. Find e^s?

(Ans : 1)

22. A producer supplies 200 units of x at ₹ 10/ unit. If e^s=2 find its supply at ₹ 11/ unit. (Ans: 240)

23. Price e^s of X = 2. A producer supplies it at ₹ 10/ unit. How muct the price/ unit has to be reduced so that supply will fall by 40% ? (Ans: 8)

24. A producer carns of ₹ 1000 at a price of ₹ 10/ unit of a good. (given e^s = 0⁰8). If the price falls by 10% find his total revenve at that reduced price. (Ans: 828)

25. The price of a good rises by 30%. Find percentage increase in its supply, if e^s = 2. (Ans: 60%)

26. The supply of a good rises by 25%. If the initial price is ₹ 4/ unit and e^s = 1 then find the increased price.

(Ans: 5)

27. e^s of good x is 2.5. If its price rises from ₹ 2 to ₹ 3/ unit and the initial supply was 100 units, find the supply at increased price. (Ans: 150)

ANSWER KEY

True/ Falls

1. False	2. False	3. True	4. True	5. True	6. False
7. True	8. False	9. False	10. True	11. True	12. True
13. True	14. True	15. False	16. True	17. True	18. False
19. True	20. False	21. True	22. False	23. False	24. False
25. False	26. False	27. True	28. True	29. False	

Multiple Choice

1. b) maximum	2. c) more than AP	3. a) AP rises
4. a) AP	5. b) inversely U-shaped	6. b) rises
7. c) TFC, TC	8. a) MC	9. b) TVC
10. c) variable proportion	11. b) more than AC	12. d) neither 'a' nor 'b'
13. c) variable cost	14. d) AFC = ATC-AVC	15. a) AFC
16. a) rises	17. c) 5	18. c) both 'a' & 'b'
19. d) all of the above	20. c) fall in factor price	21. a) rise in price
22. a) change in price	23. b) parallel to y axis	24. c) 0°
25. b) rise in factor price.		

Very short answer type question

1. No, because AP is the ratio between TP & quantity of labour & TP never be Zero.

2. Yes, when TP falls, MP becomes negative.

3. TP, AP & MP curves are inversely U-shaped in general

4. It refers to a situation when output is increased by only increasing one of the variable factor (say labour), keeping the other factors fixed.

5. It refers to a situation when output is increasing by increasing all the factors simultaneously in the same proportion.

6. It is the actual money expenditure on inputs or payments made to outsiders for hiring their factor services.

   Example: Payment for raw material, transportation, publicity etc.

7. It is the estimated value of the inputs supplied by the owners including normal profit.

   Example: Rent if own land, wage for self labour etc.

8. MC is not related with FC and MC affected only by change in variable cost.

9. AFC falls with the rise in output

10. MR = 0 when TR is maximum & MR becomes negative when TR falls.

11. When MR = 0, TR becomes maximum

12. Supply curve is upward sloping curve.

13. Due to change in other factors except price.

   Like : use of premitive technology, rise in factor price etc.

14. Rise in factor price

15. When the rate of change in quantity suppiled of a good is more than the rate of change in its price.

Numerical Question

Production

1. AP: 8, 8, 8, 7.25, 5.8, 4.16

    MP: 8, 8, 8, 5, 0, -4

2. TP: 20, 36, 48, 56, 60, 60, 56

    AP: 20, 18, 16, 14, 12, 10, 8

3. TP: 10, 22, 36, 48, 56, 60

    AP: 10, 11, 12, 12, 11, 10

4. a) MP: 10, 18, 7, 0, -4

    Phase 1:   up to 2 unit of labour

    Phase 2:   up to 4 unit of labour

    Phase 3:   6th unit of labour

4. b) Phase 1:   up to 2 unit of labour

    Phase 2:   3 to 4 unit of labour

    Phase 3:   5 unit of labour

Cost

5.   FC: 40 at all level of output

     VC: 0, 60, 80, 90, 110, 150

6.   TC: 60, 76, 82, 89, 102, 108

7.   MC: -, 60, 52, 68, 76

     AVC: -, 60, 56, 60, 64

8.   FC: 120 at all level of output

     VC: 0, 30, 50, 66, 80, 100, 150

     FC: -, 120, 60, 40, 30, 24, 20

AVC: -, 30, 25, 22, 20, 20, 25

MC: -, 30, 20, 16, 14, 20, 50

9. VC: 10, 25, 50

TC: 30, 45, 70

10. MC: 60, 20, 10, 15, 35, 70

11. TC: 550, 660, 790, 940, 1150, 1460

TVC: 150, 260, 390, 540, 750, 1060

AFC: 400, 200, 133.33, 100, 80, 66.67

AVC: 150, 130, 130, 135, 150, 176.67

AC: 550, 330, 263.33, 235, 230, 243.33

12. MC: 13, 9, 8, 10, 15, 17

Revenue

13. TR: 5, 8, 9, 8

AR: 5, 4, 3, 2

MR: 5, 3, 1, -1

14. TR: 25, 46, 63, 76, 90, 90

MR: 25, 21, 17, 13, 14, 0

15. TR: -, 14, 24, 31, 36, 36, 33, 28

AR: -, 14, 12, 10.33, 9, 7.20, 5.50, 4

16. AR: 10, 9, 8, 6, 4

MR: 10, 8, 6, 0, -4

TR: 10, 18, 24, 24, 20

17. Price: 7, 5, 3, 1

output: 1, 2, 3, 4

TR: 7, 10, 9, 4

MR: 7, 3, -1, -5

Theory of Firm under Perfect Competiton

The theory of firm under perfect competition:-

In the previous chapter we have studied the production function and the cost curves of firm. In this chapter and the next two chapters our area of discussion will be the equilibrium of a firm or a producer and different market structures where the firms operate.

4.1 Market structure:

In the general sense, all the places of purchase and sale of goods and services in different areas are called market. However in economics, the term market is used in a different sense. In economic sense, a market is not a particular place or locality where goods are bought and sold. A market for a particular commodity can be regional, national or international. According to Prof. Samuelson, a market is a mechanism through which buyers and sellers interact to determine price and quantity of good and services.

Market can be classified from different perspectives. For example, based on the degree of competition, the market can be classified into three categories.

a)   Perfectly competitive market

b)   Monopoly market

c)   Imperfectly competitive market.

Imperfectly competitive market again classified into different forms.

4.2   Perfectly competitive market or market under perfect competition:

4.2.1 Definition:

A market in which a large number of buyers and sellers compete for the transaction of the homogeneous product is called a perfectly competitive market

Perfectly competitive market is such a market structure where competition among the rivalry firms is completely absent. It is an ideal market structure which rarely exists in the real world. A good example of perfect competition is the agriculture market, foreign exchange market. Although perfect competition is an unusual phenomenon, but theoretically it is very important in determining the price in economics.

4.2.2 Features of perfect competition and there implication:

There is said to be perfect competition in an industry or in a market when certain conditions or assumptions are satisfied. These conditions or assumptions are divided into two groups:

a) Conditions of pure competition.

b) Conditions of perfect market

A) Conditions of pure competition:

1. A large number of buyers and sellers:

There are so many buyers and sellers in a perfectly competitive market or industry. The number of buyers and sellers is so large that each buyer buys a very small portion of the total goods and each seller sales a very small portion of the total goods in the market.

The implication of this assumption is that no buyer or seller can individually influence the price of a product in the market. It means the buyers and sellers are price taker, not the price maker in this market.

2. Homogeneous product:

The product that are sold in this market are identical. When one unit of the product is a perfect substitute for another is called homogeneous product.

The economic implication is that no seller can increase or decrease the price of a product. Because, as the product are homogeneous, if the seller increase the price of his product, the buyers will move to another seller. Again the sellers does not decrease the price. Because he can sell all his product at current price. So the price of the product of each firm is same or fixed. As a result, average revenue(AR) is constant, which is equal to market fixed price. Since, the product is homogeneous, there is no questions of any advertising cost.

3. Free entry and exit:

In a perfectly competitive market, new buyers and sellers can enter the market without any barrier in the long run but not in the short run. And also old buyers and sellers can leave the market without any restrictions. As the resources are perfectly moveable, so free entry or exit is almost cost free.

The economic implication of this assumption is that the firms make only normal profit in the long run. It is because if there is super-normal profit in the industry in the long run, it attracts new firms and they enter the market and as result the amount of profit reduced to normal profit. On the other hand, if there is loss in the industry then some existing firms leave the market and the price of the product of remaining firms is equal to the average cost (AC) in the end, so they ensure normal profit.

B) Conditions of perfect market-

4) Perfect knowledge:

In the perfectly competitive market, every buyer and seller has perfect knowledge of the market.

The economic implication of this assumption is that there is only one price in the market and all products are traded at that price. Sellers do not get any chance to increase the price of the product as the buyers have complete knowledge about the market.

On the other hand, all firms with a perfect knowledge of the factors market have the equal opportunity to use same type of factors of production and technology. As a result, no firm gets the benefit of production cost and the cost structure of all the firms is uniform.

5. **Perfect mobility of factors of production:**

The factors of production used in the perfectly competitive market are completely moveable. This means that factors can move freely from one place to another or from one used to another.

6. **Absence of transport cost and selling cost:**

In perfect competition the firms only bear the production cost. No expenses are incurred in this market for sale and transportation of product. This condition or assumption is taken to keep the price of product stable.

According to many economists if there are a large number of buyers and sellers, homogeneous product is traded and buyers and sellers could freely enter or exit in market, is called pure competition. On the other hand, a perfect competitive market has all the features described above.

4.2.3 Total revenue, Average revenue, Marginal revenue curve, Dimand curve, Price line, Selling curve of firms under perfectly competitive market.

Total Revenue : A firm that sells it's output at market price and earn revenue is called total revenue (TR). In perfect competition if a firm sells quantity Q at fixed price P, then its Total Revenue, $TR = P \times Q$. In this market, total income increases at the same rate as output at fixed price increases, and at the zero level of production, total revenue is also zero. So in perfect competition, firm's TR curve is a straight line upwards sloping from the origin. The slope of this curve is price (P).

Average Revenue : The per unit revenue of a firm is called average revenue (AR). The average revenue is obtained by dividing the total revenue (TR) of the firm by the amount of sales (Q).

$$AR = \frac{TR}{Q} = \frac{P \times Q}{Q} = P$$

So the price (P) of the product and average income (AR) of a firm are same. Therefore, AR curve and price line of a firm are same. At this particular fixed price, a firm can sale any amount of its output. So, this price line is the demand curve (d) of the buyers and the selling curve of the sellers.

So, Demand (d) = price (p) = Average Revenue (AR) = Marginal Revenue (MR).

As the price is fixed in perfect competition, so AR curve or demand curve is infinitely elastic and parallel to x-axis.

Marginal Revenue: Marginal revenue is addition made to total revenue when one more unit of output is sold. In perfect competition, if a firm increase its output from Q_1 to Q_2 at fixed price, then,

$$MR = \frac{PQ_2 - PQ_1}{Q_2 - Q_1} = \frac{P(Q_2 - Q_1)}{Q_2 - Q_1} = P$$

So, AR and MR curve of a firm are indifferent in this market.
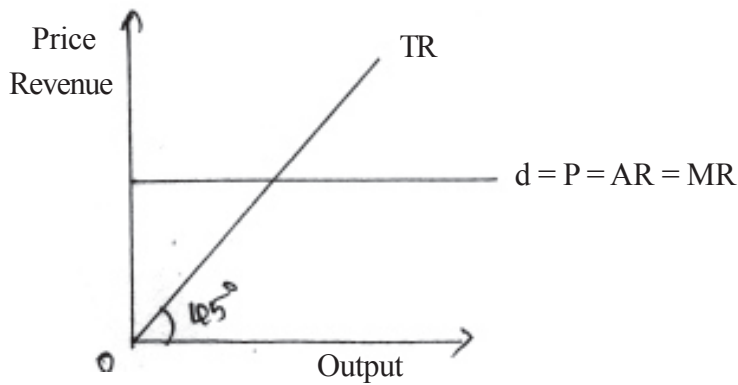
Fig : Fixed curves under PCM

4.2.4 Price determination in the perfectly competitive market:

Each individual buyer and seller is Price taker under perfectly competitive market. The price of the commodity is determined by the intersection of the market demand and supply curves of the commodity. Market demand curve is the sum of the individual demand curves of different buyers. This market demand curve is downward to the right like individual demand curve. On the other hand, the sum of the individual supply curves of different firms is the market supply curve. The market supply curve is upwards to the right just like the individual supply curve. The price at which market demand curve (DD) & market supply curve (SS) are equal is called equilibrium price or market price (P).

Fig.: DD and SS curves intersect each other at equilibrium point E. So OP is the equilibrium market price and OQ is the equilibrium output of the product
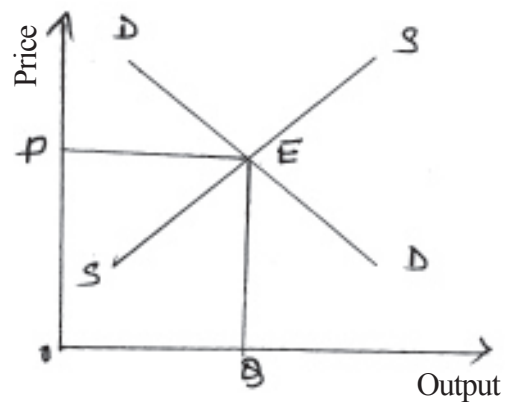
4.3. Equilibrium of producer or firm-conditions of profit maximisation:

The main objective of a producer or a firm is to achieve maximum profit. Profit is the difference between total revenue (TR) and total cost (TC). So a firm is in equilibrium position when it achieve maximum profit. Then the difference between total revenue and total cost is maximum.

The equilibrium or profit maximisation conditions of a firm can be classified in two approaches-

a)   Total revenue- total cost approach

b)   Marginal revenue- marginal cost approach

4.3.1 TR-TC approach and determination of equilibrium of a firm in perfect competition:

In a perfectly competitive market as the main objective of a firm is to earn maximum profit, so he is in equilibrium when he achieves maximum profit. So with TR-TC curve, equilibrium conditions of a profit maximising firm are-

i)   The vertical distance between TR and TC is maximum, where both TR and TC curves have the same slope.

ii) Profits fall if one more unit of output is produced. The Firm's equilibrium conditions with TR-TC curves is graphically shown in the following figures.

TR = TR is the total revenue curve which is a straight line, upwards sloping from the origin. The slope of the curve, is the price (P).

TC = Total cost curve is inverse 's' shaped, starting from the level of fixed cost, reflecting the law of variable proportion.

TT Curve = Total profit curve, it is obtained by subtracting TC curve from TR curve at each level of output.

OX = At this level of output, the vertical distance between TR and TC curve is maximum . So it is the level of maximum profit.

Point A and B = Break- even points and at these points TR = TC and profit is zero.

Losses = The firm incurs losses when TC is more than TR. This occurs from origin till X_A output level and beyond X_B output level. Thus the firm is in equilibrium at OX level of output where it earns maximum profit.
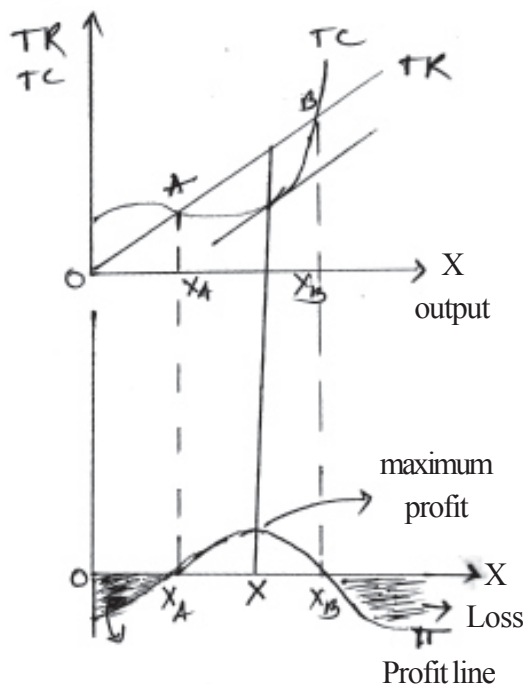
4.3.2 MR-MC Approach and determination of equilibrium of a farm in perfect competition.

It is more important to analyse the equilibrium or the profit maximum conditions of the firm with the marginal revenue- marginal cost approach. To achieve the maximum profit, the firm produces at a particular level of output, which is the equilibrium level of output of the firm. Any level of output more or less than this particular level, does not ensure the maximum profit of the firm. In this situation, the three conditions of profit maximisation that must hold are:

i)   The market price (P) is equal to the marginal cost at equilibrium level of output.

ii) The marginal cost is non- decreasing at equilibrium output.

iii) In the short run P> AVC at equilibrium output and in the long run, P> AC at equilibrium output.

Condition 1: P= MC

In a perfectly competitive market for a profit maximising firm, profit is the difference between TR and TC. As long as the change in total revenue (MR) is more than the change in total cost (MC), production will increase and with it profit will increase. That means if MR(p)>MC both production and profit have the potential to increase. Again, if MR (p) <MC, then due to increase in costs the firm incurs losses and they will not produce. In his situation the firm will achieve stability and maximise its profit only when MR, i.e, price (p) is equal to MC.

In this figure at output OQ_1, price CQ_1 is greater than magical cost EQ_1. As the price is greater, so firm has the potentially to increase output and profit. Again at output OQ_2, price FQ_2 is less than marginal cost DQ_2. As price is less than cost, so firm will not produce at this level, rather reduce production. So Considering both the aspects it can be said that OQ_0 is the desirable production level of the firm where P = MC and profit is maximum.
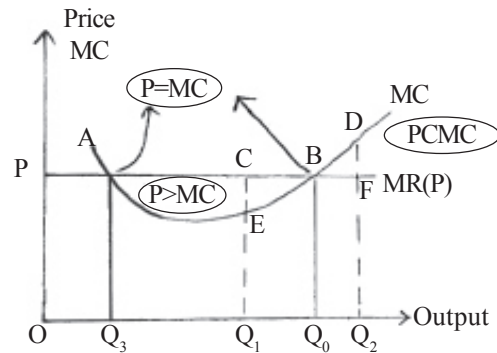
Fig: Condition-1 and Condition 2

Condition 2: MC is non decreasing at equilibrium output.

According to the profit maximisation condition of firm in a perfectly competitive market, MC Curve of firm is non decreasing at equilibrium output.That means, the slope of MC curve cannot be negative.

In the previous figure the first condition of profit maximisation P = MC is satisfied at both the point A and B. But at point A, output OQ_3, MC curve is decreasing. So there is a possibility to increase both production and profit as cost is decreasing.

On the other hand, at point B, output OQ_0, P = MC and MC curve is non-decreasing. That means, after that level, it is not profitable to increase output, as cost (MC) is more than price. So, at point B, OQ_0 is the desirable level of output where MC curve is increasing.

Condition 3:

i)   In short run, P> AVC

ii)   In long run, P> AC.

i)   In a perfectly competitive market, the firms will not produce at the level where average variable cost (AVC) is greater than price (P) in the short run. Because as soon as the market price is lower than AVC, it will discourage the firm to produce bearing fixed costs as well as variable costs (like, cost of labour, electricity, raw materials etc.)

In the figure, at OQ_0 level of output, E is the equilibrium point where condition-1 and condition-2 are satisfied. At this point, price (EQ_0) is greater than AVC (BQ_0). In this situation, if the firm want to produce at the level OQ_0, then it has to bear total fixed cost as well as variable cost, equal to the area of □ ABEP. So firm will not produce if price is less than AVC. That means, the firm will produce only when P>AVC in the short run.
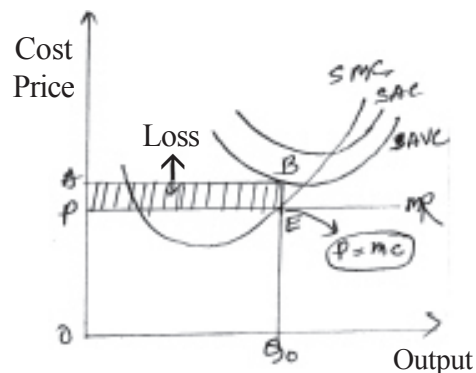
ii)   In a perfectly competitive market, the firm will not produce at the level where average cost (AC) is greater than market price (p). It is because, as there is free entry and exit of firms in the long run, so the firms only earn normal profit. In this situation, in the longrun if P<AC, the farms will face losses and they will force to stop production and leave the market.

In the figure, at OQ_0 level of output, E is the equilibrium point where condition-1 & condition-2 are satisfied. At this point, price (EQ_0) is more than average cost (BQ_0). In this situation the firm has to bear variable cost equal to area of ☐ABEP at OQ_0 level of output. So, firm will not produce if price is less than AC. That means the firm will produce only when P>AC, in the long run.
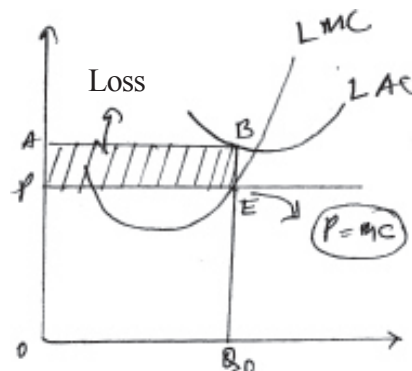
4.4   Short run equilibrium under perfect competition- four situations can take place in the short run.

Situation-1: Supernormal profit

In the perfectly competitive market the firm will earn supernormal profit in the short run when P=MR = MC>AC.

In the figure, E_1 is the equilibrium point where the necessary and sufficient conditions of equilibrium are satisfied. Here, firm earns supernormal profit as P>AC.

Supernormal profit = Total revenue – Total cost

$= ☐ \ OQ_1E_1P – ☐ OQ_1BA$

$= ☐ \ ABE_1P$
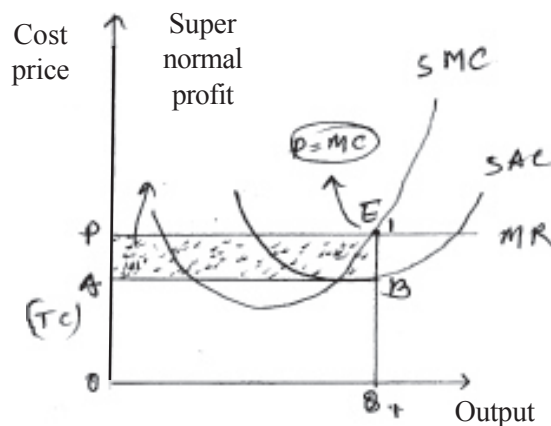
So, at equilibrium point E, through producing OQ_1 Level of output the firm earns ☐ ABE_1P, Supernormal profit.

Situation 2: break even point and normal profit:

The break even point of a firm arises when TR equals TC or AR equals AC. At this point firm is earning zero economic profit. The minimum profit which a firm needs to run his business, is called normal profit. Normal profit of firm is included in his total cost. At break even point the market price passes through the minimum point of AC curve.

In the figure, E_2 is the equilibrium point or break even point where $P = MR = M = AR = AC$. Here, as $P = AC$, the Super normal profit of firm is zero and it earns normal profit.

Profit = TR–TC

$= \square\, OQ_2E_2P - \square\, OQ_2E_2P$

$= 0$ (Zero)

Situation-3 : Total Losses Minimised

This situation occurs when the price is so low that it does not cover fully the average fixed cost (AFC). Then P<AC but P>AVC, that means, though price is less than average cost but greater than average variable cost. In this situation the firm still continues to produce even though there are losses because at least AVC is being covered by price.

In the figure, E_3 is the point of equilibrium. Here price OP is less than AC (BQ_3) so it incurs losses. But still price is greater than AVC (FQ_3), So it continues to produce.

Total Loss = TC – TR

$= \square\quad OQ_3BA - \square\quad OQ_3E_3P$

$= \square\quad PE_3BA.$

Situation-4: Shutdown point

This situation occurs when the price so low that it cannot cover the fixed cost at all. The losses are incurred which are equal to fixed costs. The revenue just covers variable costs. The price line passes through the minimum point of the AVC curve.

In the adjacent figure, at price OP, E_4 is the point of equilibrium.

Total loss = TC – TR

$= \square\quad OQ_4BA - \square\quad OQ_4\,E_4P$

$= \square\quad PE_4BA.$
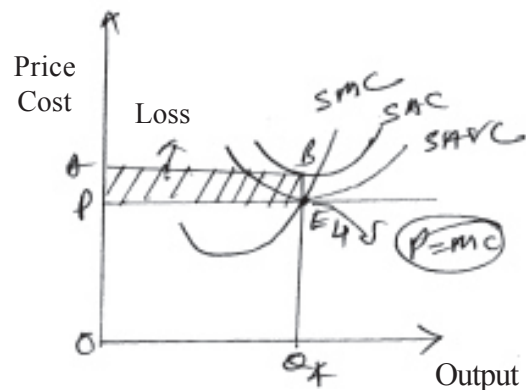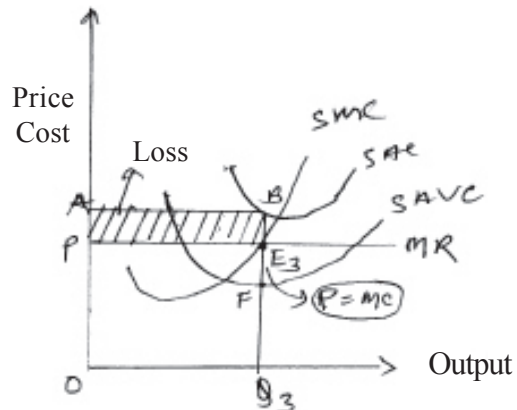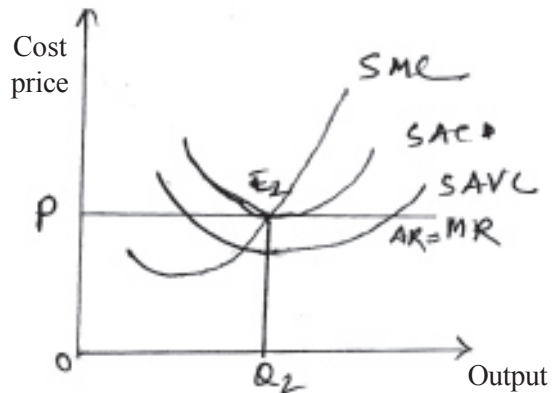
Loss per unit is equal to average fixed cost (equal to BE_4). Point E_4 called shutdown point. For Price below OP the firm will produce nothing at all because the loss will be greater than fixed costs.

4.5   Supply curve of a firm under perfect competition:

Short run supply curve

A firm's supply curve in the short run is the rising part of the SMC curve from and above the minimum AVC. So at all prices strictly less than the minimum AVC, output is zero. In the adjacent figure, the bold line represents the short run supply curve of the firm.
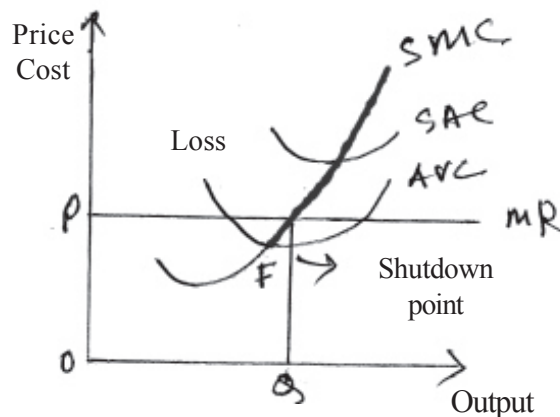
Long run supply curve :

A firm's long run supply curve is the rising part of the LMC curve from and above the minimum LAC curve. So, in the long run at all prices strictly less than the minimum LAC, output is zero. In the adjacent figure, the bold part of LMC curve represents the long run supply curve of the firm.
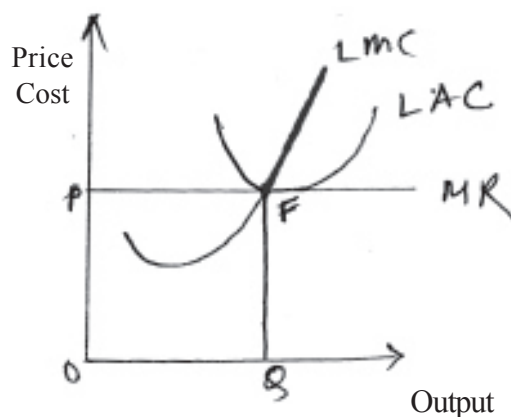
Exercise

1.1 True/False:-

1. As there are large number of buyers and sellers so they are price taker, not price maker.
2. In the long run firms earn supernormal profit in a PCM (Perfect Competitive Marker).
3. There is only production cost in a PCM.
4. There is perfect mobility of factors of production in a PCM.
5. In a PCM, the rate at which production increases, total revenue is also increase at the same rate.
6. The demand curve and the price line are different in a PCM.
7. In a PCM, Marginal revenue curve is parallel to x-axis.
8. In a PCM, If market price is greater than marginal cost, then production will increase.
9. In a PCM, AR = AC is the condition of break-even point.
10. The long run supply curve is MC curve of a perfectly competitive firm.

1.2 Fill in the blanks:-

1. In a perfectly competitive market the short run supply curve is the MC curve above the _____ point.
2. In a PCM, if p=AVC is the position of _____ point.
3. In a PCM, if P>AVC but P<AC then firm faces the situation of _____.
4. In break-even point, market price is equal to _____.
5. In equilibrium, MC curve is _____.
6. In a PCM, the intersection of market demand and market supply determines _____.
7. In a PCM, demand curve is _____ elastic.
8. In a PCM, selling cost is _____.
9. As homogeneous product is sold in a PCM, so the price of the product is _____.
10. In a PCM, in the long run if there is supernormal profit, then _____ enter.

1.3 MCQ:

1. How many types of market are there, according to the degree of competition-
   a) Two   b) Three   c) Four   d) Five.
2. Which of the following is not the condition of a pure competition-
   a) A large number of buyers and sellers   b) Homogeneous product
   c) Free entry & exit   d) Absence of transport cost.
3. Which of the following is not applicable in case of total revenue-
   a) P×Q   b) AR × Q   c) MR × Q   d) All are correct.

4. AR curve is same as-
      a) Price line   b) MR curve   c) Demand curve   d) All of these.

5. Which of the following condition is not applicable for short run equilibrium of a firm-
      a) MR = MC   b) slope of MC>O   c) P>AVC   d) P>AC.

6. In a PCM, if AVC<P then firm earns-
      a) Supernormal profit   b) Normal profit   c) Losses   d) All of these

7. In case of break-even point-
      a) P = AC   b) P = AVC   c) P<AC   d) P>AVC.

8. In case of shutdown profit-
      a) P = AVC   b) P = AC   c) P>AVC   d) P>AC.

9. A large number of sellers sell homogeneous product, it means-
      a) Horizontal supply curve   b) Lots of losses of all sellers
      c) Inability of sellers to change price   d) Vertical demand curve.

10. Which of the following is not a example of PCM-
      a) Agriculture market   b) Stock market
      c) Foreign exchange market   d) Labour market.

1.4 Very short question :   **1 mark each**

1. Define perfect competition.
2. Write two features of pure competition.
3. What is homogeneous product?
4. Write the difference between super normal profit and normal profit.
5. What is called the slope of the total revenue curve of a PCM?
6. What is the shope of the demand curve in a PCM?
7. What is average revenue in a PCM?
8. How price is determined in a PCM?
9. How the profit of a firm is determined?
10. Write the condition of equilibrium of TR-TC approach in a PCM.
11. Write the conditional difference between short run and long run equilibrium of a firm.
12. Write the necessary and the sufficient condition of equilibrium of a perfectly competitive firm.
13. Write the condition of supernormal profit of a firm in short run.
14. What is break-even point?
15. In which case the firm will not leave the market even if they incur losses?
16. What do you mean by shutdown point?

17. Which is the short run and long run supply curve in a PCM ?

2. Answer the following questions : (3/4 marks)

1. Write the features of a perfectly competitive market (PCM).

2. How price is determined in a PCM?

3. In a perfectly competitive market buyers and sellers are price taker, not price maker-discuss.

4. Why market price is fixed in a PCM?

5. "In the long run perfectly competitive firms earn normal profit| – Discuss.

6. Mention the differences between pure competition and perfect competition.

7. Determine the demand curve in a perfectly competitive market.

8. Discuss the shape of total revenue curve in a PCM.

9. Write the relationship among the average revenue, marginal revenue and market price in a PCM.

10. Determine the maximum profit of a perfectly competitive firm using TR-TC approach.

11. Write the short run and long run conditions of the firm under a PCM.

12. How the short run equilibrium of a firm is determined with the help of marginal revenue and marginal cost curves?

13. Will the production continue even if the firm losses in the short run? Justify

    Or, do you think the perfectly competitive firm stops production if it cannot pay fixed costs in the short run? Justify.

14. Explain the long term equilibrium position of the firm with diagram.

15. How the short run supply curve of a firm is determined in a PCM?

16. Discuss the concept of break-even point and shutdown point of a perfectly competitive form with diagram.

17. Follow question number- 8,9,10 & 11 of the exercise in text book.

Answer

True/False:

1. True 2. False 3. True 4. True 5. True 6. False 7. True 8. False 9. True 10. False.

Fill up the blanks:

1.	Shutdown	2.	Shutdown
3.	Total loss minimised	4.	Average cost
5.	Non decreasing	6.	Price
7.	Perfectly	8.	Absent
9.	Fixed	10.	New firms

MCQ:

1. b   2. d
3. d   4. d
5. d   6. d
7. a   8. a
9. c   10. d

Answer of one more question:

1. A market in which a large number of buyers and sellers compete for the transaction of the homogeneous product is called a perfectly competitive market.

2. i) A large number of buyers and sellers. ii) Homogenous product

3. When one unit of product is a perfect substitute for another is called homogeneous product.

4. Super normal profit is a matter of short run whereas, normal profit is a matter of both short run & long run.

5. Price

6. Parallel to x-axis

7. The per unit revenue of a firm is called $AR = \dfrac{TR}{Q}$

8. The intersection of market demand and market supply curves determines price.

9. Profit = Total Revenue- Total cost.

10. When the vertical distance between TR and TC curve is maximum.

11. In a perfectly competitive market, P>AVC in short run & P>AC in long run.

12. Necessary condition: P(MR) = MC

    Sufficient condition = Slope of MC curve > 0

13. P = MR = MC>AC

14. The point at which TR = TC or AR = AC of firms is called Break-even point.

15. When P<AC but P>AVC

16. In the short run, at the point where P = AVC of a firm, is called shut down point.

17. A firm's short run supply curve is the rising part of the SMC curve from and above the minimum AVC curve.

    A firm's long run supply curve is the rising part of the LMC curve from and above the minimum LAC curve.

Market Equilibrium

1. **Concept of market equilibrium:-**

Market equilibrium is the state in which market supply and market demand balance each other and as a result price becomes stable. Equilibrium price is that price at which market demand and market supply are equal to each other.

Generally, an over supply of goods and services causes prices to go down, which results in higher demand. While an-under-supply causes prices to go up resulting in less demand.

2. **Market-equilibrium under perfect competition:-**

Market equilibrium under perfect competition determined by the forces of market demand and market supply.

Market demand refers to the sum total of demand for a commodity by all the buyers in the market and market supply refers to the sum total of supply of a commodity by all the firms in the market.

Market equilibrium is struck when, at the prevailing price in the market, quantity demanded is equal to quantity supplied.
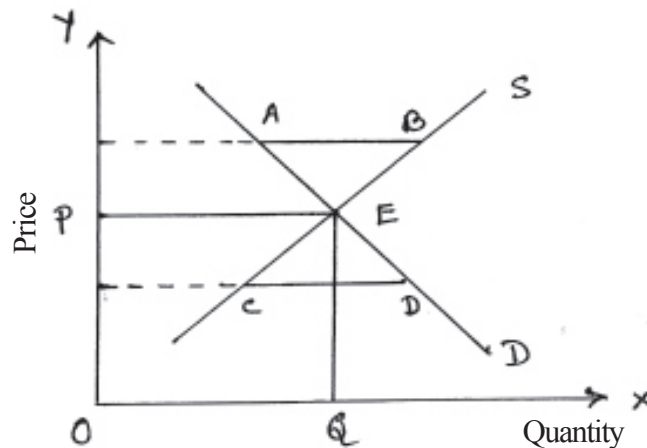
In the above fig, D is the market demand curve and S is the market supply curve. They intersect each other at point E, Where market demand = market supply. So, Here E is the equilibrium point. Here, OQ is the equilibrium quantity and OP is the equilibrium price.

If there is excess supply i.e. Market supply > market demand, which is equal to AB in the above fig. leads to a fall in the market price. Due to fall in price, quantity demanded increases from point A to point E.

At point E, Market demand is equal to market supply. Thus the equilibrium point is E and equilibrium quantity is OQ.

Again if there is excess demand i.e. market demand > market supply, which is equal to CD in the fig, leads to rise in market price. Due to rising price, quantity demanded decreases from point D to point E. At point E, Market demand is equal to market supply. Thus E is the equilibrium point where equilibrium quantity is OQ.

3.1 Shift in demand and change in market equilibrium:

There are two aspects of change or shift in demand :

i)   Increase in demand and

ii)   Decrease in demand.

i.   Increase in demand:

If there is an increase in demand, the demand curve shifts to the right from D to D_1. The new demand curve intersects the supply curve(which is unchanged) S at the point E_1. Now the new equilibrium point is E_1. The equilibrium price increases from OP to OP_1 and equilibrium quantity increases from OQ to OQ_1.
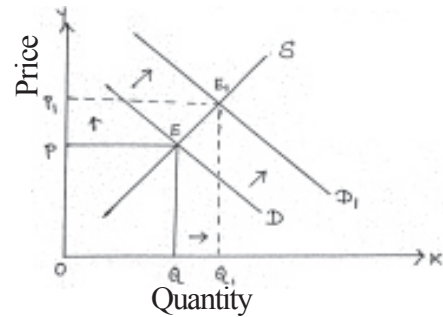
ii.   Decrease in demand:
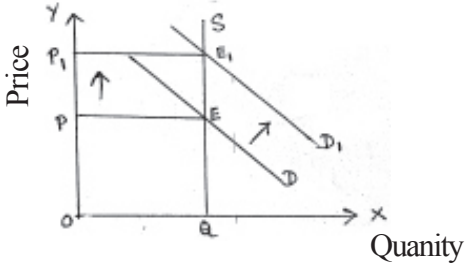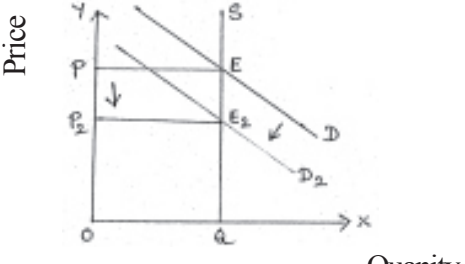
If supply remains unchanged and demand decreases the demand curve will shift to the left from D to D_2. The new demand curve D_2 intersects the supply curve at point E_2 which is the new equilibrium point. At the point E_2, The equilibrium price falls from OP to OP_2 and equilibrium quantity falls from OQ to OQ_2.
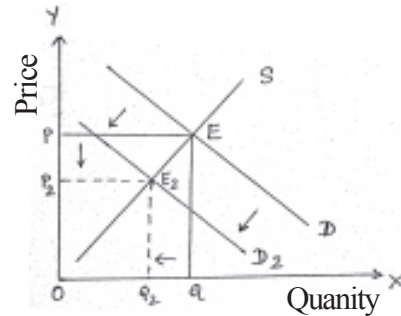
3.2 Change in demand and equilibrium price when supply is perfectly elastic:

A) Increase in demand	B) Decrease in demand
When supply is perfectly elastic if demand increases price remain unchanged and equilibrium quantity increases from OQ to OQ_1.	When supply is perfectly elastic, if demand decreases price remain unchanged but equilibrium quantity decreases from OQ to OQ_2.

3.3 Change in demand and equilibrium price when supply is perfectly inelastic:

A) Increase in demand	B) Decrease in demand
Here equilibrium price will rise from OP to OP_1 and equilibrium quantity remains unchanged at OQ.	Here equilibrium price reduces from OP to OP_2 and equilibrium quantity remains at OQ.

4.1 Shift/change in supply and market equilibrium:-

There are two aspects of change in supply :

    i)   Increase in supply   and   ii)   Decrease in supply

i)   **Increase in supply:**

If there is an increase in supply, the supply curve shifts to the right from S to S_1. The new supply curve S_1 intersects the demand curve at point E_1, which is new equilibrium point. At point E_1, the equilibrium price decreases from OP to OP_1 and equilibrium quantity increases from OQ to OQ_1.
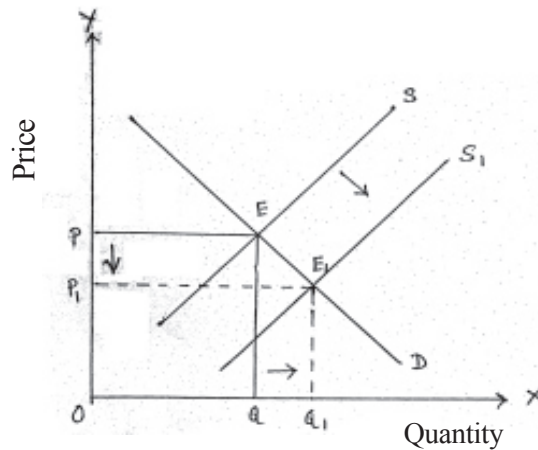
ii)   **Decrease in supply:**

If there is an decrease in supply, the supply curve shifts to the left from S to S_2. The new supply curve S_2 intersects the demand curve at point E_2, which is the new equilibrium point. At point E_2, the equilibrium price increases from OP to OP_2 and equilibrium quantity decreases from OQ to OQ_2.

4.2 Change in supply and equilibrium price when demand is perfectly elastic:-

A) Increase in Supply	B) Decrease in Supply
When demand is perfectly elastic and supply increases price remain unchanged at OP and Equilibrium quantity increases from OQ to OQ_1.	When demand is perfectly elastic and supply decreases, price remain unchanged at OP and quantity decrease from OQ to OQ_2.

4.3 Change in supply and equilibrium price when demand is perfectly inelastic:-

A) Increase in Supply	B) Decrease in Supply
When demand is perfectly elastic and supply increases equilibrium price reduces from OP to OP_1 and equilibrium quantity (OQ) remains constant.	When demand is perfectly inelastic and supply decreases, equilibrium price increases from OP to OP_2 equilibrium quantity (OQ) remains constant.

5. Simultaneous change in demand and supply and market equilibrium:

There may be simultaneous changes, both in demand and supply. It may be simultaneous increase or simultaneous decrease in demand and supply. These various situations are discussed below:

(A) Simultaneous increase in demand and supply:

The three situations of simultaneous increase in demand and supply can be described as follows :

i) Increase in demand > increase in supply:
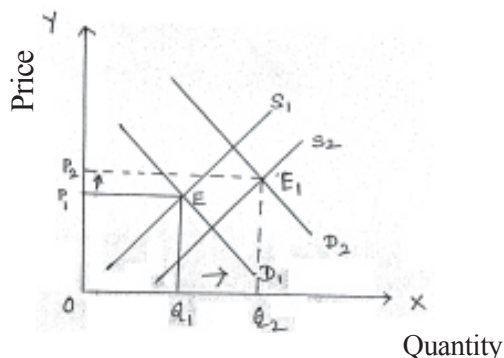
In the above fig, D_1 and S_1 are the initial demand and supply curves. E is the initial equilibrium. OP_1 is the equilibrium price and OQ_1 is the equilibrium quantity. Now the increase in demand (D_2 curve) and increase in supply (S_2 curve) intersect each other at point E_1 which is new equilibrium point. Since, increase in demand is greater than increase in supply, the new equilibrium price is OP_2 and new equilibrium quantity is OQ_2.

ii) increase in demand = increase in supply:
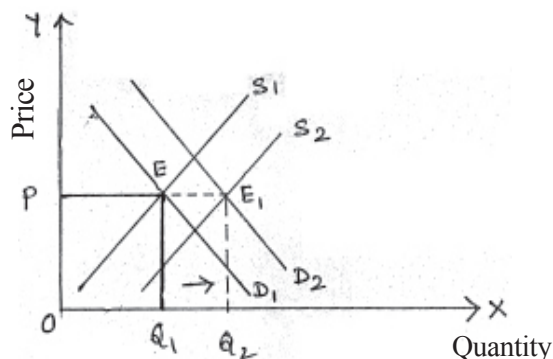
In case of increase in demand is equal to increase in supply, there is no excess demand or excess supply. Equilibrium price remain unchanged but equilibrium quantity increased from OQ_1 to OQ_2.

iii) Increase in demand < increase in supply:
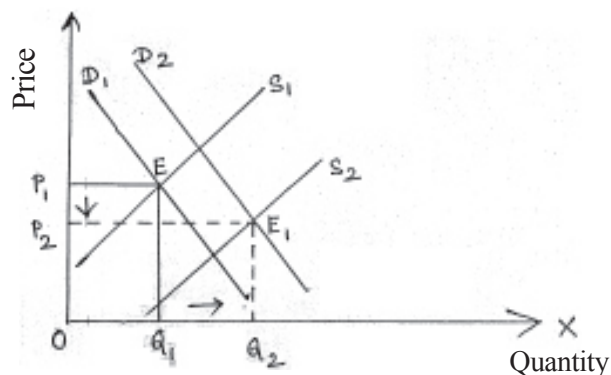
Here increase in demand is less than increase in supply. The new equilibrium is E_1 and the equilibrium price falls from OP_1 to OP_2 and the equilibrium quantity rise from OQ_1 to OQ_2.

B) Simultaneous Decrease in demand and supply:

The three situations of simultaneous decrease in demand and supply can be described as follows.

i) Decrease in demand> decrease in supply:
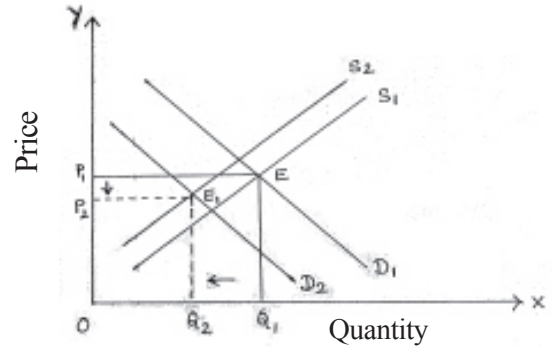
In the above fig, D_1 and S_1 are the initial demand and initial supply curve. E is the initial equilibrium point, where OP_1 is equilibrium price and OQ_1 is initial equilibrium quantity. Due to the decrease in demand and supply the new demand and supply curves are D_2 and S_2. Here, the decrease in demand is greater than the decrease in supply. The new equilibrium point is E_1, where D_2 and S_2 Intersect each other. It shows that equilibrium price falls from OP_1 to OP_2 and equilibrium quantity falls from OQ_1 to OQ_2.

ii) Decrease in demand= decrease in supply:
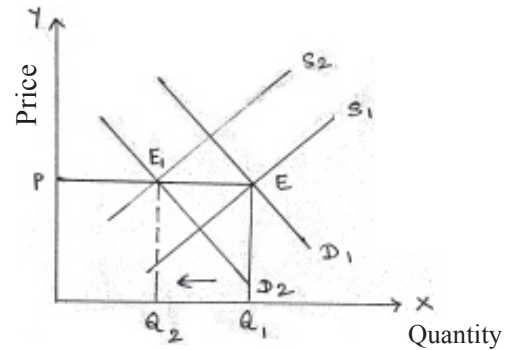
Here, decrease in demand and decrease in supply is equal. So there is no change in equilibrium price but equilibrium quantity decrease from OQ_1 to OQ_2.

iii) Decrease in demand< decrease in supply:
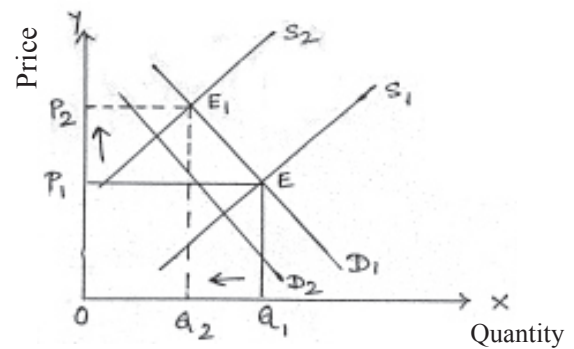
In the fig. the decrease in demand is less than the decrease in supply. The equilibrium point moves from E to E_1. The new equilibrium price rise from OP_1 to OP_2 and equilibrium quantity falls from OQ_1 to OQ_2.

6.   Simple applications:

A)   Price ceiling : The maximum price of a product fixed by the government is called price celling. Often the ceiling price is fixed which may lower than the equilibrium price. Government fixes this price to ensure the availability of essential commodities to the weaker section of the society.

Celling Price
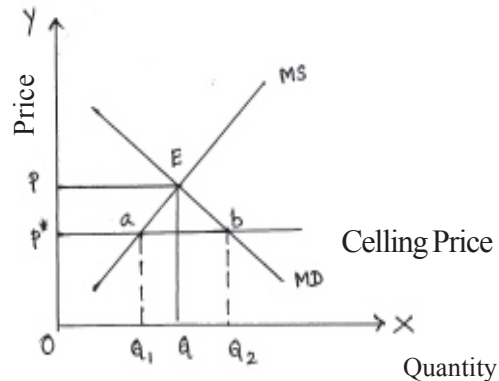
In the above fig, market demand and market supply intersects at point E which is market equilibrium. OP is the equilibrium price and OQ equilibrium quantity.

Now, government fixes the price ceiling at OP*which is lower than the equilibrium price due to the reduced price market demand extends from OQ to OQ_2 But the market supply contracts from OQ to OQ_1. In this situation there is excess demand = ab = Q_1Q_2. To overcome from this problem, rationing is the remedy. Each person is allocated a fixed quota of commodity at celling price.

B)   Price floor: The minimum price of a commodity as fixed by the government is called price floor. No one can buy the product in the market at a price lower than that floor price. Often this is equated with the support price.
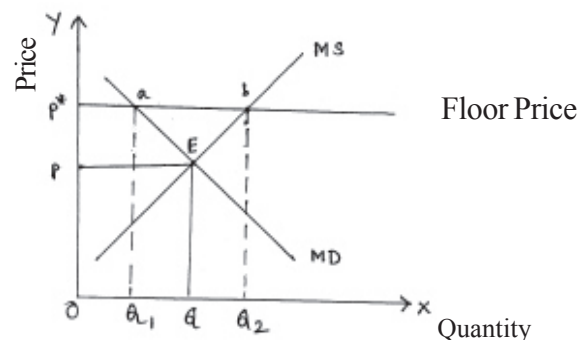
Floor Price

In the above diagram, OP is the equilibrium price and OQ is the equilibrium quantity. But government fixes OP* as the floor price which is higher than the equilibrium price. It leads to excess supply ab = Q_1Q_2. The government buys this surplus and stores it as a buffer stock.This stock may used for public distribution for the purpose of price control and unforeseen emergencies.

Exercise

A) True/False

i) Under perfect competition, firm can influence the market price.

ii) Support price regulates income of the buyers.

iii) Market demand refers to the sum total of demand for a commodity by all the buyers in the market.

iv) If there is excess demand then market demand is less than the market supply.

v) If there is no change in supply, if demand decreases the equilibrium price will fall.

vi) If there is no change in demand, if supply increases the equilibrium price will fall

B) Choose the correct answer:

i) Market equilibrium occurs when-

    a) Market demand = market supply   b) Market demand > market supply

    c) Market demand < market supply   d) None of these.

iv) In a situation of access demand market price tends to-

    a) Rise b) fall

    c) remain constant   d) none of these.

v) If Supply of a product falls equilibrium price will:

    a) Fall   b) rise

    c) remain same d) None of these.

vi) If supply curve is perfectly elastic and demand increase then the price will-

    a) Rise   b) fall

    c) remain constant   d) zero.

vii) If the demand curve is perfectly inelastic and supply increases then the equilibrium price-

    a) Falls   b) rises

    c) remains same d) none of these.

C. Filling the blanks:

i) If the supply is perfectly elastic and demand decreases the equilibrium price will be _____

ii) If the supply curve is perfectly inelastic and demand decreases the equilibrium price will be _____ .

iii) If the demand curve is perfectly elastic and supply decreases the equilibrium price will be _____ .

iv) In case of simultaneous increase in demand and supply and increase in demand is more than increase in supply then the equilibrium quantity will _____ .

v) If decrease in demand and decrease in supply is equal then the equilibrium price will be
_____ .

vi) Selling price is fixed by the _____ .

vii) The buffer stock is used for the _____ .

D. Answer the following questions: (1 mark)

i) What is market equilibrium?

ii) What is excess demand?

Iii) What is excess supply?

Iv) Define equilibrium price.

V) Define equilibrium quantity.

Vi) Define price selling.

Vii) Define price floor.

Viii) What is support price?

E. Answer the following questions: (3/4 marks)

i) How do the forces of demand and supply determine the equilibrium price?

ii) Explain the market- equilibrium under perfect competition.

iii) Explain the equilibrium price and quantity if demand increase/decrease and supply remain constant.

iv) Show with the help of diagrams the effects of equilibrium price and quantity when:

    a) Supply is perfectly elastic and demand increases.

    b) Supply is perfectly elastic and demand decreases.

    c) Supply is perfectly inelastic and demand increases.

    d) Supply is perfectly inelastic and demand decreases.

v) Explain the equilibrium price and quantity if supply increase/decrease and demand remain constant.

vi) Show with the help of diagrams the effects of equilibrium price and quantity when:

    a) Demand is perfectly elastic and supply increases

    b) Demand is perfectly elastic and supply decreases.

    c) Demand is perfectly inelastic and supply increases.

    d) Demand is perfectly inelastic end supply decreases

vii) Discuss the simultaneous change in demand and supply and market equilibrium when-

    a) Increase indemand> increase in supply

    b) Increase in demand = increase in supply.

    c) Increase in demand < increase in supply.

d) Decrease in demand > decrease in supply

e) Decrease indemand = decrease in supply.

f) Decrease indemand< decrease in supply.

viii) Write a short note:

a) Price celling

b) Price floor.

Answer

A) True/False:

i) False   ii) False   iii) True

iv) False   v) True   vi) True

B) Choose the correct answer:

i) (a)   ii) (b)   iii) (b)

iv) (c)   v) (b)

C) Fill in the blanks:

i) Same   ii) fall   iii) same

iv) increase   v) same   vi) government

vii) public distribution.

D)   **1 mark**

i) Market equilibrium is a state in which market demand and market supply are balanced each other and prices become stable.

ii) Excess demand is a situation in which market demand for a commodity is greater than its market supply.

iii) Excess Supply is a situation in which the quantity of a good or service supplied is more than the quantity demanded.

iv) The equilibrium price is the price at which quantity demanded equals quantity supplied.

v) Equilibrium quantity is that quantity which corresponds to the equilibrium price in the market.

vi) Price celling is the maximum price of a commodity that the sellers can charge from the buyers.

vii) To stabilising income of the farmers, a minimum price fixed by the government for a commodity is called price floor.

viii) The price at which the government purchase commodities to maintain a certain price level is called support price.

Non-Competitive Markets

6.1 In the general sense, market refers to a specific place where goods are bought and sold between buyers and sellers. But the meaning of market in economics is different. In economics, a market is a system by which buyers and sellers bargain for the price of a product, settle the price and transact their business-buy and sell a product or services. On the basis of competition market can be categorised in two categories-perfect competition and imperfect competition. Again there are three types of imperfect market- monopoly, monopolistic competition and oligopoly.

Market Structure

↓

Perfect Competition   **Imperfect Competition**

↓

Monopoly   **Monopolistic Competition**   **Oligopoly**
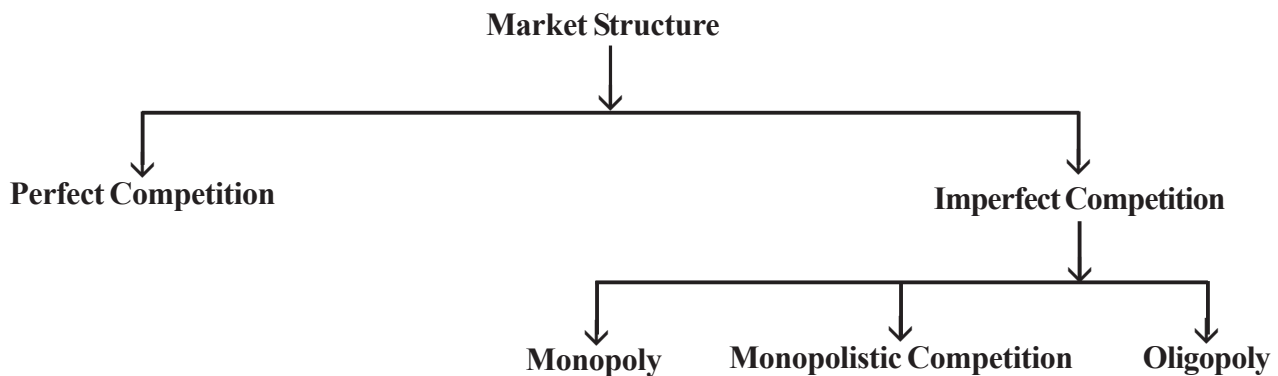
6.2   Perfect competition:

Perfect competition refers to a market situation where there are large number of buyers and sellers dealing in a homogeneous product at a price fixed by the market. e.g. Agricultural goods like wheat, rice etc.Under perfect competition, the price of the commodity is uniform and the demand curve is perfectly elastic. It means that a small change in price leads to a greater change in its quantity demanded.

Features:

i)   **Large number of buyers and sellers:**

In this market there are large number of buyers and sellers. Here an individual buyer or seller cannot influence the market price.Under perfect competition, a firm or an individual buyer both are price taker.

ii)   **Homogeneous product:**

This is one of the basic feature of perfect competition.The product that is offered for sale in the market are homogeneous (identical in every aspect like size, shape, quantity etc).

iii) Free entry and exit:

Firms are free to start producing commodity or stop production in the market i.e. entry and exit is free.

iv) Perfect knowledge about the market:

In this market, consumers and firms both have perfect knowledge of the output, inputs and their prices.

v) Perfect mobility of the factors of production :

The factors of production i.e. land, labour, capital and organisation are perfectly mobile under perfect competition. Because of this feature the production cost of the firms is equal.

vi) Absence of any other cost:

In such market there is no cost other than cost of production.There are no other expenses like advertisement cost, transportation cost etc.

6.3 Imperfect competition:

Imperfect competition occurs in a market when one of the conditions in a perfectly competitive market are left unmet. It refers to a market situation where the number of buyers and sellers are not large in comparison to perfect competition, dealing in a heterogeneous product and barriers to entry or exit in the market.There are three types of imperfect market- monopoly, monopolistic competition and oligopoly

6.3.1 Monopoly:

Monopoly refers to a market situation where there is a single seller selling a product which has no close substitute, the seller controls the price and strong barriers of entry. e.g. Railways in India. Monopoly is strictly opposite to perfect competition. In monopoly competition there is single seller who fixes its own price. If the buyers feel that the price is high, quantity demanded will be low. If the monopolist tries to increase its quantity of seiling then it can reduce its price.The demand curve of monopoly firm is downward stopping.

Features:

i) Single seller and large number of buyers:
Under monopoly competition there is only one seller and large number of buyers in the market. Here no buyer can influence the price of the product.

ii) No close substitute of the product:
As there exist only one seller for the product, it is very difficult to get an alternative product under monopoly. As there are no close substitutes, a monopolist does not face any competition.

iii) Restrictions on entry and exit:
There exist strong barriers to entry of new firms and exist of existing firms. These barriers could be due to legal restrictions like licencing, patent rights etc.

iv) Price discrimination:

A monopolist may change different price for his product from different consumers at the same time. It is known as price discrimination.

v) Price maker firm:

In monopoly competition, there is single seller who can fix whatever price he wishes to fix for his product. A monopolist thus, a price maker.

6.3.2 Monopolistic competition :

It can be treated as the Mixture of perfect competition and monopoly.Monopolistic competition refers to a market situation in which there are large number of firms which sell closely related but differentiated products. e.g. Market of products like soap, toothpaste, TV sets etc.The demand curve under monopolistic competition is downward sloping. It implies that a firm can sell more only by reducing the price of its product.

Features:

i) Large number of buyers and sellers :

Under Monopolistic competition, there exist large number of buyers and sellers, but the number of sellers are not very large as compared to perfect competition.

ii) Product differentiation :

In this competition the products which are produced by the seller are differentiated in nature.The products of each firm is differentiated from the other on the basis of colour, taste, packing, trade mark size, shape etc.

iii) Selling cost:

The expenses on advertisement and publicity are called selling cost.The producer has to spend a lot of money on advertisement just to popularise their quality, quantity and technology of the product.

iv) Free entry and exist of firms:

In monopolistic competition, firms are free to entry and exst at anytime they wish, but the entry is not as easy as perfect competition.

v) Lack of knowledge:

Due to large number of similar but differentiated products, the buyers and sellers do not have perfect knowledge about the market condition. It is very difficult for the consumer to evaluate all the products available in the market.

vi) Partial control over price:

A firm under monopolistic competition is neither a price take non price maker. However they have the partial right for controlling the price of the product.

vii) Nonprice competition:

Under monopolistic competition, firms can compete each other by offering free gifts, making favourable credit terms etc. without changing prices of their own product.

6.3.3 Oligopoly:

Oligopoly refers to a market situation in which only a few sellers are selling homogeneous or differentiated product to a large number of buyers. Oligopoly lies in between monopolistic and monopoly. e.g. Markets for auto mobile, cement, steel etc. Under oligopoly the exact behaviour pattern of the producer cannot be determined with certainity. So the demand curve faced by the oligopoly is indeterminate.

On the basis of nature of product, oligopoly can be categorised in two categories- pure oligopoly and differentiated oligopoly and on the basis of competition it can be categorised in two categories- collusive oligopoly and non-collusive oligopoly.

Types of Oligopoly

On the basis of nature of product | **On the basis of competition**

Pure Oligopoly | **Differentiated Oligopoly** | **Collusive Oligopoly** | **Non-collusive Oligopoly**
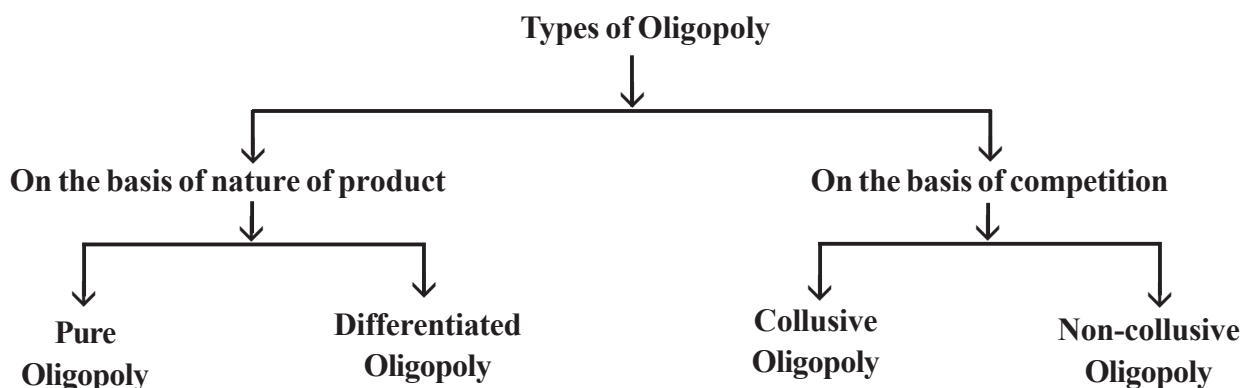
i) **Pure/Perfect oligopoly:**

The oligopoly is pure or perfect when the firms deal in homogeneous products.

ii) **Differentiated/imperfect oligopoly:**

The oligopoly is said to be imperfect, when the firms are dealing in differentiated products. i.e. The products that are closed but are not perfect substitutes.

iii) **Collusive/cooperative oligopoly:-**

Collusive oligopoly is that form of oligopoly in which all the firms decide to avoid competition and determine the price and quantity of output on the basis of cooperative behaviour

iv) **Non-collusive/ non-cooperative oligopoly:-**

Non collusive oligopoly is that form of oligopoly in which all the firms determine the price and quantity of output according to the action and reaction of the rival firms.

Duopoly:

Duopoly is the most basic form of oligopoly in which there are exactly two sellers and many buyers. Under duopoly, it is assumed that the products sold by the two farms is homogeneous and there is no substitute for it. The main feature of such market is the interdependence between the sellers. Here both the sellers closely watch the action and reactions of each other and formulate their policies accordingly e.g. Aircraft, manufacturer (Boeing and Airbus), Smartphones- Apple and Android.

Features of Oligopoly :

i)   Few firms:

In oligopoly market, there are few large firms and each farm produces a significant portion of the total output.

ii)   Interdependance:

In oligopoly market, the firms are interdependent on each other for changing price, quantity and quality of the products.

iii) Selling cost:

The expenses on advertisement and publicity are called selling cost. Due to strong competition and interdependence of the firms, they change their quantity, quality and price of the product just to popularise their product

iv)   Barriers to entry:

Under oligopoly there are some barriers like patent, requirement of large capital, control over crucial factor etc. The firms who cross the barriers can earn abnormal profit in the long run.

v)   Formulation of cartels:

Under oligopoly, firms may form a cartel and instead of independent pricing strategy Oligopoly firms prefer group discussion that will protect the interest of all the firms.

vi)   Nonprice competition:

Under oligopoly, firms can compete each other by offering free gifts, making favourable credit terms etc. Without changing price of their own product.

vii) Nature of product:

The farms under oligopoly may produce homogeneous or differentiated product.

6.4 Comparison among Perfect Competition, Monopoly, Monopolistic Competition & Oligopoly:

Basis	Perfect Competition	Monopoly	Monopolitic Competition	Oligopoly
1. Number of Buyers & sellers.	Very large number of sellers and buyers	Single seller and large number of buyers	Large number of seller and buyers	Few sellers and large number of buyers
2. Nature of Product	Homogeneous	No close substitutes	Product differentia- tion with close substitutes	Products are homogeneous or differentiated.
3. Price	Uniform price as each firm is price- taker	Price discrimina- tion is possible since the firm is price-maker	Different price because of differentiation in the product	Policy of price rigidity is followed.

4.	Knowledge about market	Perfect knowledge	Imperfect knowledge	Imperfect knowledge	imperfect knowledge.
5.	Entry and Exit of firms	Freedom to entry and exit.	Not possible	Not absolute freedom	Possible only for those firms which over come barriers like patent, requirement of huge capital etc.
6.	Selling cost	Not required	Only informative selling cost are incurred	Huge selling cost	Huge selling cost
7.	Demand curve	Perfectly elastic	Negatively slope and less elastic.	Negatively slope and more elastic.	Indeterminate demand curve
8.	Profit in the long run	Normal profit	Extra normal profit	Normal profit	Extra normal profit

Exercise

1. **State whether the following statements are True or False:**

i) There are only few sellers under oligopoly.

ii) In case of perfect competition, a firm is able to charge higher price.

iii) Under monopolistic competition, a firm can earn super normal profit even in the long run.

iv) A firm is price maker under monopoly.

v) There is no difference between a firm and industry in monopolistic competition.

vi) No selling costs are required in perfectly competitive market.

vii) A cartel is a formal collusive agreement among the firms under oligopoly.

viii) Price discrimination is associated with monopolistic competition.

ix) A monopolist can sell any quantity he likes at a price.

x) The horizontal straight line demand curve under perfect competition indicates that an individual firm has no control over price of his product.

2. **Choose the correct answer :**

i) Under monopoly, a firm has–

    a) Partial control over price   b) No control over price

    c) Full control over price   d) None of these.

ii) A farm is a price taker under–
   a) Perfect competition   b) Monopoly
   c) Oligopoly   d) Monopolistic competition.
iii) What is the nature of demand curve in case of monopolistic competition?
   a) Perfectly elastic   b) Downward slopping and more elastic
   c) Downward slopping and less elastic   d) None of these.
iv) A market situation in which there are only few sellers is called–
   a) Perfect competition   b) monopoly
   c) Monopolistic competition d) oligopoly.
v) There are a large number of buyers and sellers in–
   a) Perfect competition   b) Monopolistic competition
   c) Both (a) and (b)   d) None of these.
vi) Monopolistic competition constitutes–
   a) Single firm producing close substitutes.
   b) few firms producing differentiated substitutes.
   c) Many firm producing close substitutes.
   d) Many firm producing differentiated substitutes.
vii) Which market induces cartels ?
   a) Perfect competition   b) Monopoly
   c) Monopolistic competition d) oligopoly.
viii) In case of collusive oligopoly, prices often decided by–
   a) The firm   b) buyers
   c) both a and b   d) None of these.
ix) 'Homogeneous products' is a characteristic of–
   a) Perfect competition   b) Perfect oligopoly
   c) both a and b   d) None of these
x) Consumers get maximum variety of goods under–
   a) Perfect competition   b) Monopolistic competition
   c) Monopoly d) Duopoly
xi) 'Interdependence between firms' is a feature of –
   a) Perfect competition   b) Monopolistic competition
   c) Oligopoly   d) Monopoly
xii) Which characteristic of monopolistic competition is similar to monopoly?
   a) One seller and large number of buyers   b) demand curve slopes negatively.
   c) Freedom of entry and exit   d) Full control over price.

3. Choose the appropriate word and fill in the blank :

i) In case of monopolistic competition, _____ goods are produced.

(Homogeneous/differentiated).

ii) The demand curve for a firm under _____ is indeterminate. (Oligopoly/monopoly)

iii) A firm under _____ follows the policy at price rigidity. (Monopoly/oligopoly)

iv) There are two sellers in _____ market. (Duopoly/oligopoly)

v) In case of _____ firm has no control over the price of the product.

(Oligopoly/perfect competition)

vi) If the firms produce differentiated products then it is called _____ oligopoly.

(Perfect/imperfect)

vii) If the firms cooperate with each other in determining price or output or both, it is called _____ oligopoly. (Collusive/non-collusive)

viii) In case of _____ there is perfect knowledge among buyers and sellers.

(Monopoly/perfect competition)

ix) _____ Market structure is opposite of perfect competition. (Monopoly/oligopoly)

x) Compared with monopolistic competition, a firm's demand curve under monopoly is _____ elastic. (More/less)

4. Very short answer type questions: **(1 mark)**

i) What do you mean by the term 'market'?

ii) Define perfect competition.

iii) What do you mean by monopoly?

iv) Define monopolistic competition.

v) Define oligopoly.

vi) When a firm is called price maker?

vii) Define duopoly.

viii) What is the shape of demand curve under monopolistic competition?

ix) What do you mean by homogeneous product?

x) What is price taker firm?

xi) What is meant by price discrimination?

xii) Why are selling costs not incurred in perfect competition?

xiii) State any two features of monopolistic competition.

xiv) What is pure oligopoly?

xv)   What is imperfect oligopoly?

xvi)   State any two features of perfect competition.

xvii)   What is collusive oligopoly?

xviii) What is non-collusive oligopoly?

xix)   State any two features of monopoly?

xx)   State any two features of oligopoly.

xxi)   What is product differentiation?

xxii)   What is cartel?

xxiii) State one feature of duopoly.

xxiv)   State the characteristic which makes monopolistic competition different from perfect competition.

xxv)   What do you mean by bilateral monopoly?

xxvi)   Define monopsony.

xxvii) What is duopsony?

xxviii) What is oligopsony?

xxix)   What is 'selling cost' under monopolistic competition?

xxx)   What is mean by price rigidity?

Answers:

1. **i)** True   **ii)** False   **iii)** False   **iv)** True

    v) False   **vi)** True   **vii)** True   **viii)** False

    ix) False   **x)** True.

2. **i)** c   **ii)** a   **iii)** b   **iv)** d

    v) c   **vi)** d   **vii)** d   **viii)** d

    ix) c   **x)** b   **xi)** c   **xii)** b

3. **i)** Differentiated   **ii)** Oligopoly   **iii)** Oligopoly   **iv)** Duopoly

    v) Perfect competition   **vi)** Imperfect   **vii)** Collusive   **viii)** Perfect competition

    ix) Monopoly   **x)** less.

4. i) A market is a system of arrangement by which buyers and sellers bargain for price of a commodity, settle the price and transact their business- buy and sell a product or services.

ii) Perfect competition refers to a market situation where there are very large number of sellers and buyers dealing in a homogeneous product at a price fixed by the market.

iii) Monopoly refers to a market situation where there is a single seller selling a product which has no close substitutes, the seller controls the price and strong barriers to entry.

iv) Monopolistic competition refers to a market situation in which there are large number of firms which sell closely related but differentiated products.

v) Oligopoly refers to a market situation in which only a few sellers are selling homogeneous or differentiated products to a large number of buyers.

vi) A farm is called price maker when the price of the commodity is determined by the firm itself.

vii) Duopoly is the most basic form of oligopoly in which there are exactly two sellers and many buyers.

viii) Downward slopping.

ix) It refers to a product which is identical in all respects like quality, quantity, colour, size etc.

x) A farm is called price taker when the price of the commodity is determined by the industry.

xi) Price discrimination refers to the practise of charging different prices from different buyers at the same time for the same product.

xii) There exists perfect knowledge among buyers and sellers.

xiii) a) Large number of buyers and sellers.

b) Huge selling cost.

xiv) The oligopoly is perfect or pure when the firms deal in homogeneous product.

xv) The oligopoly is said to be imperfect or differentiated, when the firms deal in heterogeneous products i.e. Products that are closed but are not perfect substitutes.

xvi) a) Very large number of buyers and sellers

b) Products are homogeneous in nature.

xvii) Collusive oligopoly which all the firms decide to avoid competition and determine the price and quantity of output on the basis of cooperative behaviour.

xviii) Non-collusive oligopoly is that form of oligopoly in which all the firms determine the price and quantity of output according to the action and reaction of the rival firms.

xix) a) Single seller and large number of buyers.

b) Price discrimination is possible, since the firm is price maker.

xx) a) Few sellers and large number of buyers.

b) Products are homogeneous or differentiated.

xxi) It means that the product of each individual firm is differentiated from the product of other firms on the basis of brand, size, colour, shape etc.

xxii) Cartel is a group of firms which jointly set there output and price so as to exercise monopoly power.

xxiii) Two sellers and large number of buyers.

xxiv) Product differentiation.

xxv) Bilateral Monopoly refers to a market situation in which there is only one buyer and one seller.

xxvi) It refers to a market situation when there is single buyer and many sellers.

xxvii) It refers to a market situation in which there are only two buyers and many sellers.

xxviii) Oligopsony refers to a market situation in which there are few buyers and many sellers.

xxix) Selling cost refers to all the expenses incurred on marketing sales promotion and advertising of the product.

xxx) It means that even if demand changes, there will be no change in the price of commodity.

Part-B : Statistics for Economics

Class - XI

<div align="right">**CHAPTER-1**</div>

Introduction

In this chapter we will discuss about the subject matter of statistics and its significance in economics.Starting with the concept of economics we will discuss about the meaning and scope of statistics.

1. **Concept of economics** :-

    Economics is a social science concerned with the production , distribution and consumption of goods and services . It studies how individuals , government , nation make choices about how to allocate resouces.

    Few important concepts of economic discussion :-

    Consumer :- A consumer is one who consumes goods and services for the satisfaction of his wants.

    Consumption :- Consumption is defined as the use of goods and services for the direct satisfaction of our wants .

    Producer :   A producer is some one who produces and supplies goods and services .

    Production : Production is the transformation of inputs (land , labour , capital , raw materials) into output .

    Saving : The income which is not spent on consumption is defined as saving .

    Investment : Investment is an expenditure by the producers on the perchase of such assets which help to generate income .

    Distribution : Distribution means to spread the product throughout the market place such that a large number of people can buy it .

    Economic Activity : Economic activities are undertaken to produce goods and services so that human wants are satisfied .

    Economic problem :   The economic problem is the issue of scarcity and how best to produce and distribute these scare resources .

2. **What is statistics** :-   Statistics is a branch of applied mathematics that concerns the collection, organisation, analysis, interpretation and presentation of data .

    In view of vastness of the subject matter , statistics is defined both in singular sense and plural sense.

(A) **Statistics - In singular sense** :-

    In singular sense, statistics means statistical methods . It refers to techniques or methods relating to collection, classification, presentation, analysis and interpretation of quantitative data .

(B) **Statistics–In plural sense** :-

In plural sense statistics refers to information in terms of numbers or numerical data , such as population statistics, employment statistics, statistics concerning public expernditure etc .

Features of statistics in the plural sense:- Main features of statistics in terms of plural sense or numerical data are as follows :-

i) **Aggregate of Facts** :- No conclusion can be drawn from a single number . Conclusion can be drawn from aggregate number of facts .

ii) **Numerically Expressed** :- Statistics are expressed in terms of numbers .

iii) **Multiplicity of causes** :- Statistics are not affected by any single factor, but are influenced by many factors.

iv) Reasonable Accuracy .

v) Mutually related and comparable .

vi) Pre-determined objective .

vii) Enumerated and Estimated

viii) Collected in a systematic Manner.

3. **Statistical Tools** :

Statistical tools refers to the methods or techniques used for the collection , organisation and presentation of data , as well as for the analysis and interpretation of data. Tables, graphs, diagrams are the well -known statistical tools for presentation of data.

4. **Limitations of statistics** :   Some notable limitations of statistics are as follows :

i) **Study of numerical facts only** :- It does not study qualitative phenomena like honesty, wisdom, health etc. Statistics studies only numerical terms .

ii) **Studies of aggregate only** :- Statistics only studies the aggregates of quantitative facts. It cannot study any particular unit.

iii) **Results are true only on an average** :- Most statistical findings are true only as average. The findings are not always valied under all conditions .

iv) **Can be used only by the experts** :- The persons who have special knowledge of statistical methods can use statistics.

v) **Homogeneity of data , an essential requirement** :- To compare data, it is essential that statistics are uniform in quality.

5. **What statistics Does** :-

Statistics has emerged as the lifeline of economics . It is because of the growing use of statistics by the economists.

Following are the main functions of statistics in economics -

i) **Quantitative expression of economic problems** :- Any economic problem like unemployment, price rise or shrinking exports , the economists is to understand its magnitude through its quantitative expression.

ii) **Construction of economic Theories** :- Economic theory is an established statistical relationship between different sets of statistical data . The construction of economic theories are not possible without statistics.

iii) **Economic Forecasting** :- Economists do forecasting through statistical studies . On studying the behaviour of price level over several years, the economists can make statistical forecasting about the trend of price level in near future.

iv) **Formulation of policies** :- To formulate government policies the finance department takes the help of statistics to make any decision like tax rate fixation etc.

v) **Economic Equilibrium** :- Any kind of equilibrium in economics (e .g market equilibrium) comes through the use of statistical methods.

Thus , so much is the significance of statistics in economics that Marshall had a concede that– "Statistics are the straw out of which I , like every other economist, have to make bricks".

6 . **Subject matter of statistics** :-

There are two components :–

a) **Descriptive statistics** : Descriptive statistics refers to those methods which are used for the collection, presentation and analysis of data. These methods relate to measurement of central tendencies (average mean, median, mode), measurement of dispersion (mean deviation , statandard deviation), measurement of correlation etc.

b) **Inferential statistics** : Inferential statistics refers to all such methods by which conclusion are drawn relating to the universe or population on the basis of a given sample . In statistics , the term universe or population refers to the aggregate of all items or units relating to any subject.

Excercise

(A) True / False

1) The data collected without any objective is called numbers .

2) Statistics studies only quantitative variables .

3) All numerical information is statistics .

4) 'A cow has 4 legs' is a statement of statistics .

5) Macro economics deals with economic issues at the level of economy as a whole .

(B) Choose the correct answer :-

1) The aggregate of data is called :-

a) Statistics   b) editing of data .   c) analysis of data d) collection of data .

2) Which of the following is an economic activity :

a) Production   b) consumption   c) Distribution   d) All of these .

3) Process of converting raw meterials in goods is called :

a) Production   b) Saving   c) investment   d) exchange

4) The part of income which is not consumed is called :

a) investment   b) saving   c) consumption   d) production

5) Which of the following indicates a stage of statistical study :

a) Collection of data   b) Presentation of data   c) Analysis of data   d) All of these .

(C) **Fill in the blanks** :-

1) A _____ is one who consumes goods and services .

2) _____ statistics refers those methods which are used for the collection of data .

3) In statistics , the term _____ refers to the aggregate of all items relating to any subject

4) In _____ sense statistics refers to information in terms of numerical data .

5) _____ studies economic activity of an individual unit .

(D) **Answer the following questions** :   **1mark**

1) Define economics .

2) Who is a consumer ?

3) Define consumption .

4) Who is a producer .

5) What is production .

6) What is saving ?

7) What is investment ?

8) Define statistics as a singular sense .

9) Define statistics as a plural sense .

10) What is statistical tool ?

(E) **Answer the following questions** :-   **3 / 4 Marks**

1) Write the features of statistics in the plural sense .

2) What are the main limitations of statistics ?

3) Explain the functions of statistics in economics .

4) Write the difference between Descriptive statistics and Inferential statistics .

Answer

(A) True / False

1) True   2) True   3) False   4) False   5) True

(B) Choose the correct answer

1) (a)   2) (d)   3) (a)   4) (b)   5) (d)

(C) Fill in the blanks :

1) Consumer 2) Descriptive 3) Universe 4) Plural

5) Microeconomics

(D) Short answer :

1) Economics is the study of scarcity and its implications for the use of resources , production of goods and services and welfare over time .

2) Who consumes goods and services for the satisfaction of his wants is called consumer .

3) Consumption is defined as the use of goods and services by a household .

4) Who produces and sells goods and services for the generation of income is called producer

5) Production is the process of converting raw material into useful things .

6) Saving is that part of income which is not consumed .

7) Investment is the process of buying assets or items that help to generate income in the long run .

8) In singular sense , statistics means statistical methods .

9) In plural sense , statistics refers to information in terms of numbers or numerical data .

10) Statistical tools refers to the methods used for the collection , organisation and presentation of data .

Collection of Data

The purpose of data collection is to understand , analyse , and explain a socio - economic problem. For example, the problem of unemployment or the problem of poverty . We also try to understand the causes behind the problems and also the possible solutions .

In statistics , the data are the individual pieces of factual information recorded , and it is used for the purpose of the analysis process .

2.1 **Sources of data** :

There are two sources of collection of data :

i) **Primary source** :

Primary source of data implies collection of data from its source of origin . It offers you first - hand quantative information relating to statistical study .

ii) **Secondary source** :

Secondary source implies that the desired statistical information already exists and you are simply to collect it from its concerned agency or the department .

\# **Primary data** : Data collected by the investigator for his own purpose , for the first time , from begining to end are called primary data . These are collected from the source of origin .

\# **Secondary data** : Secondary data are those which are already in existence and which have been collected , for some other purpose other than the answering of the question in hand .

2.1.1 Difference between primary data and secondary data :-

Primary data	Secondary data
i) Data collected by the investigator for his own purpose , for the first time.	i) Data already in existance , and which have collected for some other purpose.
ii) These are collected from the source of origin.	ii) These are collected from the secondary source .
iii) These are first hand data .	iii) These are second hand data .
iv) Primary data are always related to a specific objective of the investigator .	iv) Secondary data already collected for some other purpose .

2.1.2 **Collection of data from primary sources** :-

Some well-known methods of collection of primary data are -

(a)   Personal interview ,

(b)   Information from local sources ,

(c)   Information through Questionnaires ,

(d)   Indirect oral Investigation

(e)   Telephonic Interview .

(a)   Personal interview :-   Here data are personally collected by the investigator from the respondents . Here the investigor establishes direct relation with the persons from whom the information is to be obtained .

Merits :   i)   Data have a high degree of originality .

ii)   Data is fairly accurate .

iii)   Reliability of the data is not doubted .

iv)   This method is fairly elastic .

Demerits :   i)   Difficult to cover big areas .

ii)   There may be personal bias of the investigator

iii)   Very expensive .

b)   **Information from local sources** :-   In this case investigator appoints local persons at different places . They collect information in their own way and furnish the same to the investigator .

Merits :   i)   This method is quite economical in terms of time and money .

ii)   A fairly wide coverage of investigation .

iii)   There is a continuity .

Demerits :   i)   The data may lose its originality .

ii)   There is a lack of uniformity of data .

iii)   This method may suffers from personal bias of the correspondents .

c)   **Indirect oral Investigation :–**   This is the method by which information is obtained orally from people who are expected to posses the necessary information and not from the persons regarding whom the information needed .

Merits :   i)   This method covers wide area of investigation .

ii)   It saves labour and time as well as money .

iii)   This is a simple approach of data collection .

Demerits :   i)   Relatively less accurate .

ii)   Possibility of personal bias of the witnesses.

iii)   The conclusion may be doubtfull due to carelessness of information with the witnesses.

(d) **Information through Questionnaires** :– Under this method , the investigator prepares a questionnaire keeping in view the objective of the enquiry .

There are two ways of collection data -

i) **Mailing surveys :**– Here questionnairs are mailed to the respondents . The respondents answer the questions and returns to the investigator .

ii) **Enumerator's method :**– In this case , the enumerator himself approaches the respondent with the questionnaires . Here the enumerators themselves fill the questionnaire after seeking information from the respondents .

Merits : i) This method allows wide coverage of the area of study .

ii) Original , fairly and reliable .

iii) It have the merit of completeness .

Demerits : i) If the respondents are biased , then the information will also be biased .

ii) Not suitable for private investigation .

● **Qualities of a Good Questionnaire** :-

The following points should be followed while designing a good questionnaire :-

1) The number of questions asked should be limited .

2) The questions asked should be simple , clear and short .

3) The questions should be logically arranged .

4) The questions which are too personal in nature should be avoided .

5) Questions involving calculations by the respondent must be avoided .

6) To verify any pariticular information , questions may be set in such a way that the verification may be possible .

7) A questionnaire must show clear instructions for filling the form .

(e) **Telephonic Interviews** :– According to this mode of data collection , the investigator seeks the desired information from the respondents over the telephone .

Merits :– i) This method involves relatively lower cost in terms of time and efforts .

ii) In short time it can cover wide area .

iii) High response rate .

Demerits :– i) Limited use .

ii) Personal bias .

iii) Limited access .

2.1.3 Collection of Secondary data :–

There are two main sources of secondary data :

a) Published sources   b) Unpublised sources .

a) **Published sources :–** Some of the published sources of secondary data are :

i) Government Publications : Government organisations and departments of central and state govt . publish current information along with statistical facts is a reliable source. Some notable government publications are Annual survey of Industries, Agricultural statistics of India , RBI , Labour Gazette etc.

ii) Semi-Government Publications :- Municipalties and Metropolitan councils publish data relating education , health , births and deaths etc .

iii) Reports of committees and commissions :- Finance commission , planning commissions are some notable commissions in India which supply statistical information in their reports .

iv) Some of big trade associations collect and publish data on various aspects of trading activity.

v) Various universities and research institutions publish informations.

vi) Many Journals and newspapers (The economic Times) supply large variety of informations.

vii) International organisations like UNO, IMF, world Bank etc also publish all lot of statistical information.

b) **Unpublished sources** :–

Sometimes research scholars , universities , trade institutions and labour bureaus collect data but not publish it . This unpublished data can be used as a secodary data .

Limitation of Secondary data :–

As secondary data are collected by others , one needed to very careful while using these data . Some limitations of using secondary data are–

i) These may not have been collected by proper statistical techniques .

ii) These may not be suitable for the concerned enquiry .

iii) These may not relate to the present times .

iv) These may be biased and hence not reliable .

Two important sources of secondary data in India :–

i) **Census of India :-** Census of India is a very comprehensive source of secondary data. It includes statatistical information on the following parameters ;

a) Size, growth rate and distribution of population in India.

b) Population projection.

c) Density of population.

d) Sex composition of population.

e) State of literacy .

ii) **Reports and publication of National sample survey office (NSS0) :–**

NSSO is another important source of secondary data in India . It offers statistical information regarding– Land and Livestock Holdings , Housing condition and Migration, Employment and unemployment status in India , consumer expenditure , sources of household income in India etc .

2.2   Census Method :

Census method is that method in which data are collected covering every item of the univers or population relating to the problem under investigation . "Census of population" is the most suitable example of the census method .

Merits :–   i)   Results based on census method are acurate and highly reliable .

ii)   Results are less biased .

iii)   Informations are more meaningfull and exhaustive .

iv)   Helpful in study of complex investigation .

Demerits :–   i)   Census method is very costly .

ii)   It requires large number of mapower .

iii)   If the universe comprises a large number of items , this method face verious difficulties.

2.3   Sample Method :- Sample method is that method in which data is collected about the sample on a group of items taken from the population for examination and conclusions are drawn on their basis .

Merits :–   i)   Sample method of investigation is economical .

ii)   Time saving method .

iii)   Errours can be easily identified .

iv)   More feasible for large investigation .

v)   More scientific .

Demirits :–   i)   It is only a parital investigation of the universe .

ii)   The study may give wrong conclusion .

iii)   Difficulty in selecting representative sample .

iv)   Some-time the universe may be so diverse that it becomes difficult to frame a sample.

Methods of sampling :–

Methods of sampling

Random sampling	Non- Random sampling
→ Lottery Method	→ Purposive sampling
	→ Stratified sampling
	→ Systematic sampling
→ Tables of Random Numbers	→ Quota sampling
	→ Convenience sampling
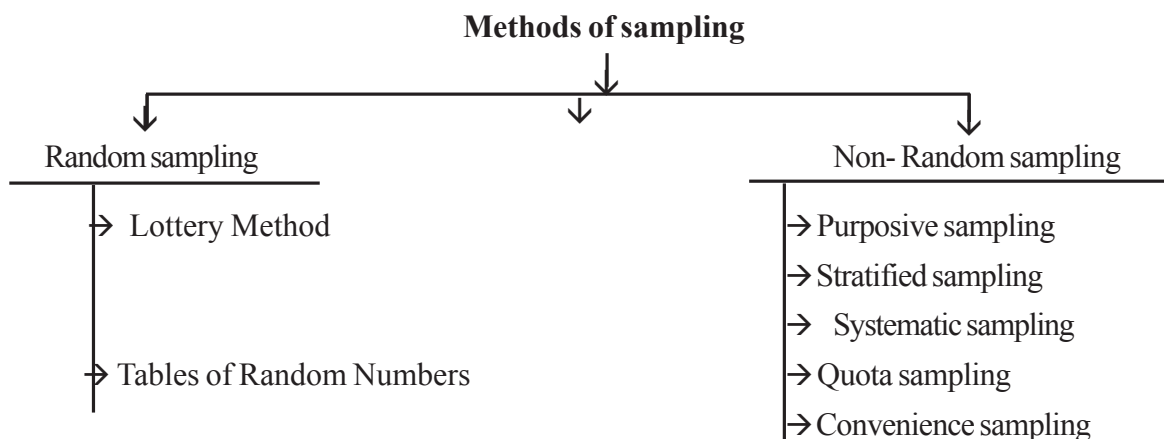
2.3.1 Random sampling : –

Random sampling is that method of sampling in which each and every item of the universe has equal chance of being selected in the sample . This method is used particularily when various items of the universe are homogenous or identical to each other . This method is impartial and economical . Random sampling may be done in any of the following ways :-

a) **Lottery Method** :– In this method , paper-slips are made for each item of the universe and shuffled in a box. Then impartially, some of the slips are drawn to form a sample of the universe.

b) **Tables of Random Numbers** :- Some statisticians have prepared a set of tables called Tables of Random Numbers . A sample is framed with referance to these tables . Out of all these tables Tippets Table is most widely used . Using 4000 figures, Tippet has involved 1000 numbers comprising of four units each. For the use of this method, all items of the universe are first arranged in an order. Then using Tippet's Table the required number of items are selected as are needed for a sample.

Merits :–   i)   This method is free from personal bias of investigation.

ii)   Each and every item has equal chances of being selected.

iii)   The universe gets fairly represented by the sample.

iv)   Very simple and staright forward method .

Demerits :–   i)   This method does not guarantee proportionate representation of different items in the universe.

ii)   It does not give weightage to certain important items in the universe.

2.3.2 Non - Random sampling :- Non - random sampling includes all those methods of sampling in which all the units of   population do not have equal probability of being selected in the sample study. Some well known non-random sampling are :-

a) **Purposive sampling** :- Purposive sampling is that method in which the investigator himself makes the choice of the sample items which in his opinion are the best representative of the universe .

Meits :–   i)   It is a very simple technique of selection of the sample items.

ii)   This method is flexible .

Demirits :–   i)   Possibility of personal bias in the selaction of items.

ii)   Reliability of the results becomes doubtful.

b) **Stratified sampling** :– According to this method , population is divided into different strata having different characteristics and some of the items are selected from each strata, so that the entire population gets represented .

Merits :–   i)   This method covers diverse characteristics of the population.

ii)   A comparative analysis of the data possible.

iii)   Reliable and gives meaningful results.

Demerits:–   i)   This has a limited scope.

ii)   Possibility of bias at the time of classification of the population into different strata.

c) **Systematic sampling :–** According to this method , units of the population are numerically , geographically and aphabetically arranged . Every nth item of the numbered items is selected as a sample item .

Merits :-   i)   This is a very simple method .

ii)   There is heardly any possiblity of personal bias in this method .

Demerits :-   i)   Every item in the population does not get equal chance .

ii)   If all the items are homogeneous , this method serves no specific purpose .

d) **Quota sampling** :– In this method , the population is divided into different groups according to different characterics of the population . The investigator selects the fixed number of items from each group to frame a sample .

e) **Convenience sampling** :– In this method , sampling is done by the investigator in such a manner that suits his convenience. This method is the simplest and least expensive, but unscientific and unreliable

2.4   Statistical Errors :–

Statistical errors are broadly classified into two parts–

a) Sampling errors, and   b) Non-sampling errors.

a) **Sampling errors** :–   Sampling error is a statistical error that occurs when an analyst does not select a sample that represents entire population of data .

b) **Non-sampling Errors** :–   The main non-sampling errors are–

i) **Error in Data Acquisition** : Such errors occur when responses are not recorded correctly .

ii) **Error of Non- response** :   When the respondents do not offer the required information then it is called error of non -response.

iii) **Error of sampling Bias :–** It occurs when a part of target population cannot be included in the choice of sample .

Excercise

A) True or False

1) Every item of population is included under the random sampling method .

2) Sampling method is more expensive .

3) Census method need large number of enumerators .

4) Census method is very costly .

5) Random sampling is free from personal bias of investigator .

6) Purposive sampling is flexible .

7) Telephonic interview has no personal bias .

8) Data collected from trade instututions is a unpublished sources .

9) Primary data involve more time and more expenses .

10) Unpublished numerical information may be used as secondary data .

B) Choose the correct answer :-

1) The method is used to estimate the population is India -

  a) Census method   b) Sampling method

  c) Both (a) and (b)   d) None of these .

2) Reliability of sampling data depends on :

  a) Size of the sample   b) method of sampling

  c) training of enumeratros   d) all of these .

3) _____ data are collected by the investigator himself

  a) Secondary   b) Primary

  c) both (a) and (b)   d) none of these .

4) A good questionnaire should be :-

  a) in order   b) brief

  b) complete   d) all of these .

5) Primary data is more useful when

  a) High degree of accuracy is required .

  b) Less time is available

  c) Source of origine is not important .

  d) All of these .

6) Data collected from census report of India is

  a) primary data   b) secondary data .

  c) Sample data   d) None of these .

7) Indirect oral investigation method suffers from

   a) biasness   b) doubtful conclusions

   c) inaccuracy   d) all the above

8) Data collected from government publication is

   a) Secondary data   b) Primary data

   c) both (a) and (b) d) Neither (a) nor (b)

C) Fill in the Blanks :-

1) _____ method is suitable for small size of population .

2) _____ method is suitable for large size of population .

3) _____ data collect from the source of origin .

4) In case of _____ surveys , questionnaires are mailed to the respondants .

5) RBI is a notable _____ publication .

6) The full form of NSSO is _____ .

7) In _____ sampling each and every item has equal chances of being selected .

8) In _____ sampling method population is divided into different groups .

9) _____ is a person who helps the investigator in collecting the data .

D) Answer the following questions :-   **1 mark**

1) what is data ?

2) What is primary data ?

3) What is secondary data ?

4) Define primary source of data .

5) Define Secondary source of data .

6) What is census method ?

7) What is sample method ?

8) Define Random sampling ?

9) Define Non- Random sampling .

10) What do you mean by personal interview ?

E) Answer the following questions :-   **3/4 Marks**

1) What are the main sources of data ?

2) Write the difference between primary data and secondary data .

3) What are the main methods of collecting primary data ?

4) What is direct personal investigation ? Give the merits and demerits of it .

5) What are the qualities of a good questionnaire ?

6) Describe the main sources of secondary data .

7) What are the limitations of secondary data ?

8) Write short note– a) Census of India   b) NSSO

9) What is census method ? Write its merits and demerits .

10) What is sample method ? Write its merits and demerits .

11) Explain the methods of random sampling .

12) Explain the various method of non - Random sampling .

13) Discuss about the statistical errors .

F)   Answer the following questions :-   **6 marks**

1)   Enumerate the various methods of collecting primary data . Discuss the merits and demerits of them .

2)   Discuss various methods of sampling . Explain the merits and demerits of every method .

3)   What are the advantages and disadvantages of collecting primary data by personal interview and Telephonic Interview method ?

4)   Describe the method - Information through questionnaires . Also write the qualities of good questionnaire .

Answers

A)   True / False :-

1)   False   2) False   3) True

4) True   5) True   6) True

7) False   8) True   9) True

10) True

B)   Choose the correct answer :-

1) (a)   2) (d)   3) (b)   4) (d)

5) (a)   6) (b)   7) (d)   8) (a)

C)   Fill up the blanks :-

1)   Censes   2)   Sampling

3)   Primary   4)   mailing

5)   government   6)   National sample survey office

7)   Random   8)   Quota

9)   Enumerator

D)   Short Answer :-

1)   Data are the individual pieces of factual information recorded , and it is used for the purpose of the analysis process .

2)   Data collected by the investigator for his own purpose , for the first time are called primary data .

3)   Data which are already in existence and which have been collected for some other purpose are called secondary data .

4)   Primary source of data implies collection of data from its source of origin .

5)   Secondary source implies that the desired statistical information already exists .

6)   In census method data are collected covering every item of the universe relating to the problem under investigation .

7)   Sample method is that method in which data is collected about the sample on a group of items taken from the population for examination .

8)   In Random sampling each and every item of the universe has equal chance of being selected in the sample.

9)   In case of Non- Random sampling all units of population do not have the equal probability of being selected in the sample study .

10)   In case of personal interview , data are personally collected by the investigator from the respondents.

Organisation of Data

An investigator collects data mainly for an investigation purpose. He / she collects data from different sources. Such kind of data are Raw data . To attain any particular conclusion and to represent these data systematically and to do systematic statistical analysis , organisation of data and then classification of data are needed .

3.1 **Defination :-** Here, Raw data means data that has not been processed for use i.e Raw data are highly disorganised data .

Classification of data is arranging or organising data (things) into groups or classes based on same criteria or common characteristics .

Organisation of data refers to the systematic editing , arrangements and classification of facts and figures (raw data) in such a form that comparison of masses of similar data may be facilitated and further analysis may be possible .

3.2 **Some basic characteristics of classification are :-**

1. Collected data are divided into various groups .

2. Data are grouped or classified on the basis of their similarities (Homogeneity)

3. Clarity and suitability are also maintained during the process of classification .

3.3 **Objectives of classification :-**

1. Classification makes the data simple , precise and brief . It is easy to use the data on the basis of need.

2. Classification process clearly reveals the similarities and dissimilarities among the collected data .

3. After classification , it is easy to compare the data and analyse properly .

4. Classification helps the investigator to arrange the data scientifically and present them through table or otherways .

5. Classification of objects saves our valuable time and effort .

3.4 **Characteristics of a good classification are :-**

1. Classification of data should be very comprehensive . i.e each and every raw data is arranged on the basis of some criteria .

2. The Data which are included in a particular section must be similar or homogeneous .

3. The classification of the data should be clear and useful for further use .

4. The classification of the raw data must be done on the basis of the purpose of investigation .

3.5   Type of Classification :-

The groups or classes of a classification are done in various ways . The raw data is classified depending on the purpose .

i)   **Chronological Classification** :-

When data are classified either in ascending or in desending order with reference to time such as year, quarter, months, weeks etc, then such a classification is called chronological classification .

Example :- Population in India (in crores)

Year	Population
1951	35.7
1961	43.8
1971	54.6
1981	68.4

ii)   **Geographical or spatial classification** :-

When the data is classified according to certain geographical location such as country , states, cities , districts etc , then such a classification is called geographical or spatial classification .

Example :- **Population of India in different states(2001)**

States	Population(crores)
U.P	16.5
Bihar	8.3
Maharastra	9.6
Hariyana	2.2

iii)   **Qualitative Classification :-**

When the data is classified on the basis of some qualities or attributes like nationality, gender, religion, marital status etc., such classification of data is called qualitative classification .

Example :- Qualilative Classification of population

```
                         Population
                             ↓
        ┌────────────────────┴────────────────────┐
        ↓                                          ↓
       Male                                      Female
        ↓                                          ↓
   ┌────┴────┐                              ┌──────┴──────┐
   ↓         ↓                              ↓             ↓
 Married  Unmarried                     Married      Unmarried
```
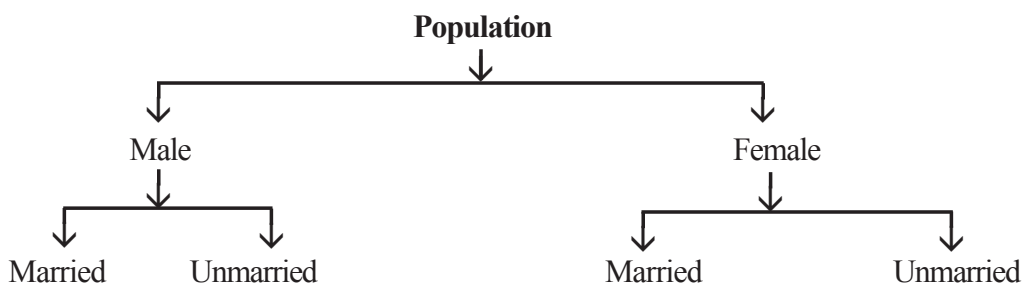
iv)   **Quantitative Classification** :-

When data are classified into classes or groups on the basis of their quantitative nature like height, weight, age, income or marks etc , such classification of data is called quantitative classification .

Example :- Arrangement of students on the basis of age groups

Age groups	No of students
0-10	2
10-20	3
20-30	2
30-40	5
Total	12

3.6   The Concept of Variable :-

Discrete Variable and Continuous Variable–

A characteristic which is capable of being measured and changes its value over time is called a variable . It can be expressed by x , y , z .

A variable refers to that quantity index which is subject to change and which can be measured by some unit like age , height , marks , salary , pension etc .

The variable can be divided into two ways :-

i) Discrete vaiable   ii) Continuous variable .

Discrete variable :- Variable which are able to taking only exact value and not any fractional value are termed as discrete varibles .i.e A discrete variable can take only certain value and it jumps from one value to another finite value .

Example :- Number of children in family can be 1 , 2 , 3 or 4 but not 1.5 , 2.5 .

Continuous variable :- The variables which are capable to take all the possible values (integers, fraction or values that are not fraction) in a give specified range are termed as continuous variable .

Example :-

Height	$1 , 2 , 3 , 4$ (integral value)
Weight	$\frac{1}{2} , \frac{1}{3} , \frac{1}{4}$ (fraction value)
Distance	$\sqrt{2} = 1.4\ 14 , \sqrt{3} = 1.732$
Measarements	$\sqrt{7} = 2.645$ (infinite number)

So, In case of discrete variable , data is obtained by counting but in case of continuous variable , data is obtained by measurement.

Discrete varibale	Continuous variable
1. Discrete variable is capable of taking only exact value and not any fractonal value	1. Continuous variables is capable of taking all the possible values in a given range .
2. Here , Data is obtained by counting .	2. Here , data is obtained by measurement .
3. Example– Number of worker, Number of children .	3. Examples– Height , weight of individuals ,

3.7   Statistical series :-

Statistical series means those data which are presented or arranged in some specific order and sequeuces. Statistical series can be classified into :-

i)   Individual series .

ii)   Discrete series .

iii)   Continuous series .

i)   Indivisual series : Individual series are those series in which the items are listed singular manner .

Example :- If Marks of ten students are given then their marks would be listed individualy .

Sl No	Marks
1	30
2	20
3	22
4	27
5	25
6	21
7	34
8	36
9	38
10	40

ii)   Discrete series (frequency Array) :- A discrete series or frequency array is that series in which data are presented in a way that exact measurements of items are clearly shown ..

Example :- Marks receive by the students (30 Nos)

Exact Marks	No of students (frequency)
25	5
30	10
35	2
40	3
45	5
50	5
Total–	**30 Nos**

iii) Continuous series :-

A continuous series is that series which represents continuous variables , showing range of values of different items of the series . Here , frequencies are given along with the value of the variable in the form of class intervals .

Example :-

Marks obtained	No of students (frequency)
0 - 10	3
10 -20	4
20 -30	3
30 -40	5
40 -50	2
50 -60	3
Total –	**20 Nos**

3.8 Some important terms :-

Class :- Class means group of numbers in which items are placed such as 0 - 10 , 10 - 20 , 20 - 30 etc

Frequency :- It refers to the number of times a given value appears in a distribution or in a particular class.

Frequency distribution :- A frequency distribution is a comprehensive way to classify raw data of a quantitative variable , generally , it is expressed through a table , how the different values of a variable are distributed in different classes .

Class frequency :- The term class frequency means the number of values exist in a particular class

Class limit :- Class limits are the two ends of a class .

The lowest value of the class is called lower class limit and the highest value of the class is called upper class limit.

Example :- Let a class limit of the class is (10 -20). Then lower class limit is 10 and upper class limit is 20.

Class interval or class width :-

Class interval or class width is the difference between the upper class limit and the lower class limit .

Example :- For the class 10 -20 , the class interval or width is (20 - 10) = 10

Mid point or Mid value :-

Mid point is the central point of a class interval or class .

$$\text{Mid point or class Mark} = \frac{Upper \text{ class limit + lower class limit}}{2}$$

If the class is (10 -20) , than Mid point is $= \frac{10 + 20}{2} = \frac{30}{2} = 15$

Frequency Curve :- Frequency curve is a graphic representation of a frequency distribution .

Example :- Frequency distribution table of Marks and students .

Class Marks	Frequency (students)
0 - 10	2
10 - 20	4
20 - 30	8
30 - 40	10
40 - 50	2
50 - 60	1

To obtain the frequency curve we plot the class (Marks) on the x - axis and frequency (students) on the y-axis .
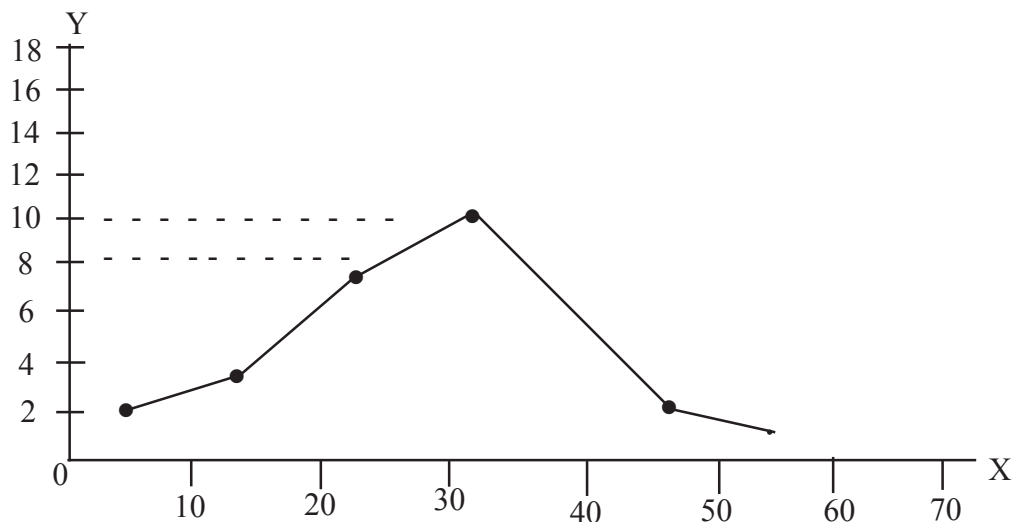
Diagramatic presentation of frequency distribution data .

Range :- Range is the difference between the largest and the smallest value of a variable .

The Range of a frequency distribution can be defined as the difference between the lower limit of the first class interval and the upper limit of the last class interval .

Example :-   Let the class interval–

00 - 10 , 10 - 20 , 20 -30 , 30 - 40 , 40 - 50

then Range is 50 - 00 = 50

After obtaining the value of range , it becomes easier to determine the number of classes once we decide the class interval . Range is the sum of all class intervals .

If the class intervals are equal then Range is the product of the number of classes and class interval of a class .

Range = Number of class × class interval .

Range shows the spread of variable or frequency distribution .

3.9   Various types of continuous series :-

i)   Exclusive series :- Exclusive method is used to form exclusive series of continuous series . Here , the upper class limit of one class equals the lower class limit of the next class .

Example :- 0 -10 , 10 - 20 , 20 - 30 , 30 -40 , 40 - 50 etc .

ii)   Inclusive series :- Inclusive method is used to form Inclusive series of continuous series . Here , It does not exclude the upper class limit in a class interval . It includes the upper class limit in a class . Thus both class limit are the part of the class interval .

So , upper class limit of a class interval is not equals the lower class limit of the next class .

Example :- O - 9 , 10 -19 , 20 -29 , 30 - 39 .

3.10   Conversion of inclusive series to exclusive series :-

In case of inclusive series , no continuity is maintained when the classes are made . Sometimes it becomes necessary to have exclusive classes (continuity) to apply some statistical tools . So , Inclusive classes need to be transformed into exclusive classes .

Steps to convert inclusive series into exclusive series :-

Steps 1 :- Find the difference between upper limit of a class interval and lower limit of a next class interval .

Step 2 :- Add half of this difference to the upper limit of each class interval and substract same amount from the lower limit of each class interval .

Example :- An inclusive series is converted to Exclusive series

Inclusive series		Exclusive series	
Income of labour	No of Labour	Income of labour	No of labour
01 - 99	2	0.5 - 99.5	2
100 - 199	4	99.5 - 199.5	4
200 - 299	3	199.5 - 299.5	3
300 - 399	2	299.5 - 399.5	2
400 - 499	1	399.5 - 499.5	1

[Gap between upper and

lower class limit = 100 – 99 = 1]

So 0.5 will be subracted from the lower limit of all the classes and added to the upper limit of all classes.

3.11   Cumulative frequency series :-

Cumulative frequency series is a modification of the simple frequency distribution . It is obtained by successively adding the frequency of the values of the classes .

It is abbreviated as c.f

There two types of c.f

i)   'Less than ' cumulative frequency distribution .

ii)   'More than' cumulative frequency distribution .

'Less than' cumulative frequency distribution .

Example :- A frequency distribution

Age	0 -10	10 -20	20-30	30-40
No of person	3	2	4	1

Less than cumulative frequency distribution

Age	No of person (c.f)
Less than 10	3
Less than 20	3+2 = 5
Less than 30	3+2+4 = 9
Less than 40	3+2+4+1 = 10

The ' less than' c.f are associated with upper class limit .

'More than cummulative frequency distribution

Age	No of person(c.f)
More than 'O'	3+2+4+1 = 10
More than '10'	2+ 4 + 1 = 7
More than '20'	4 + 1 = 5
More than '30'	1

The " More than" c.f are associated with lower class limit .

3.12 i)   Loss of Information :-

Classification of data summarises the raw data making it concise and comprehensive , but it does not show the details that are found in raw data . So , there is loss of same information .

ii)   Frequency Arrary :-

For a discrete variable , the classification of its data is known as frequency Arrary .

iii)   Bivariate frequency distribution :-

A Bivariate Frequency distribution is the frequency distribution of two variable together.

Example, when data classified on the basis of weight and height of the persons , the distribution is known as Bivariate frequency distribution .

Example :-

Weight	Height→ (3-4) ft	(4-5) ft	(5-6) ft	Total
30-40 kg	3	2	9	14
40-50 kg	–	1	7	8
50-60 kg	2	4	3	9
60-70 kg	2	-	2	4
	7	7	21	**35 Nos**

 Bivariate frequency distribution .

Construction of Discrete frequency distribution by Tally marks :-

At first , a table should be formed with three columns 3 headings such as variable , Tally Marks and Frequecy .

Then , all possible value are written in the variable column .

A tally mark devoted by (|, ||, |||, ||||, ⅣⅡ) is noted for every observation against its value.

Now after calculation of Tally marks , frequency colume is filled up against the variable (Numerical value)

Example :- Marks obtained by 12 students out of '5 marks' in an examination are.

4 , 4 , 3 , 2 , 2 , 4 , 1 , 5 , 5 , 3 , 4 , 4

We get the following frequency distribution

Marks(x)	Tally Marks	Frequency (f)
1	|	1
2	||	2
3	||	2
4	ⅣⅡ	5
5	||	2
Total student		**12 Nos.**

Similarly , Continuous frequency distribution can be formed using Tally Marks .

Excercise

1.1   Whether the statements are true / false ?

1.   Raw data are collected by investigator during the investigation .

2.   Chronological classification is done on the basis of time period .

3.   Continuous variable can obtain only complet value but not fractions .

4.   Cumulative frequency is the frequency of a particular class only .

5.   Discrete variable can obtain only fractions .

6.   The number of variable or items that come under any class is considered as class frequency.

7.   Both the upper and the lower limits are included in the inclusive method .

8.   Classification on the basis of gender and religion come under the geographical classification

9.   Height and wieght are the basis of quantitative classification of Raw data .

10.   In a class (15-25), Mid point is 25.

1.2 . **Multiple choice question (MCQ)**

1.   Which is the objective of classification ?

a) Simplification   b) Briefness   c) Comparability   d) All of these .

2.   Formula for finding mid value is given by .

a) $l_2 - l_1$ b) $\dfrac{l_2 - l_1}{2}$ c) $l_1 + l_2$ d) $\dfrac{l_1 + l_2}{2}$.

3.   A series in which every class interval excludes items corresponding to its upper limit is called :-

a) exclusive series   b) inclusive series   c) both (a) and (b)   d) None of these .

4.   According to tally bar method , which is the symble represents the frequency of five ?

a) ✓   b) |||||| c) ⩗   d) None of these

5.   Annual income of a person is–

a) A continuous variable   b) A discrete variable .

c) An atribute   d) either (b) or (c)

6.   The upper class limit of class - interval is considered for calculating

a) 'Less than' cumulative frequency.   b) 'More than' cumulative frequency .

c)   Relative frequency   d) None of these .

7.   'Nationality' of a student is –

a) An qualitative aspect   b) A discrete variable

c) A continuous variable   d) either (a) or (c)

8. Tally marks determines
   a) Class width
   b) Class - boundary
   c) Class limit
   d) Class frequency .

9. In a indivdual series each variate value has
   a) Same frequency
   b) frequency one
   c) Varied frequency
   d) frequency two .

10. If the class Mark or 'Mid point' of a distribution is 26 then the class interval is
    a) 23.5 - 28.5
    b) 23 - 28
    c) 22.5 - 30.5
    d) None of these .

1.3 Fill in the gaps :-

1. The population is a _____ as it depicts a series of values for different years .

2. _____ variable can take both integral values and fractional values .

3. Class interval = upper class limit _____ lower class limit .

4. _____ are the two ends of a class .

5. Range = Number of classes × _____ .

6. _____ is the sum of all class intervals .

7. Class Mid point or class Mask = Upper class limit + Lower class limit

8. In a class 10 -20 , 20 -30 , 30 -40 , Range is _____ .

9. When data is categorised according to time it is called _____ .

10. _____ refer to those data which are presented in some order and sequence .

1.4 Very short question :-

1. What is a class ?

2. What is meant by a variable ?

3. What do you mean by classification of data ?

4. What is meant by class - interval ?

5. Which series exclude the upper limit of the class interval ?

6. Which series includes the upper limit of the class interval ?

7. Define qualitative classification .

8. Define the term " frequency" .

9. Which series have class interval ?

10. What is class mid - point ?

11. What do you mean by Array ?

12. What do you mean by raw data ?

2. **Short Answer Type Question :** (3/4 Mark)

1. State the different methods of classification .
2. Briefly explain two objectives of classification of data .
3. What is meant by individual series ? Give example .
4. Discuss the meaning of continuous series ? Give example .
5. Distinguish between inclusive method and exclusive method of classification .
6. What do you mean by cumulative frequency ?
7. What do you mean by discrete series ? Give example .

3. **Answer the following Question:** (6 Mark)

i) Discuss the essentials of a good classification .
ii) Briefly discuss the following concepts
   a) class interval   b) Range   c) Frequency   d) Mid value
iii) Discuss the method of constructing a discrete frequency distribution through Tally Marks .
iv) How to convert a inclusive series into exclusive series ?

Answer

1.1 **Answer to the true / False question** :-

1. True	6. True
2. True	7. True
3. False	8. False
4. False	9. True
5. False	10. False

1.2 **Answer to the MCQ**

1. (d)   2. (d)   3. (a)   4.(c)   5. (b)   6. (a)   7. (a)   8. (d)   9. (b)   10. (a)

1.3 **Answer to the fill in the gaps** :-

1. Time series
2. Continuous
3. minus
4. class limit
5. class interval
6. Range
7. 2

8. 30

9. Chronological

10. Statistical series .

1.4 Answer to the very short Questions :-

1. Class means group of numbers in which items are placed such as 0 -10 , 10-20 , 20-30 etc.

2. A characteristic which is capable of being measured and changes its value over time is called a variable . It is expressed by x , y , z .

3. Classification of data is arranging or organsing data (things) into groups or classes based on some criteria or common characteristics .

4. Class interval or class width is the difference between the upper class limit and the lower class limit of class . Example of (0 -20) is a class .

   $\therefore$ Class interval (20– 0) = 20

5. Exclusive series .

6. Inclusive series .

7. When the data is classified on the basis of some qualities or attributes like nationality, gender, religion etc, such classification is called qualitative classification .

8. It refers to the number of times a given value appears in a distribution or in a class .

9. Continuous series .

10. Mid value or class Mark $= \dfrac{Upper \text{ class limit } - \text{ lower class limit}}{2}$

11. The presentation of individual series either in ascending order or in decencding order is known as Array .

12. Raw data are highly disorganised data .

Presentation of Data

4.1 The presentation of data means exhibition of data in a clear and attaractive manner so that the data can be easily understood and analysed . This chapter deals with presentation of data precisely so that the voluminous data collected could be made usable readily and are easily comprehended . There are four forms of presentation of data - Textual or descriptive presentation , Tabular presentation , Diagramatic presentation and Graphical presentation.

Presentation of Data

| Textual Presentation | Tabular presentation | Diagramatic presentation | Graphical presentation |
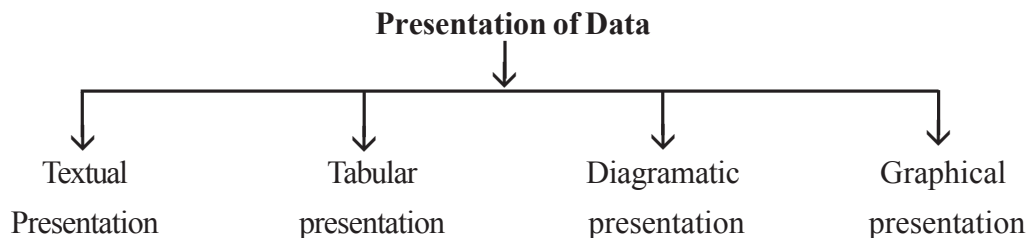
4.2 Textual presentation : In textual presentation , data are described within the text . When the quantity of data is not too large this form of presentation is more suitable .

Example : In a bandh call given on 8th september 2005 protesting the hike in price of petrol and diesel, 5 petrol pumps were found open and 17 were closed whereas 2 schools were closed and remaining 9 schools were found open in a town of Bihar .

In this example , data have been presented only in the text .

Advantages of Textual Presentation :-
    i)   This method is effective when the quantity of data is less .

    ii)   It provides ample amount of information and details .

    iii)   This method is a combination of figures and facts which makes it easy to understand .

Disadvantages of Textual Presentation :-
    i)   This method of presentation is ineffective when the quantity of data is too large .

    ii)   One has to go through the complete text of presentation for comprehension .

    iii)   If the data is not presented with proper facts and figures, it may lead to wrong analysis .

    iv)   It becomes difficult for the reader to draw conclusions from the data .

4.3   Tabular presentation of data :-

In a tabular presentation , data are presented in rows (read harizontally) and columns (read vartically). Tabulation is a scientific process to present the classified data in an orderly manner so that data are clearly understood .

4.3.1 Objectives of Tabulation :

i)   It simplifies the complex data .

ii)   It facilitates statistical analysis .

iii)   It facilitates comparison of data .

iv)   It provides reference for further analysis .

v)   It economises space .

4.3.2 Tabulation of Data and parts of a Table :

To construct a table , it is important to understand the various parts of a good statistical table . When all these parts are put together in a systematically ordered manner , it forms a table . The important components of a statistical table are as follows .

Table number : Table number is assigned to a table for identification purpose . It is given at the top or at the begining of the title of the table .

Title :- The title of a table narrates about the contents of the table . It has to be clear , brief and carefully worded so that the interpretation made from the table are clear and free from ambiguity . It finds place at the head of the table succeeding the table number or just below it .

Head note : If the title of the table does not give complete information , a head note supplements the title by explaining the table or the parts of the table .

Caption or column Heading : At the top of each column in a table , a column designation is given to explain figures of the column . This is called caption or column heading .

Stubs or Row Heading : Like caption or column heading , each row of the table has to be given a heading. The designations of the rows are also called stubs or stub items , and the complete   self column is known as stub column .

Body of the table : Body of a table is the main part and it contains the actual data. Location of any one figure / data in the table is fixed and determined by the row and column of the table   .

Unit of Measurement : The unit of measurement of the figures in the table (actual data)   should always be stated along with the title . If different units are there for rows and columns of the table, these units must be stated along with 'stubs' or 'captions' .

Source : It is a brief statement or phrase indicating the sources of data presented in the table. Source is generally written at the bottom of the table .

Note : It is the last part of the table . It explains the specific features of the data content of the table which is not self explanatory and has not been explained earlier.

Table Number → Title →

Table 4.1 Population of India according to workers and non-workers by gender and location, 2001

(Crore)

Column Headings/Captions →

Units

Row Headings/stubs →

Body of the table ←

Location	Gender	Workers			Non-worker	Total
		Main	**Marginal**	**Total**		
Rural	Male	17	3	20	18	38
	Female	6	5	11	25	36
	Total	23	8	31	43	74
Urban	Male	7	1	8	7	15
	Female	1	0	1	12	13
	Total	8	1	9	19	28
All	Male	24	4	28	25	53
	Female	7	5	12	37	49
	Total	31	9	40	62	102

Source : Census of India 2001

Source

Note : Figures are rounded to nearest crore

Note

4.3.3 Types of Table : Statistical tables can be classified into various categories depending upon the basis of their classification.

Kinds of Table

- According to purpose
  - General Purpose Table
  - Special purpose Table
- According to originality
  - Original Table
  - Derived Table
- According to construction
  - Simple or one-way Table
  - Complex Table
    - Double or two way table
    - Treble Table or Three way Table
    - Manifold Table
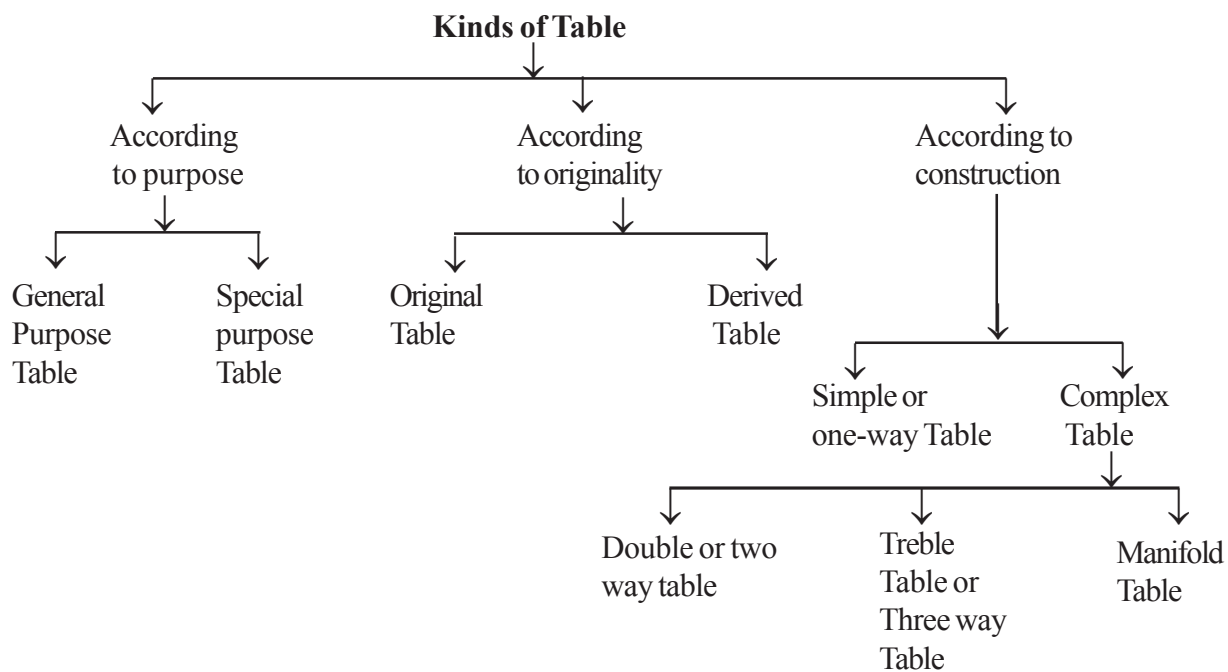
i) **Tables according to purpose** : According to purpose , there are two kinds of tables–

    a) General purpose table : General purpose table contains detailed information for general use . Such tables used extensively in the reports of the government departments . These tables are known as reference table.

    b) Special purpose table : These tables prepared for specific purpose . These tables do not provide detailed information and do not show those figures which are not related to some specific objectives . That is why these tables are also called summary table .

ii) **Tables according to originality :** On the basis of originality, tables are of two kinds .

    a) Original table : An original table is that in which data are presented in the same form and manner which they are collected . It is also known as primary table .

    b) Derived table : A derived table is that , in which data are first converted into ratios or percentage and then presented . It is also known as derivative table .

iii) **Tables according to construction :** On the basis of construction , tables may be of two types–

    a) Simple table : A simple table is that which shows only one characteristic of data . It is easy to construct and simple to understand.

Table 4.2

Number of students in a school

Class	Number of students
(vi)	60
(vii)	50
(viii)	40
(ix)	50
(x)	70
Total	**270**

    b) **Complex Table** : A complex table is that which shows more than one characteristic of data . On the basis of characteristic , complex table can be divided into three parts–

i) **Double or Two way table** : A double or two way table is that which shows two characteristics of data.

Table 4.3

Number of students in a school (According to sex and class)

Class	Number of students		Total
	Boys	Girls	
vi	40	20	60
vii	35	15	50
viii	20	20	40
ix	20	30	50
x	55	15	70
Total	**170**	**100**	**270**

ii)   **Treble table** : A trible table is that which shows three characteristics of data .

Table 4.4

Number of students in a school (According to sex , class and Habitation)

Class	Boys			Girls			Total		
	Rural	Urban	Total	Rural	Urban	Total	Rural	Urban	Total
vi	10	30	40	10	10	20	20	40	60
vii	05	30	35	05	10	15	10	40	50
viii	10	10	20	05	15	20	15	25	40
ix	05	15	20	10	20	30	15	35	50
x	15	40	55	05	10	15	20	50	70
Total	45	125	170	35	65	100	80	190	270

iii)   **Manifold table** : A manifold table is that which shows more than three characteristics of data .

Table 4.5

Number of students in a shcool (According to their sex, class, habitation and marks)

Class	Boys				Girls				Total
	Rural		Urban		Rural		Urban		
	Below 50%	Above 50%	Below 50%	Above 50%	Below 50%	Above 50%	Below 50%	Above 50%	
vi	04	06	10	20	03	07	05	05	60
vii	01	04	12	18	03	02	04	06	50
viii	03	07	02	08	01	04	04	11	40
ix	02	03	04	11	02	08	09	11	50
x	06	09	11	29	02	03	02	08	70
Total	16	29	39	86	11	24	24	41	270

4.3.4 Classification of Data and Tabular Presentation :

The most important advantage of tabulation is that it organise data for further statistical treatment and decision making. Classification used in tabulation is of four kinds–

a)   **Qualitative Classification :** When classification is done according to attributes , such as social status , physical status , nationality etc . it is called qualitative classification . Tabular presentation of qualitative classification can be given as follows :

Table 4.6

Literacy rate in India by sex and location (percent)

Sex	Location		Total
	Rural	Urban	
Male	79	90	82
Female	59	80	65
Total –	68	84	74

Source : Census of India 2011 (Literacy rates relate to population aged 7 years and above) .

b)   **Quantitative Classification** : In quantitative classification , the data are classified on the basis of characteristics which are quantative in nature . In other words those characteristics can be measured quantatively, e.g age , height , production , income etc. are quantitative characteristics .Tabular presentation of quantitative classification can be given as follows :

Table 4.7

Distribution of 542 respondents by their age in election study in Bihar

Age group (year)	Number of respondents	Percent
20 - 30	3	0.55
30 -40	61	11.25
40 - 50	132	24.35
50 - 60	153	28.24
60 - 70	140	25.83
70 - 80	51	9.41
80 - 90	02	0.37
Total	**542**	**100.00**

Source : Assembly election , patna central constituency , 2005 . A study conducted by A . N Sinha , Institute of social study , patna .

c)   **Temporal classification** : In this classification time becomes the classifying variable and data are categorised according to time . Time may be in hours , days , weeks , months , years etc . Tabular presentation of temporal classification can be given as follows .

Table 4.8

Yearly sales of a tea shop form 1995 to 2000

Year	Sales (₹ in lakhs)
1995	79.2
1996	81.3
1997	82.4
1998	80.5
1999	100.2
2000	91.2

Data source : Unpublished data

d)   **Spatial Classification** : When classification is done on the basis of place , it is called spatial classification . The place may be a village, town, block , district, state, country etc . Tabular presentation of spatial classification can be given as follows :

Table 4.9

Export from India to rest of the world in 2013 - 14 as share of total export (percent)

Destination	Export share
USA	12.5
Germany	2.4
Other EU	10.9
UK	3.1
Japan	2.2
Russia	0.7
China	4.7
West Asia Gulf coop. Council	15.3
Other Asia	20.4
Others	18.8
Total	**100.00**

(Total Exports : US $ 314 .40 billion)

Advantages of Tabular presentation of data :
i)   Tabular presentation is the most simplest form of data presentation where data are easily understood.

ii) The Tabular presentation facilitates comparison of data by presenting the data properly in rows and columns .

iii) It is very easy to analyse the data from tables .

iv) Tabulation highlights important characteristics of data .

v) Tabular presentation is very economical interms of time, money and efforts involved in data presentation .

Disadvantages of Tabular presentation of data :

i) Preparing tables require proper understanding of data , otherwise they will be misleading .

ii) Tabular presentation is not so attaractive as compared to diagramatic presentation .

iii) It is not so useful to give comparative analysis as compared to graphical presentation

iv) Tables donot leave ever lasting effect on mind of the reader .

4.4 Diagramatic presentation of Data :

When data is presented in the form of diagrams , it is called diagramatic presentation of data . This methods provides the quickest understanding of the actual situation to be explained by data in comparison to tabular or textual prsentation .

Diagramatic Prsentation

Bar Diagram   **Pie Diagram**

| Simple Bar Diagram | Multiple Bar Diagram | Component Bar Diagram or Sub - Divided Bar Diagram | Percentage Bar Diagram | Broken Scale Bar Diagram | Bilateral or Deviation bar Diagram |
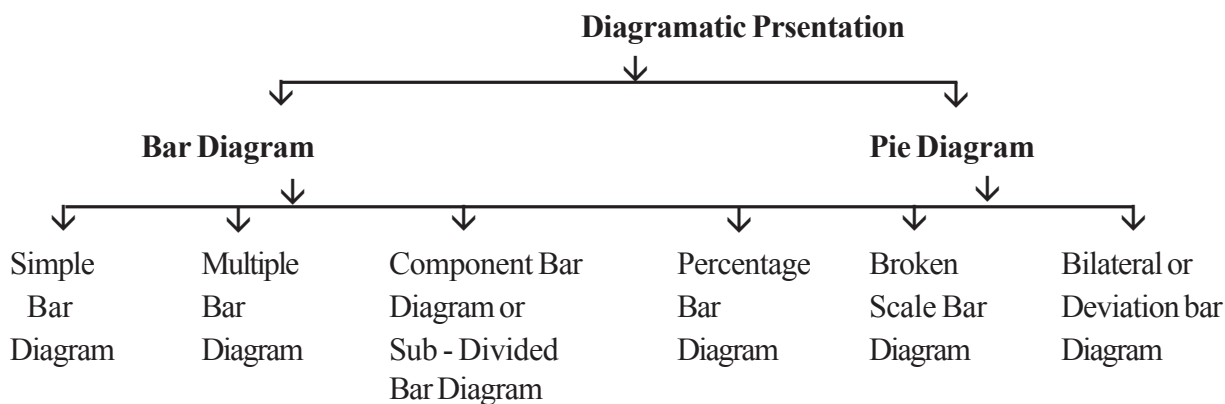
4.4.1 Bar Diagrams : Bar diagrams are those diagrams in which data are presented in the from of bars or rectangles . It may be 'vertical' or ' horizontal' . In bar diagram , bar shows the value of the varibale .

4.4.2 Features of bar diagram :

i) The length or height of the bars differs according to the values of the variable .

ii) Bar diagrams are of the same width .

iii) Bars are equidistant from each other .

iv) Bars may be vertical or horizontal .

v) All bars are based on same common line .

vi) To make bars attractive , various shades or colours may be used .

4.4.3 Types of Bar Diagrams : Bar diagrams are of six types–

i) **Simple Bar Diagram :** It is the simplest form of bar diagram which represent only one variable or characteristic of the data . Simple bar diagram can be either vertical or horizontal .

Example : Present the following information in a simple bar diagram (vertical and Horizontal)

Years	Exports (₹ in lakhs)
2006	30
2007	50
2008	40
2009	70
2010	60
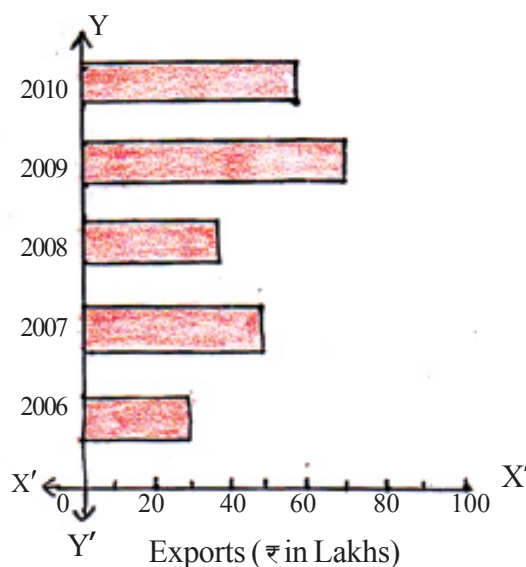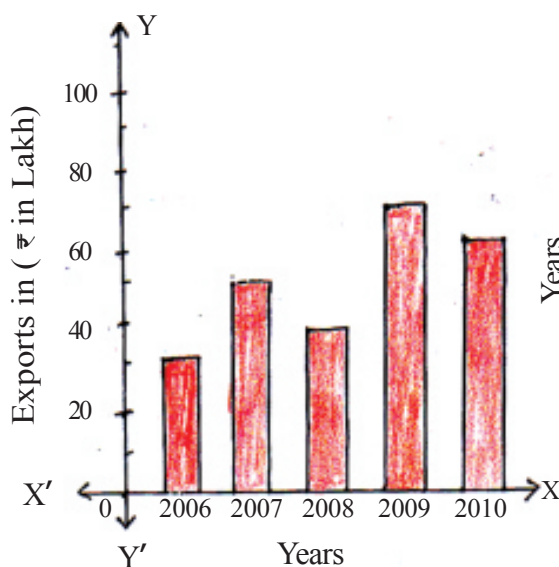
ii) **Multiple Bar Diagram :** Multiple bar diagrams are used for comparing two or more sets of data. e.g exports for different years, marks obtained in different subjects in different classes etc.

Example : Present the following data on distribution of students according to the streams and year in a college, in a multiple bar diagram.

Year	Arts	Commerce	Science
2006	300	500	700
2007	500	200	900
2008	700	400	700
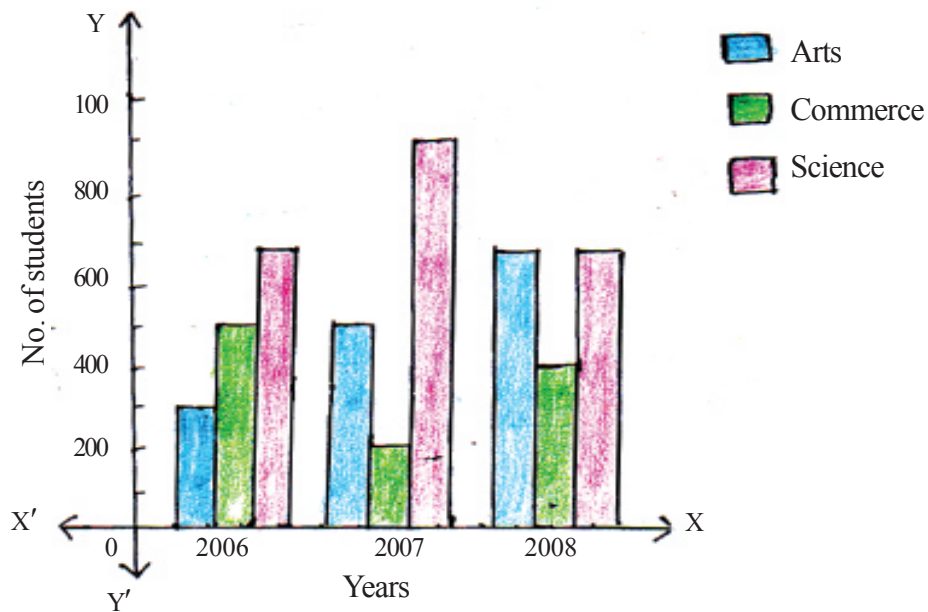
This diagram can also be presented in the following manner–
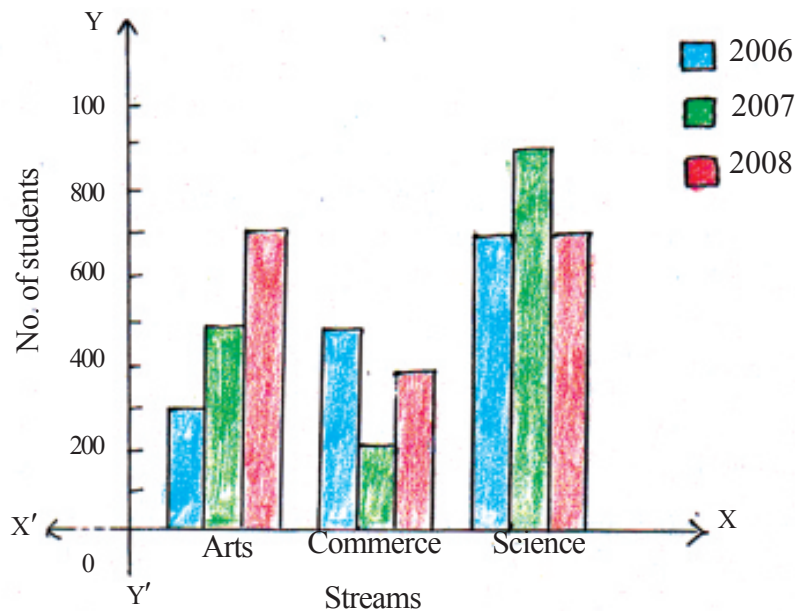
iii)   **Component or sub divided bar diagram :**

Component bar diagrams or charts , also called sub-divided bar diagrams. It is very useful in comparing the size of different component parts (the elements of parts which a thing is made up of) and also for showing the relationship among integral parts .

Example : Present the following data on distribution of students according to the stream in a college, in a subdivided bar diagram.

Years	Arts	Commerece	Science	Total
2010	700	300	400	1400
2011	300	300	500	1100
2012	400	200	700	1300
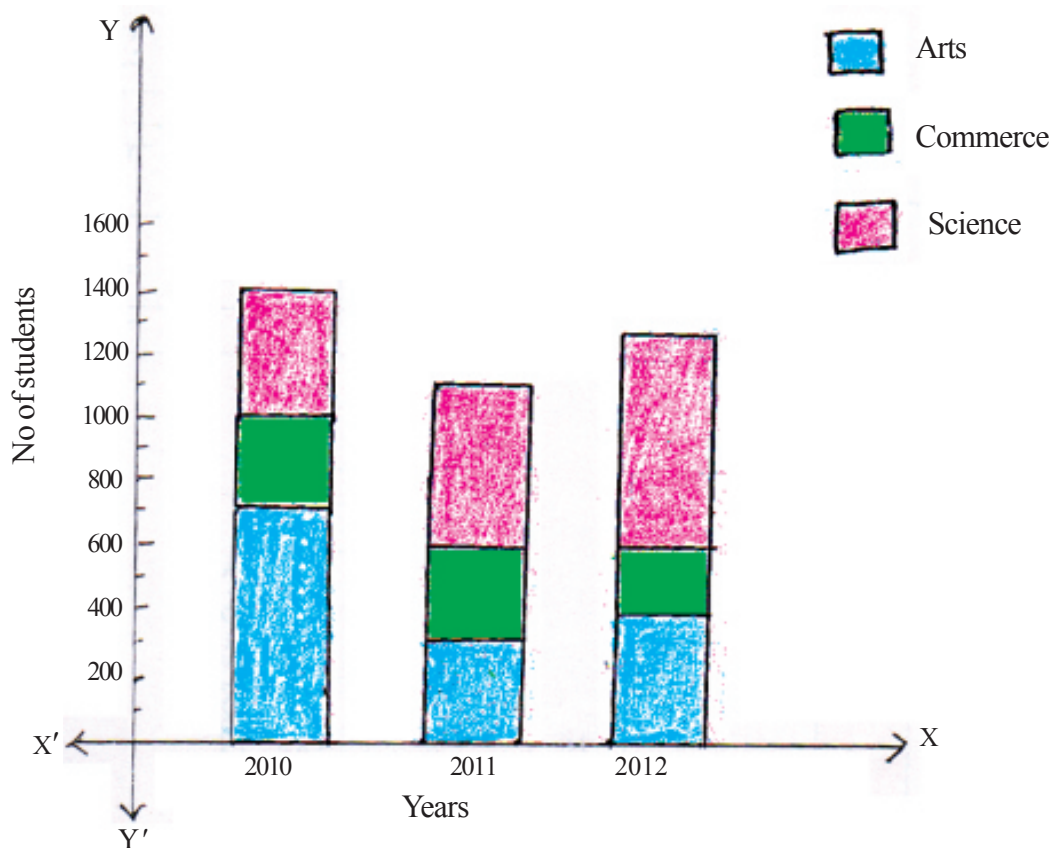
iv)   **Percentage Bar Diagram** : These diagram show simultaneously different parts of the values of a set of data in terms of percentages . In those diagrams height of all the bars will be 100 and the various segement of the bar will vary in height or length according to their percentage value of the table . It is generally used when the values are of high magnitude .

Example : Present the following data on wheat and rice production in percentage bar diagram.

Years	2010	2011	2012
Wheat production (in quintals)	200	600	400
Rice production (in quintals)	300	400	100

Solution :

Production	Year					
	Production in 2010		Production in 2011		Production in 2012	
	in quintals	% age	in quintals	% age	in quintals	% age
Wheat	200	40	600	60	400	80
Rice	300	60	400	40	100	20
Total–	**500**	**100 %**	**1000**	**100%**	**500**	**100 %**
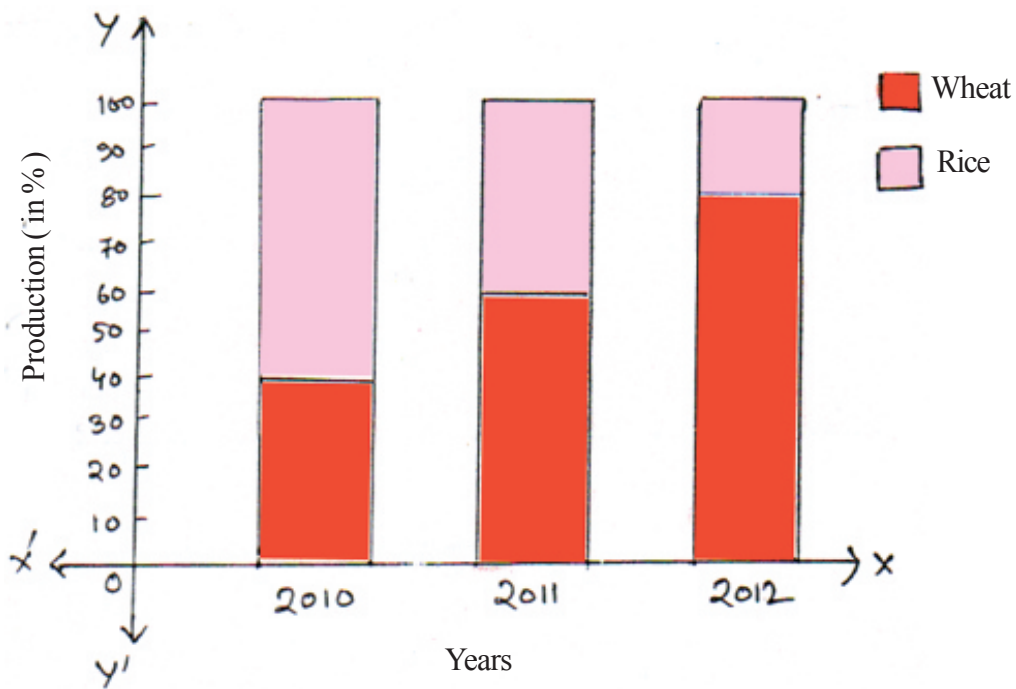
v)   **Broken scale Bar Diagram :**   This diagram is used when value of some variable is very high or low as compared to others . Here height or largest bar may be broken to adjust scale . The vlaue of each bar is written on the top of the bar .

Example : Present the following data of production of a company in Broken scale bar diagram.

Year	2015	2016	2017	2018
Production (in quintals)	30	50	60	2500
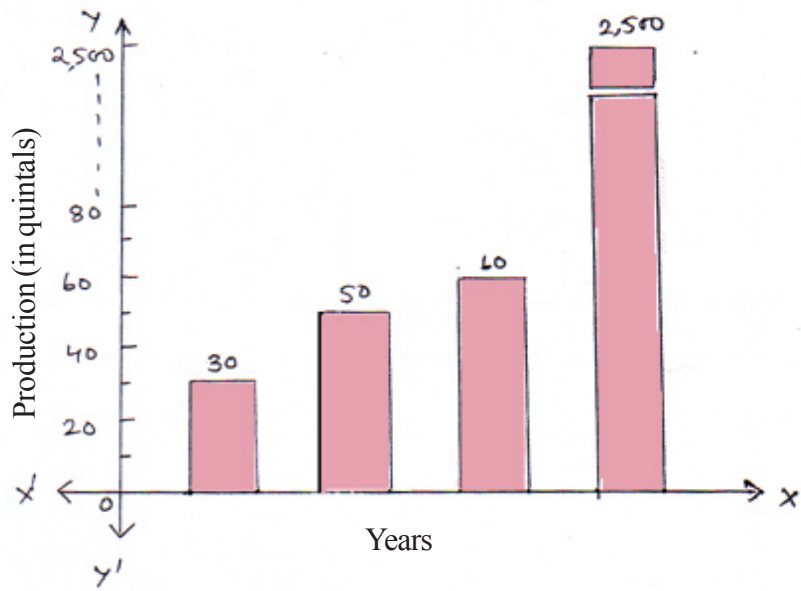
vi) Deviation or Bilateral bar diagram :

Deviation bar diagrams represents two different attributes of data . These diagrams can have both positive and negative values . Positive values are shown above the x axis (base line) and negative values below the base line .

Example : Represent the following data relating to net profit and loss of a company for a period of five years by deviation bar diagram.

Year	2016	2017	2018	2019	2020
Net profit / Loss (₹ in crore)	250	100	(–) 50	150	(–) 100
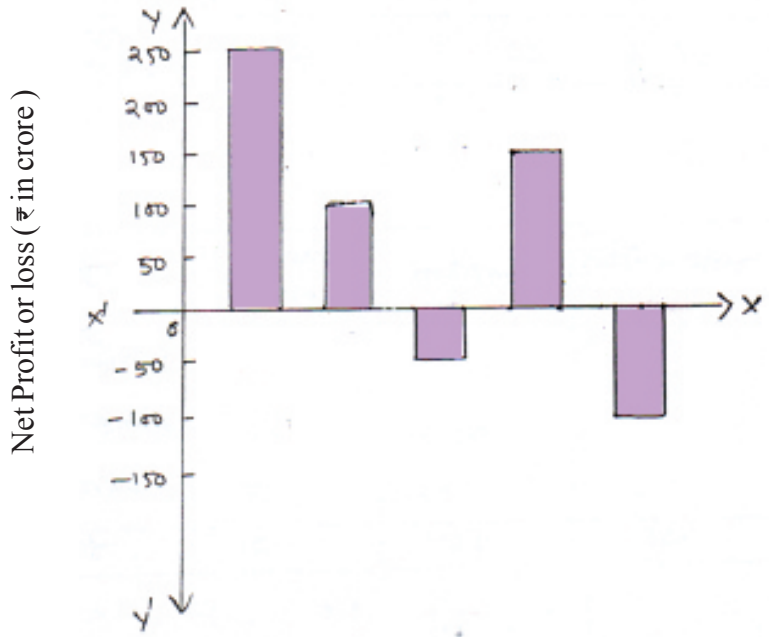
4.4.4 Pie Diagram : Pie diagram is also a component diagram , but unlike a bar diagram , here it is a circle whose area is propartionally divided among the components it represents . It is also called a pie chart . The circle is divided into as many parts as there are components by drawing straight lines from the centre of the circumference.

Example : Present ditribution of Indian population by their working status , using the data given below in pie diagram.

Status	Marginal worker	Main worker	Non- worker
Population (crore)	12	36	73

Solution :

Status	Population (crore)	Percent	Angular Component
Marginal Worker	12	9.9	36º
Main worker	36	29.8	107º
Non - worker	73	60.3	217º
Total	**121**	**100.0**	**360º**

$$\left[\text{Hints } \frac{12}{121} \times 100 = 9.91 = 9.9, \ 9.9 \times 3.6 = 35.64 \cong 36^o \right] \text{ or } \left[\frac{12}{121} \times 360° = 35.6° = 36° \right]$$

12 Crore

73 Crore

36 Crore

■ Marginal Worker

■ Main Worker

■ Non- Worker
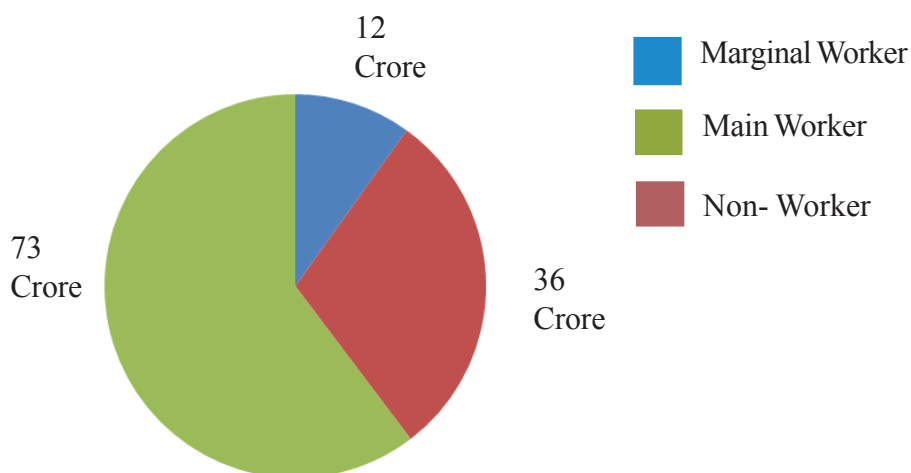
Advantages of Diagramatic presentation :

i) Diagrams are able to attract the attention of a reader .

ii) It makes it easier to understand the data .

iii) Diagrams make it easier to compare data .

iv)   Diagrams are accepted universely .

v)   It represents large volume of complex data in a simplified manner .

Disadvantages of Diagramatic Presentation :

i)   Diagrams can represents only limited information .

ii)   Diagrams can be used for comparative studies .

iii)   Diagrams are not capable of further analysis .

4.5   Graphical presentation of data :

Graphical presentation of data is another method of presenting the data and it is much more accurate and appropriate than the diagramatic presentation . It is a technique of presenting statistical data on a graph paper to clearly visualize and simplify data set .

4.5.1   Types of Graphs : There are two types of graphs - frequency distribution graphs and Arithmatic line graphs or time series graphs.

Types of Graphs

Frequency Distribution Graphs   Arithmatic Line Graphs or Time series Graphs
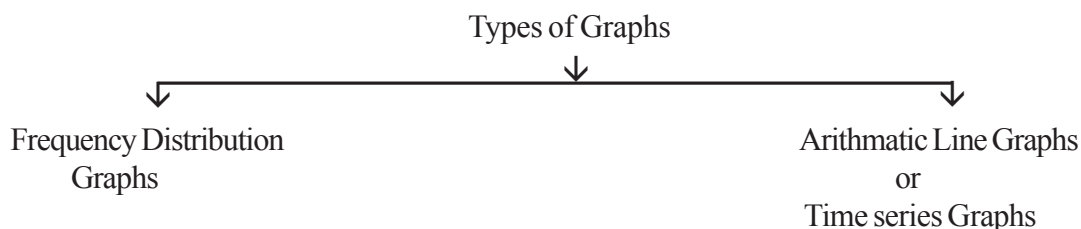
4.5.2   Frequency Distribution Graphs : Frequency distribution graphs are those graphs which are drawn with the help of frequency . Here the frequencies are ploted along with y axis and class limits on x axis. The most common form of graphs of a frequency distribution are– Line frequency graph, Histogram, frequency polygon, frequency curve and ogive.

Frequency Distribution Graphs

Line frequency graph   Histogram   Frequency   Frequency   Ogive
                              polygon   curve
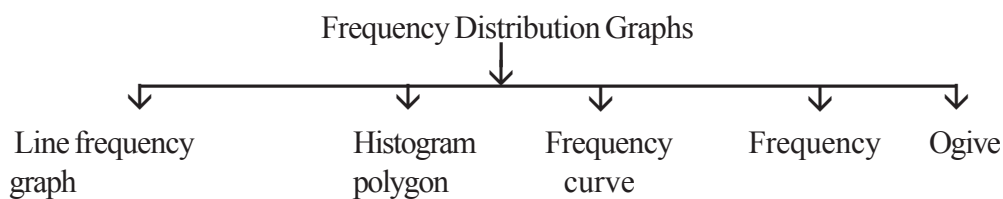
i)   Line frequency graph : Such graphs are used to represents discrete series . In those graphs values of the variables are taken on x axis and corresponding frequencies are taken on y axis .

Example : prepare a line frequency graph from the following data.

Weight (in kg)	51	52	53	54	55
No of . students	10	05	15	20	25
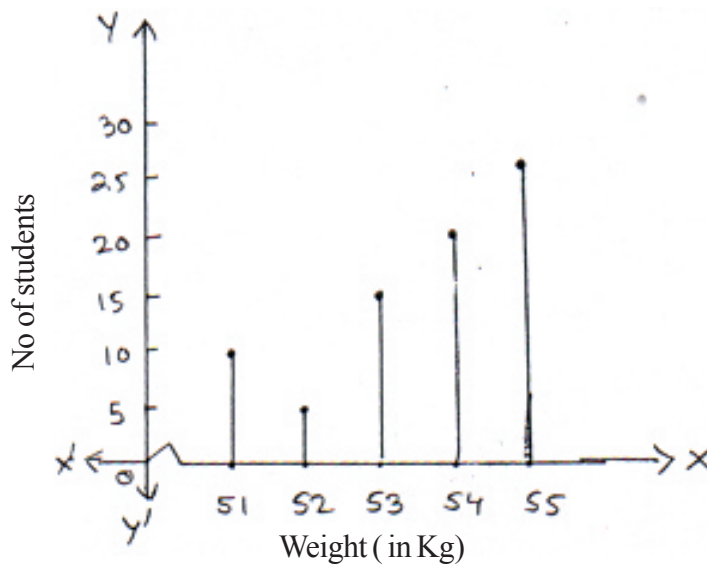
Scale 1 cm = 1 Kg. on X-axis
1 cm = 5 students on Y- axis

No of students

Weight (in Kg)

Histogram : A Histogram is a graphical presentation of a frequency distribution of a contineous series . It is a two dimensional diagram . It is a set of rectangles with base as the intervals between class boundaries (along x - axis) and with areas proportional to the class frequency.

The technique of constructing histogram is illustrated in the following cases.

i)   When equal class intervals are given :

In this case , the height of each rectangle is taken to be equal to the frequency of the corresponding class .

Example : Prepare Histrogram for the following data.

Marks	0 - 10	10 -20	20 - 30	30-40	40 - 50
No of students	20	40	70	60	30
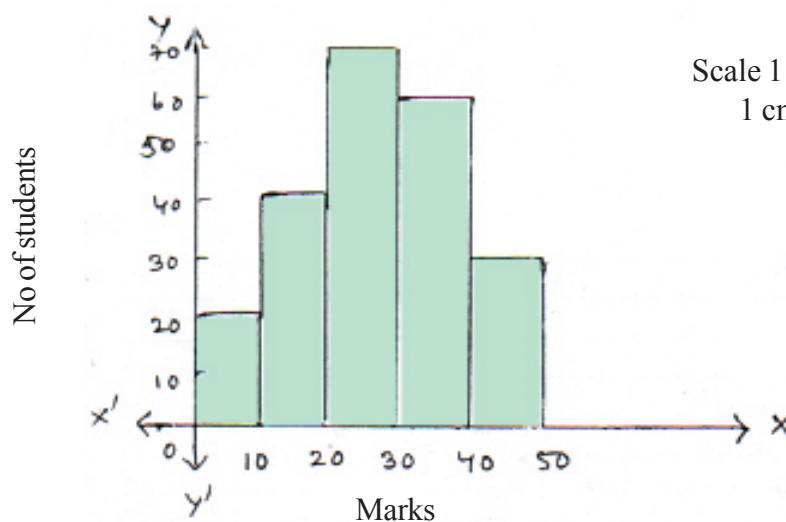
Scale 1 cm = 10 Marks on X - axis
1 cm = 10 students on Y - axis

No of students

Marks

ii)   When Unequal class intervals are given :

In this case , frequencies are first adjusted before constracting the histogram . The frequency distribution is adjusted in accordance to equal class width .

Example : Construct histogram for the following data.

Marks	0 -10	10 -20	20 - 30	30-60	60-80	80 - 90	90-100
No of students	7	5	10	18	4	3	9

Solution : Here width of lowest class interval is 10

$\therefore$ Adjustment factor for $30 - 60 = \dfrac{30}{10} = 3$

Adjustment factor for $60 - 80 = \dfrac{20}{10} = 2$

$\therefore$ Adjusted frequency for $30 - 60 = \dfrac{18}{3} = 6$

and Adjusted frequency for $60 - 80 = \dfrac{4}{2} = 2$

Marks	No of students	Adjusted frequency for Histogram
0 - 10	7	7
10 -20	5	5
20 -30	10	10
30 - 60	18	6
60 - 80	4	2
80 - 90	3	3
90 - 100	9	9
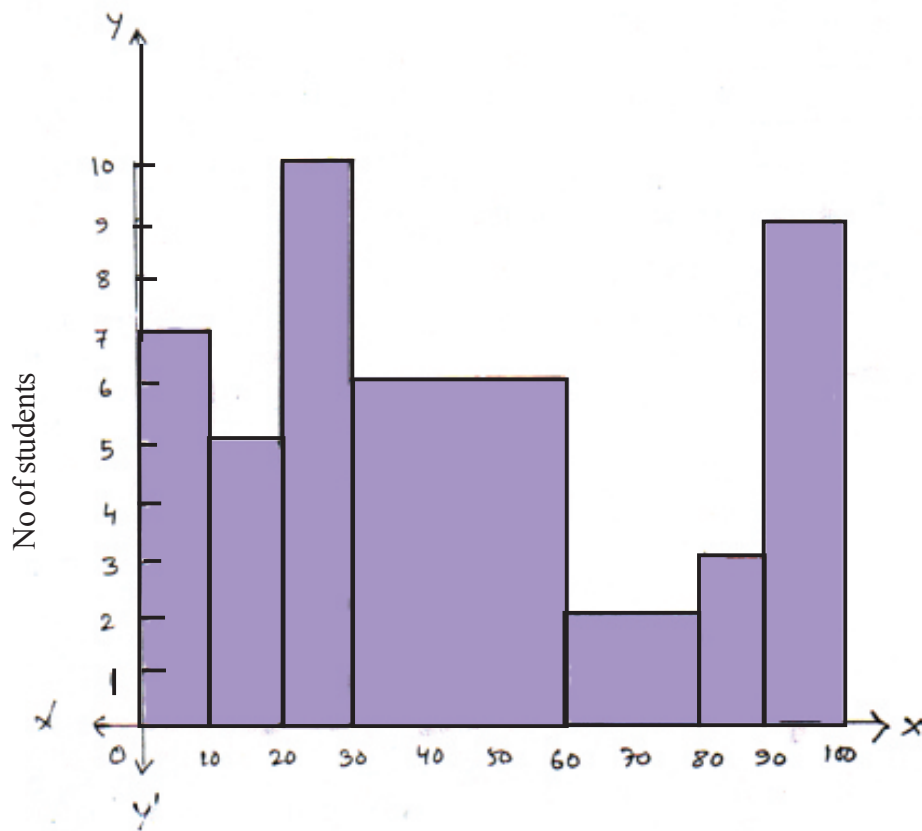
Marks

Scale 1 cm = 1 marks on
(X - axis)
1 cm = 1 students
on (Y - axis)

iii)   **When mid points are given :**

If the mid points of various classes are given is place of class– intervals, then these must be converted into class intervals.

Example : Construct a histogram for the following frequency distribution.

Mid - Points	15	25	35	45	55
Frquency	6	10	15	8	12

Solution : Here the difference between two mid points is 10 .

Half of the difference i.e. $\dfrac{10}{2} = 5$ will be added and substracted from each mid point to get the class intervals.

Classes	10 - 20	20 -30	30 - 40	40 - 50	50 - 60
Frequency	6	10	15	8	12

Scale 1 cm = 10 classes (X - axis)
1 cm = 1 frequency (Y - axis)
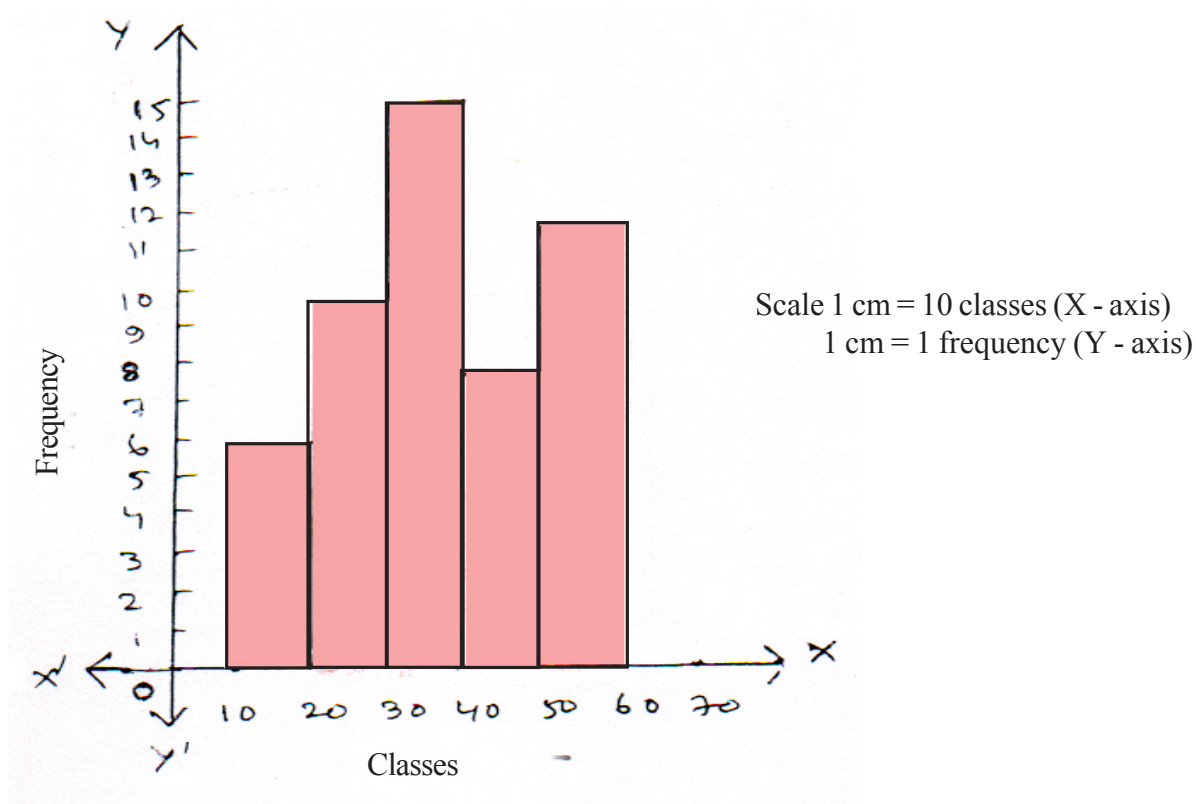
iv)   When class intervals are inclusive : In this case, inclusive series of class interval should be first converted into exclusive series.

Example : Draw a histogram from the following series.

Marks	10 -19	20 - 29	30 - 39	40 - 49	50 - 59
No of student	5	15	10	25	30

Solution : The difference between upper limit of a class interval and lower limit of the next class interval is 1.

Half of the difference i.e $\frac{1}{2}$ = 0.5 will be subtracted from lower limits of all the classes and added to the upper limits of all class.

Marks	9.5-19.5	19.5-29.5	29.5-39.5	39.5-49.5	49.5-59.5
No of students	5	15	0	25	30

Scale 1 cm = 1 marks on (X - axis)
1 cm = 1 students on (Y - axis)
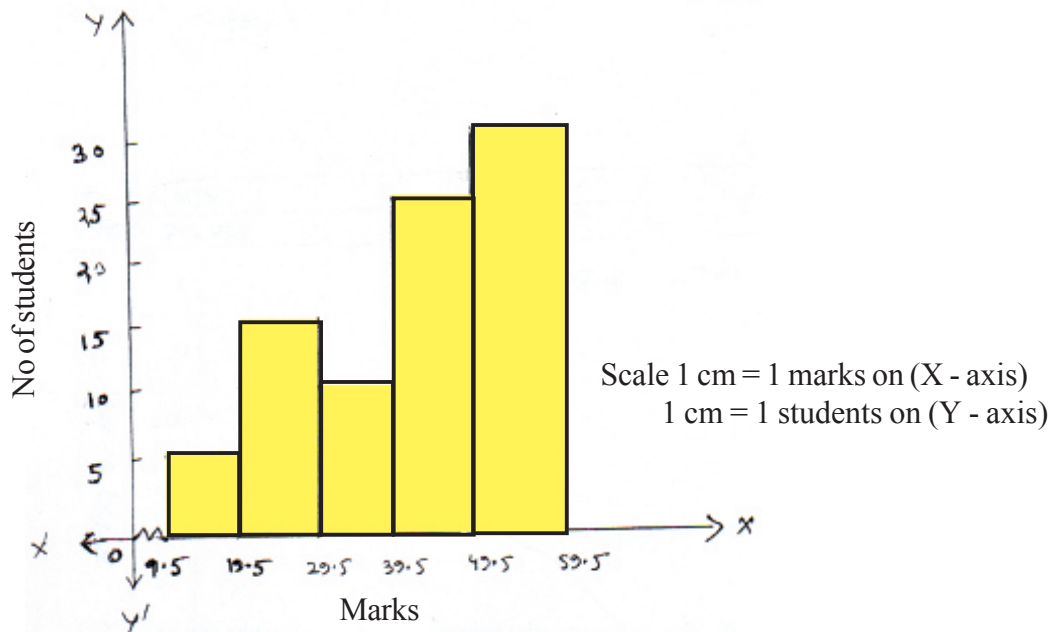
Frequency polygon : A frequency polygon is a plane bounded by straight lines , usually four or more lines. Frequency polygon is an alternative to histogram and is also derived from histogram itself . If we connect the mid points of the top of each rectangles of the histograms by straight line that is called frequency polygon.

Frequency polygon can be constructed either for discrete series or for contineous series. Again a frequency polygon for a contineous frequency distribution can be drawn in two ways - with the help of Histogram and without the help of Histogram .

Explain : Construct frequency polygon for the following data .

No of Rooms	1	2	3	4	5
No of Houses	5	15	20	10	5

Scale 1 cm = 1 marks on (X - axis)
1 cm = 1 students on (Y - axis)
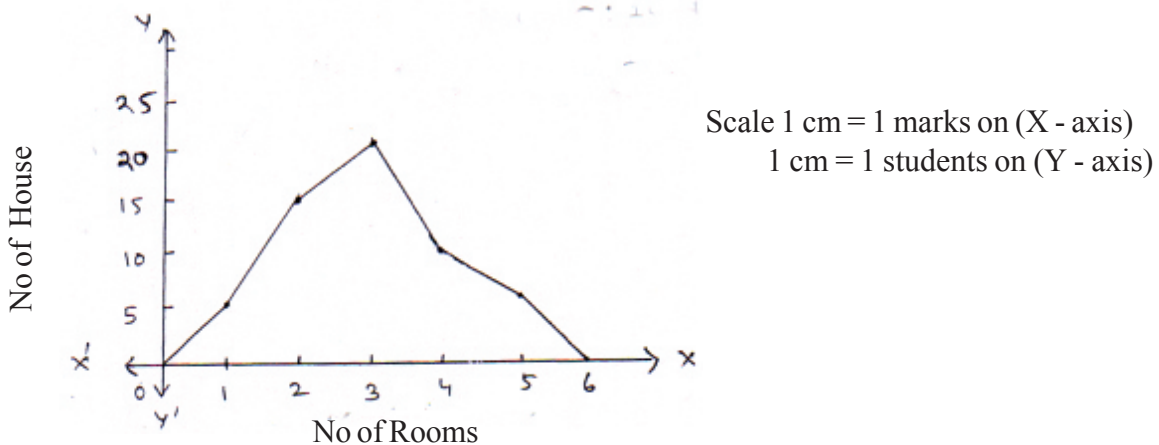
Example : Construct frequency polygon for the following data with Historgram and without Histogram.

Marks	20- 30	30 - 40	40 - 50	50 - 60	60 - 70
No of Students	15	25	20	10	5

Scale 1 cm = 10 marks (X - axis)
1 cm = 5 students (Y - axis)
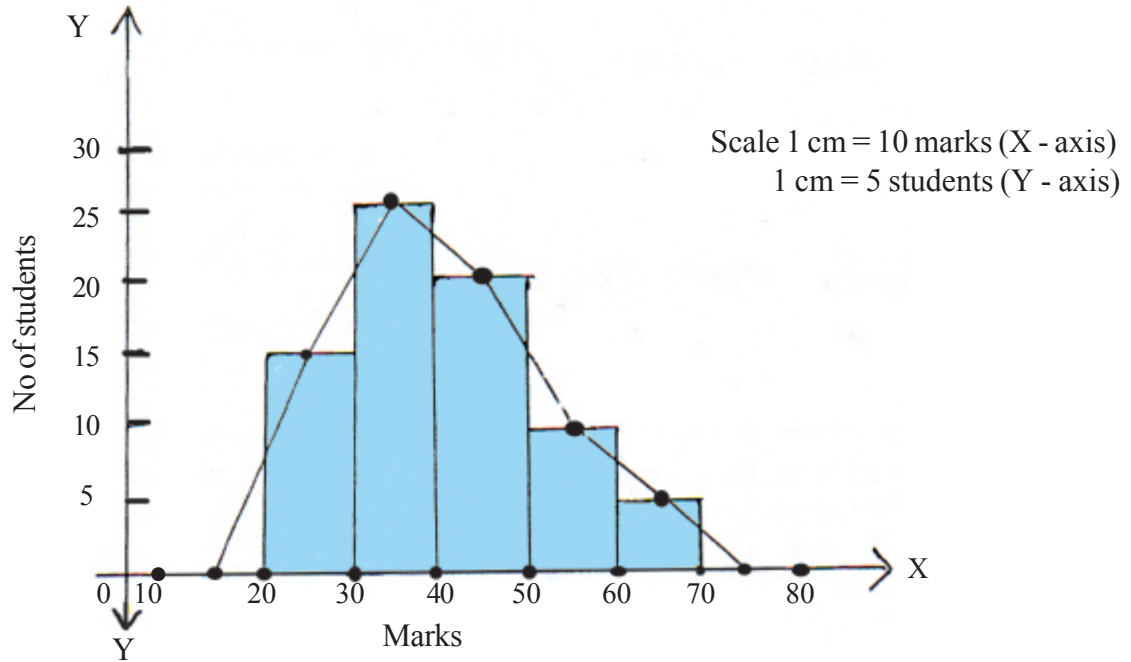
Without Histogram :

Marks (Mid- points)	25	35	45	55	65
No of students	15	25	20	10	5

Scale 1 cm = 10 marks (X - axis)
1 cm = 5 students (Y - axis)
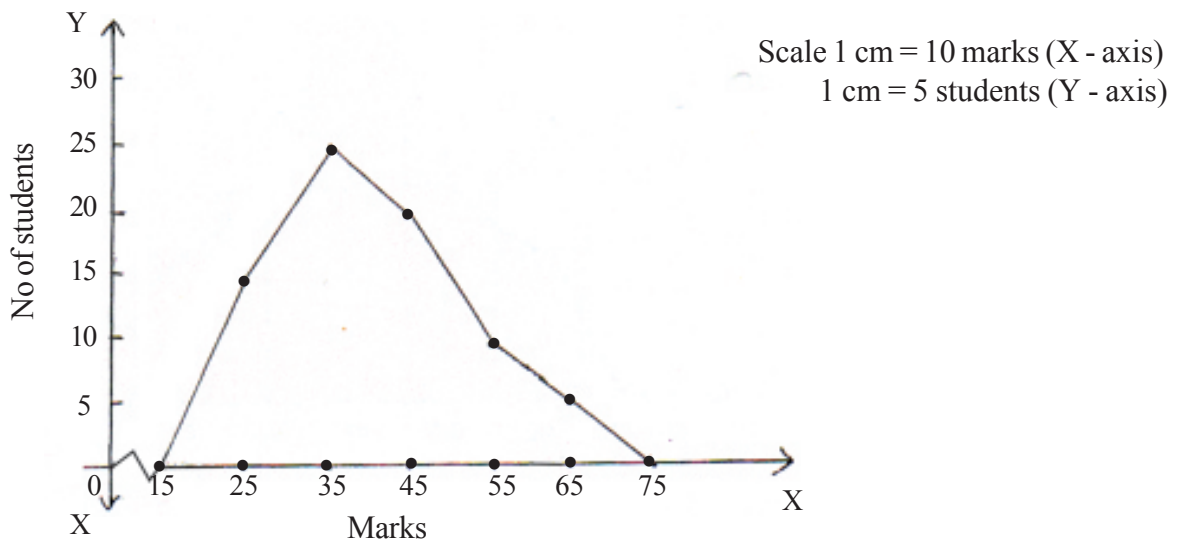
Frequency Curve : A frequency curve is a curve which is plotted by joining the mid points of the tops of rectangles in a histogram through free hand smooth curve and not by straight line. It is also known as smoothed frequency curve .

We can construct frequency curve with histogram and without histogram .

With Histogram :

Example : Draw a frequency curve from the following distribution.

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	5	10	15	30	15	10

Scale 1 cm = 10 classes interval (X - axis)
1 cm = 5 frequency (Y - axis)
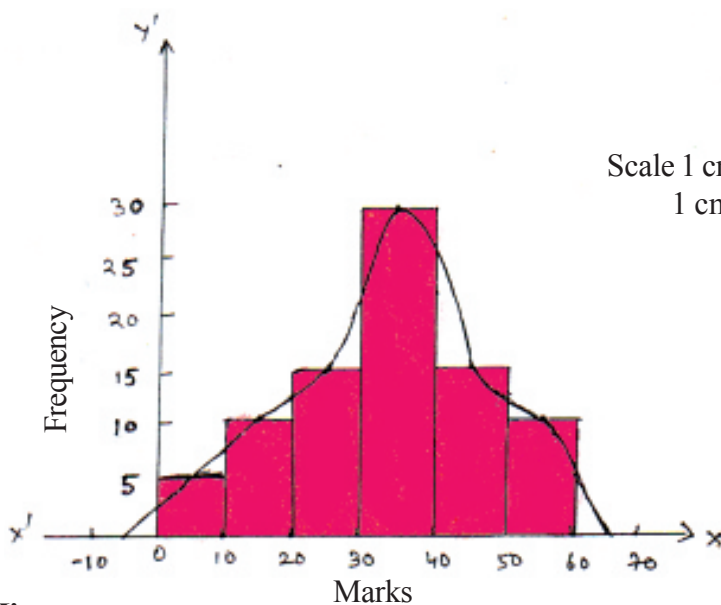
Without Histogram :

Class interval (Mid points)	5	15	25	35	45	55
Frequency	5	10	15	30	15	10

Scale 1 cm = 1 Class interval (X - axis)
1 cm = 5 Frequency (Y - axis)

Class intervals (Mid point)
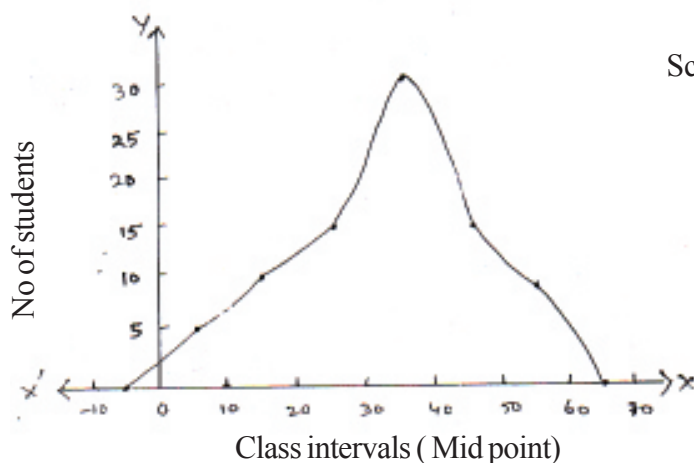
Ogive or cumulative frequency curve :

It is the curve which is constructed by plotting cumulative frequency data on the graph paper in the form of a smooth curve .

Since there are two types of cumulative frequency distribution viz, 'less then' cumulative frequency and 'more than' cumulative frequency we have accordingly two types of ogives - Less than ogive and More than ogive .

Less than ogive : In this method cumulative frequencies are calculated starting with the upper limits of the classes.

Example : Construct a 'Less than' ogive from the data given below.

Income	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No of Person	5	10	20	15	5

Solution :-

Income (Rs 0000's)	No of person
Less than 10	5
Less than 20	15
Less than 30	35
Less than 40	50
Less than 50	55

Scale 1 cm = ₹ 10,000 (X-axis)
1 cm = 10 Person (Y-axis)
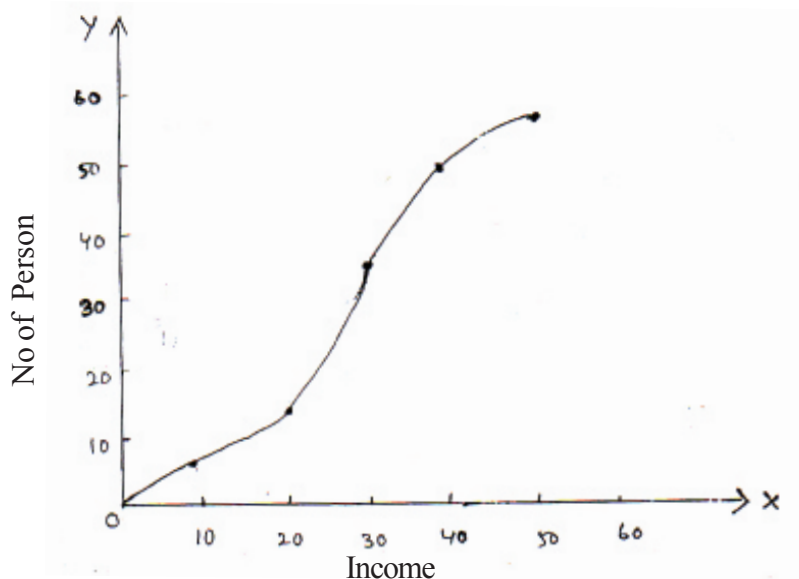
More than ogive : In this method cumulative frequencies are calculated starting with lower limits of the classes.

Example : Construct a 'More than' ogive from the data given below.

Income	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No of Person	5	10	20	15	5

Solution :

Income (Rs 000's)	No of person
More than 0	55
More than 10	50
More than 20	40
More than 30	20
More than 40	5
More than 50	0

Scale 1 cm = Rs 10,000 (X - axis)
1 cm = 10 person (Y - axis)
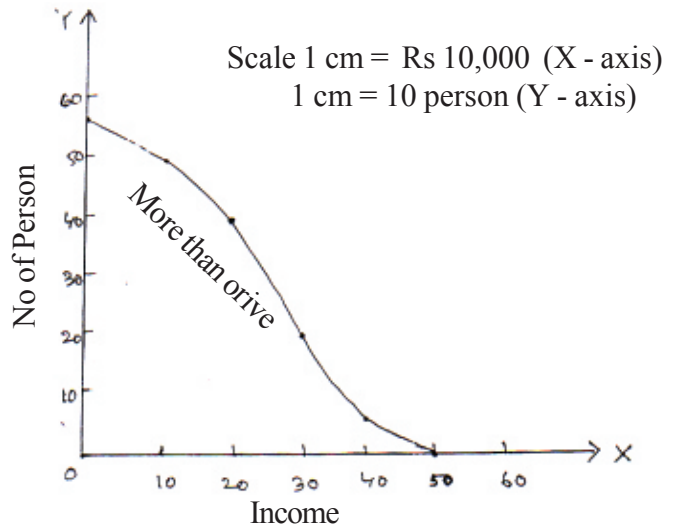
Example : Draw, 'Less than ogive' and 'more than ogive' in the same graph . Also determine the value of median .

Income	0-10	10-20	20-30	30-40	40-50
No of Person	5	15	25	3	2

Solution :

Less than method		More than method	
Income (Rs 0000's)	No of person	Income (Rs 0000's)	No of person
Less than 0	0	More than 0	50
Less than 10	5	More than 10	45
Less than 20	20	More than 20	30
Less than 30	45	More than 30	5
Less than 40	48	More than 40	2
Less than 50	50	More than 50	0
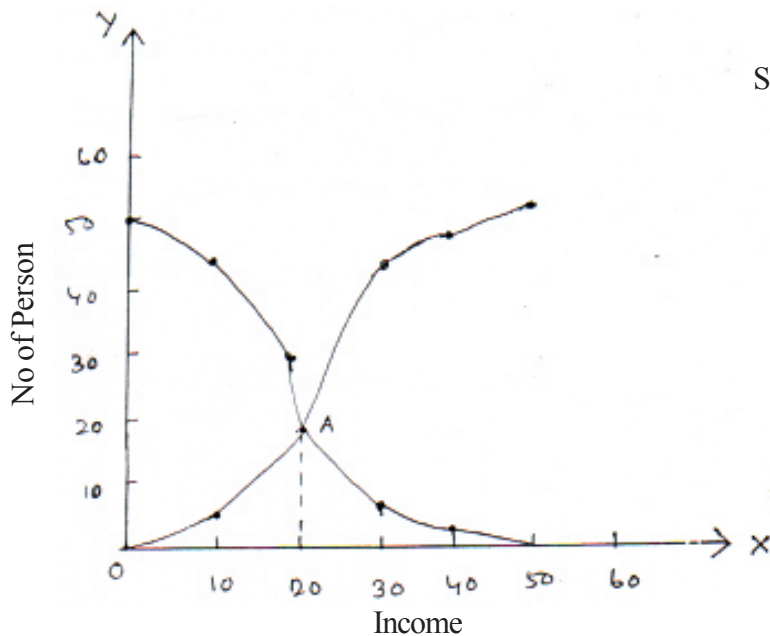
Median is the value of the variable on x - axis obtained by drawing the perpendicular from the point of intersection of the ogives (point A). Here median is 20.

4.5.3 Arithmatic Line Graphs :

An arithmatic line graph is also called time series graph . In this graph time (hour, day / date, week, month, year etc) is ploted along x-axis and the value of variable (time series data) along y - axis . A line graph by joining these plotted points , thus obtained is called arithmatic line graph. (time series graph). It helps in understanding the trend, periodicity , etc in a long term time series data.

Types of Time series Graphs : There are two types of time series graphs - one variable graphs and two or more than two variables graphs.

Time Series Graphs

One variable Graphs   Two or more than two variables Graphs
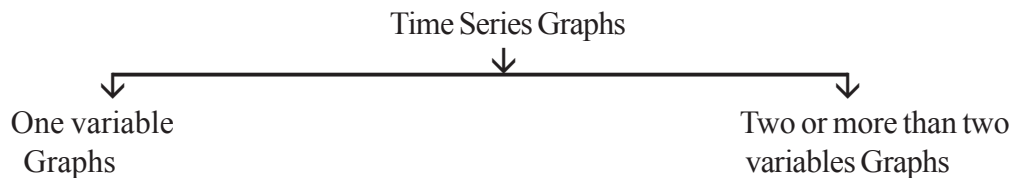
i)   **One variable Graphs :** These graphs represent values of only one variable on a graph paper with respect to some time period .

Example : Construct time series graph for the following data.

Years	2010	2011	2012	2013	2014
Exports (Rs in lakh)	15	20	35	15	10
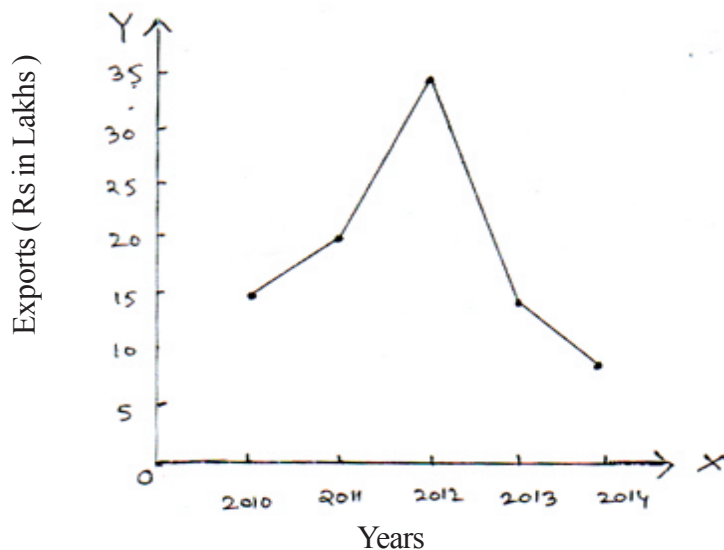
ii)   Two or more than two variable graphs : These graphs represent value of two or more than two variables on a graph paper with respect to same time period .

Example : Construct a time series graph to present the following data.

Years	2014	2015	2016	2017	2018	2019	2020
Imports (Rs in Lakhs)	1000	1800	2500	2100	2700	3800	5000
Exports (Rs in Lakhs)	2100	3000	4000	3500	4500	5300	6900
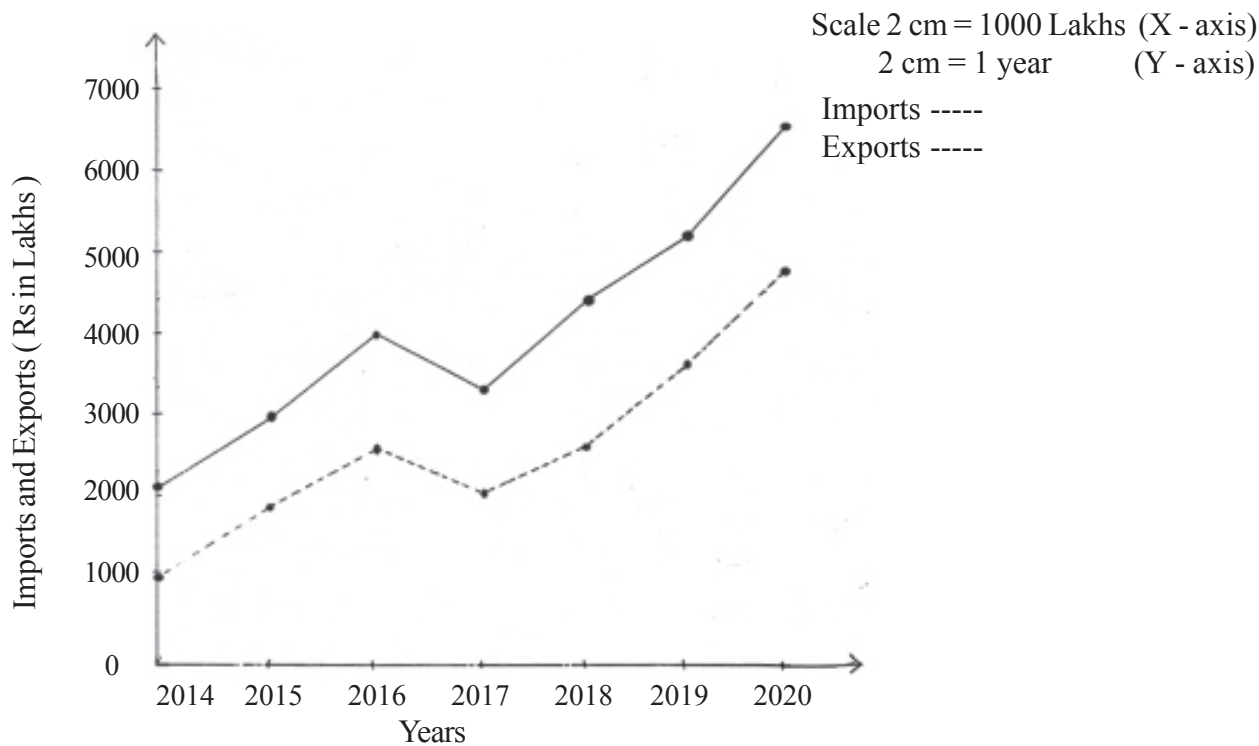
Imports -----
Exports -----

Advantages of graphical presentation :

i)   Graphical presentations are always attractive and impressive.

ii)   Graphs present the complex data in a simple way.

iii)   There is not a requirement of specialised knowledge to understand and comprehand the graps .

iv)   Graphs provide easy comparison of two or more phenomena.

v)   Graphs are widely used methods of presentation of data.

Dis-advantages of graphical presentation :

i)   Only a limted set of data can be presented in the form of a graph .

ii)   A small error in measurement of scale may create large difference in the shape of graph . Hence it can give misleading results .

iii)   It may not be always easy to arrive at final conclusions after seeing the graphs. Graphs offer only preliminary conclusions .

iv)   Graphs show the tendencies of data . Hence they cannot be used to test the accuracy of the data .

Exercise

1.   State whether the following statement are True or False .

i)   The value of median can be determined with the help of an ogive .

ii)   In the third quadrant , the values of both x and y are positive .

iii)   Arithmatic line graphs are also known as Time series graphs .

iv)   Frequency curve is a simple form of frequency polygon which is drawn by free hand smooth curves .

v)   In case of more than ogive , the cumulative total tends to increase .

vi)   Bars in bar diagrams are equidistant from each other .

vii)   Percentage bar diagrams present only part values of a set of data .

viii)   Pie diagrams are two dimensional diagrams .

ix)   In case of tabulation , source of information is given just above the footnote .

x)   General purpose table is also known as summary table .

2.   Multiple choice questions : choose the correct asnwer :

i)   An orderly arrangement of data in columns and rows are called–

        a) Tabulation   b) Classification   c) Investigation   d) None of these .

ii) When two types of information is obtained from a table , then it is called–

a) simple table   b) Double table   c) Treble table   d) None of these .

iii) Pie diagram is also know as–

a)   Pie chart   b) Angular circle diagram

c)   both (a) and (b)   d) None of those .

iv) The column heading of a table is known as–

a) stubs   b) sub-title   c) Reference note   d) caption

v) The objective of diagramatic presentation of data is–

a) summarisation   b) presentation   c) considerisation   d) all of these .

vi) The total angle at the centre of a pie chart is –

a) 90° b) 180° c) 270° d) 360°

vii) Bar diagram is a -

a)   One dimensional diagram   b) Two dimensional diagram

c)   Three dimensional diagram   d) None of those .

viii) The essential condition for construction of histogram is–

a)   Discrete series   b) contineous series   c) both a and b   d) None of these .

ix) A graph which is based on Time period is–

a)   Frequency graph   b) Time series graph   c) Histogram   d) None of those .

x) The graph of cumulative frequency distribution is called–

a)   a line graph   b) a histogram   c) an ogive   d) None of those .

3.   **Fill in the blanks :**

i) Data are represented on the basis of _____ in pie diagram.   (area / angle)

ii) In bar diagram bars are also called _____ (column / Row)

iii) _____ completes the information in the title of the table (Headnote / caption)

iv) General purpose tables are also called _____ (Derived table / Reference table)

v) _____ classification of tables is done one the basis of time . (Qualitative / Temporal)

vi) In case of one dimensional diagrams _____ of the bar remains the same. (Height /width .

vii) In time series graph , the year or month or days one plotted on the _____ (x-axis /y -axis)

viii) _____ bar diagrams are used generally when the values are of high magnitude.

(simple / Percentage)

ix) In case of frequency polygon , points are joined by _____ (free hand / straight line)

x) For ' Less than' ogive the cumulative frequencies are plotted against the _____ limit at the class interval.   (Lower / upper) .

xi) Mode is formed graphically by _____ (Histogram / ogive) .

4. Very short answer type questions :   **(1 mark)**

i) What do you mean by presentation of data ?

ii) What is stub ?

iii) Which part of statistical table briefly explains about the figures entered in columns ?

iv) What is complex table ?

v) Write down the names of two main parts of a table ?

vi) Why are bar diagrams said to be one - dimensional diagrams ?

vii) What is tabulation ?

viii) What is spatial classification ?

ix) What is frequency curve ?

x) Name a bar diagram where height of all the bars is same .

xi) What is frequency distribution graphs ?

xii) What is Histogram ?

xiii) What do you mean by Arithmatic line graph ?

xiv) Which graph can determine the value of median ?

xv) Which type of ogive rise upwords to the height ?

5. Answer the following questions :   **(3/4 Marks)**

i) What is meant by tabulation ? Write the objectives of tabulation .

ii) Distinguish between simple table and complex table .

iii) Write short notes on Three way Table .

iv) Distinguish between textual presentation and Tabular presentation .

v) Mentain two advantages and disadvantages of textual presentation .

vi) Mention the limitations of tabular presentation .

vii) Write short note on ' spatial classification' .

viii) Distinguish between qualitative classification and quantitative classification .

ix) State the difference between Tabulation and classification .

x) Mentain four merits of diagramatic presentation .

xi) Write short notes on 'Broken scale Bar diagram'

xii) What is the difference between subdivided bar diagram and percentage bar diagram .

xiii) State the features of bar diagram .

xiv) What do you mean by fequency distribution graph ? Mention their types .

xv) Differentiate between frequency polygon and frequency curve .

xvi) Write short note on 'Time series Graph' .

xvii) Mention three limitations of graphical presentation .

xviii) Distinguish between Bar diagram and Histogram .

xix) Using imaginary figures , prepare a simple table .

xx) Discuss in brief :   a) Title   b) Body of the table .

xxi) From the following data relating to cement production of a factory (2015-20) prepare a simple bar diagram .

Years	2015	2016	2017	2018	2019	2020
Exports (in M. Tonnes)	100	150	250	300	450	500

xxii) Draw a line frequency graph of the following data .

Marks	30	40	50	60	70
Frequency	20	15	05	10	15

xxiii) Draw a time series graph of the following data .

Year	2016	2017	2018	2019	2020
Profits (₹ in crore)	20	32	37	45	48

6.   Answer the following questions :   **(6 Marks)**

i) Briefly explain the components of a good table .

ii) Define diagramatic presentation of data . How is it different from Tabulation ?

iii) What is an ogive curve ? How is it constructed ?

iv) Make a multiple bar diagram of the following data.

Stream	No of students		
	2018	2019	2020
Arts	600	500	400
Commerce	400	450	600
Science	200	350	500

v)   Represent the following data with the help of sub divided bar diagram.

Year	Production (in '000 tonnes)		
	Wheat	Rice	Cotton
2018	40	25	10
2019	10	22	14
2020	20	14	21

vi)   The mininimum temperature of Shimla in the last weak of December is given below . Represent the data with the help of deviation bar diagram.

Date	25th Dec.	6th Dec	27th Dec.	28th Dec.	29th-Dec.	30th Dec	31st Dec
Temparuture	3	−1	−2	2	5	−2	− 4

vii) Present the following data by a percentage bar diagram .

Subject	Number of Students (in 000)		
	2011-12	2012-13	2013-14
Statistics	25	30	28
Economics	40	32	36
Mathematics	35	42	30

viii) From the following data exports of a firm prepare a broken scale bar diagram.

Years	2016	2017	2018	2019	2020
Exports (in Rs in lakh)	16	22	35	340	25

ix)   Represent the following data which shows the expenditure of family with the pie diagram

Commodity	Food	clothes	Rent	Education	Misc	Savings
Expenditure	300	125	200	110	75	90

x)   Represent the following information using histogram.

Mid values	2.5	7.5	12.5	17.5	22.5
Frequency	5	10	35	15	5

xi)   Draw a histogram for the following data .

Marks	0 - 10	10 - 20	20 - 50	50 - 70	70 - 80
No of students	8	12	28	16	12

xii) Present the following data in the form of frequency polygon using histogram.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50-60	60-70	70-80
Frequency	5	12	18	20	30	25	14	7

xiii) Present the following data in the form of polygon without histogram.

Marks obtained	10 - 20	20 - 30	30 - 40	40 - 50	50-60	60-70
No of student	5	10	20	25	15	40

xiv) Construct a frequency curve with histogram.

Wages	100 - 110	110 - 120	120 - 130	130 - 140	140 - 150
No of workers	130	160	80	120	240

xv) Draw 'Less than ogive' and 'more than ogive' from the given data . Also determine the median value

Profit (in lakhs)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
No of companies	10	20	40	25	15

Answers

1. i) True ii) False iii) True iv) True v) False vi) True vii) False viii) True
    ix) True x) False .

2. i) a ii) b iii) c iv) d v) b vi) d vii) a viii) b ix) b x) c

3. i) angles ii) column iii) Headnote iv) Reference table v) Temporal vi) Width
    vii) x- axis viii) percentage ix) Straight line x) upper xi) Histogram .

4. i) The presentation of data means exhibition of data in a clear and attractive manner so that data can be easily understood and analysed .

    ii) Stubs are titles of the rows of a table .

    iii) captions .

    iv) A complex table is that which shows more than one characteristic of data .

    v) Heading and Table number .

    vi) Because in bar diagrams , only the height represents the value of the variable .

    vii) Tabulation is a scientific process to present the classified data in an orderly manner so that data are clearly understood .

    viii) When classification is done on the basis of place , it is called spatial classification .

    ix) A frequency curve is a curve which is plotted by joining the mid points of the tops of rectangles in a histogram through freehand smoothed curve and not by straight line .

    x) Percentage bar diagram .

    xi) Frequency distribution graphs are those graphs which are drawn with the help of frequency

    xii) A histogram is a graphical presentation of a frequency distribution of a contineous series .

    xiii) The graphs represents the time series distribution are known as Arithmatic line Graphs .

    xiv) Ogive .

    xv) Less than ogive.

CHAPTER - 5

Measures of Central Tendency

In the previous chapters , we discussed about the different methods of data collection , organisation of data and different techniques of presentation of data .

Inspite of that , there is a huge need for some single measurement, which can easily describe the main characteristic of the series or data base . In this chapter, we will discuss the measures of central tendency which is a numerical method to explain the data in brief .

5.1 Meaning :

Measure of central tendency is the process to find a single value , which is used to represent an entire data. The measuring of central tendency is a way of summarising the data in the form of a typical or representative value

5.2 Objectives and functions of statistical averages :-

1. The main objective of average is to present a simple and brief picture of the data , which makes it easier to understand and remember .

2. Averages are very useful for making comparative studies as they reduce the mass of data to a single value .

3. Averages are very useful for decision making and in policy formulation .

4. Average helps to take an idea about the whole universe by means of sample data .

5.3 Essentials of an ideal Average :-

A good measure of average must have following qualities :-

1. It must be easy to understand .

2. It must be based on all the observations .

3. It should be least affected by fluctuation of sampling .

4. It should be capable of further algebric treatment .

5. It must be rigidly defined .

5.4 Kinds of statistical averages :-

There are different kinds of averages out of them, we shall discuss arithmetic average, Median and Mode .

5.4.1 Arithmetic Mean Or Mean :-

Arithmetic mean is the most commonly used measure of central tendency . Arithmetic mean is defined as the sum of the values of all observations divided by the number of observation and usually denoted by $\overline{X}$.

Let , the $X_1 , X_2 , X_3 \ldots \ldots \ldots$ Xn are Number of observations.

$$\therefore \quad \overline{X} = \frac{X_1 + X_2 + X_3 + \ldots + X_n +}{N} = \frac{\Sigma X}{N} = \frac{\text{Sum of all observation}}{\text{Number of observation}}$$

The Arithmetic mean is of two Kinds :-

    i)   Simple Arithmetic mean   ii) Weighted Arithmetic mean.

Calculation of simple arithmetic average or Mean

A.   **In case of Individual series :-** The arithmetic mean in individual series can be calculated by following methods .

    i) Direct method   ii) Shortcut or assumed mean Method   iii) Step deviation method

i)   Direct Method :- According to this method , all the units are added and then their total is divided by the number of items and the quotient becomes the mean .

Formula Mean ($\overline{X}$) $= \dfrac{\Sigma X}{N}$

        ΣX = total of all the items

        N = Number of items

Example :- Following are the marks received by 5 students

        50 , 52 , 54 , 58 , 60

Solution :-

Sl No	Marks obtain
1	50
2	52
3	54
4	58
5	60
N = 5	$\Sigma X = 274$

(Mean) $\overline{X} = \dfrac{\Sigma X}{N}$

        $= \dfrac{274}{5}$ $= 54.8$

Arithmetic Mean of Marks of 5 students is 54.8

ii)   **Shortcut Method (Assumed Mean Method) :–** If the number of observation in the data is more, then the calculation can be easier by using assumed mean method .

Here , you assume a particular figure in the data as the assume mean . Then you take deviation of the said assumed mean from each of the observation . Then , you take the summation of these deviations and divide it by the number of observation in the data .

$$\text{Formula , Mean } (\overline{X}) = A + \frac{\Sigma d}{N}$$

A = assumed mean.

d = deviation of assumed mean from individual item ,

N = Number of observation

Example :- Let , the following are the makes of 5 students

50 , 52 , 54 , 58 , 60

Solution :

Sl No	Marks	d = X - A
1	50	–4
2	52	–2
3	(54) – A	0
4	58	4
5	60	6
N = 5		$\Sigma d = 4$

Let, assumed Mean A = 54

$$\text{Mean } (\overline{X}) = A + \frac{\Sigma d}{N}$$

$$= 54 + \frac{4}{5}$$

$$= 54 + 0.8 \quad = 54.8$$

$$\therefore \text{ Mean } (\overline{X}) = 54.8$$

iii)   **Step deviation method :-** In this method deviation of observation taken from assumed mean are divided by a common factor (c) to get step deviation . $\therefore$ $d' = \left(\dfrac{X - A}{C} \right)$

$$\text{Formula , Mean } (\overline{X}) = A + \frac{\Sigma d'}{N} \times C$$

Here , A = assumed mean

$$d' = \frac{d}{c} \Big\} \quad d = (X - A), c = \text{common factor}$$

Example : Let, the marks of 5 students are

50 , 52 , 54 , 58 , 60

SL NO	Marks (X)	$d' = \dfrac{X-A}{C}$, (c = 2)
1	50	$\dfrac{50-54}{2} = \dfrac{-4}{2} = -2$
2	52	$\dfrac{-2}{2} = -1$
3	⑤④ – A	$0 / 2 = 0$
4	58	$\dfrac{4}{2} = 2$
5	60	$\dfrac{6}{2} = 3$
N = 5		$\Sigma d' = 2$

$$\text{Mean } (\overline{X}) = A + \frac{\Sigma d'}{N} \times C$$

$$= 54 + \frac{2}{5} \times 2$$

$$= 54 + \frac{4}{5}$$

$$= 54 + 0.8 \quad = 54.8$$

Arithmetic Mean ($\overline{X}$) = 54.8 of Marks of students

B) Arithmetic Mean in case of discrete series :-

i) Direct Method :- In case of discrete series, frequencies are given corresponding to different values of variables. Here, total Number of observations, (N) = sum total of frequencies

$$\because N = \Sigma f.$$

Here, In case of direct method (Discrete series), frequency, against each observation is multiple by

the value of the observation. Then, we have to divide the obtained summed up value by total number of frequency.

$$\text{Formula , Mean (X)} = \frac{\Sigma fx}{N} = \frac{\Sigma fx}{\Sigma f}$$

Example : Let the marks of students are given

Marks	5	10	12	14	16
No of student	2	1	3	3	1

Solution :

Marks(X)	No of student (f)	fx
5	2	10
10	1	10
12	3	36
14	3	42
16	1	16
	N = Σf = 10	ΣfX= 114

$$\text{Mean } (\overline{X}) = \frac{\Sigma fx}{N} = \frac{114}{10} = 11.4$$

Arithmetic mean of Marks is 11.4 .

ii) Assumed Mean (Method short Cut Method) :-

We described the assumed Mean or short cut method in individual series . In case of descrete series, we multiply each deviation value {d = (X - A)} by frequency (f) to get fd .

Here , N = Σf (sum total of frequencies)

$$\text{Formula, Mean (} \overline{X} \text{)} = A + \frac{\Sigma fx}{\Sigma f}$$

Example : Let the marks of students are .

Marks	5	10	12	14	16
No of students	2	1	3	3	1

Solution :

Marks (X)	No of students(*f*)	*d* = (X - A)	*fd*
5	2	−7	−14
10	1	−2	−2
⑫ – A	3	0	0
14	3	2	6
16	1	4	4
N = 10			Σ*f*d= - 6

Let , A = 12

$$\text{Mean } (\overline{X}) = A + \frac{\Sigma fd}{N}$$

$$= 12 + \frac{(-6)}{10}$$

$$= 12 - 0.6$$

$$= 11.4$$

∴ Arithmetic Mean of Marks obtained by students is 11.4

iii) Step deviation Method :- Here , the deviations (d) are divided by the common factor (c), which simplifies the calculation. Here $d' = \dfrac{d}{c} = \dfrac{X - A}{C}$ and $N = \Sigma f$

Formula , Mean $(\overline{X}) A = A + \dfrac{\Sigma fd'}{N} \times C$

Example :- Let the marks of students are :-

Marks	5	10	12	14	16
No of students	2	1	3	3	1

Solution :   Let , C=2

Marks(X)	No of students (f)	d = X - A	$d' = \dfrac{d}{c}$	fd'
5	2	–7	$\dfrac{-7}{2} = -3.5$	–
10	1	–2	$\dfrac{-2}{2} = -1$	–1
(12) – A	3	0	$\dfrac{-0}{2} = 0$	0
14	3	2	$\dfrac{2}{2} = 1$	3
16	1	4	$\dfrac{4}{2} = 2$	2
N = Σf = 10			Σfd = – 3	

Mean $(\overline{X}) = A + \dfrac{\Sigma \, fd'}{N} \times C$

$\qquad = 12 + \dfrac{(-3)}{10} \times 2$

$\qquad = 12 - \dfrac{6}{10}$

$\qquad = 12 - 0.6 \qquad = 11.4$

∴ Arithmetic Mean of Marks obtained by students is 11.4

C)   Arithmetic mean is case of continuous series :-

In case of continuous series , the value of a variable is grouped in various class interval, along with their respective frequencies . The process of calculating arithmetic mean in case of continuous series is same as discrete series . Here, the mid value (point) of the various class intervals are used to replace the class intervals .

i)   Direct Method :- Mid value or point of each class interval are to be find out by the formula

$\qquad$ Mid value (m) = $\dfrac{\text{Lower class limit + upper class limit}}{2}$

multiply each Mid value (m) by the corresponding frequency and find the total of the product (Σfm) and divide it by N .

Formula , Mean $(\overline{X}) = \dfrac{\Sigma\, fm}{N}$

Here, $N = \Sigma f$

Example : Let the marks of students are :

Marks	0 -10	10 -20	20 - 30	30 - 40
No of students	2	2	3	3

Solution :

Marks (X)	No of students (f)	Mid value (m)	fm
0 - 10	2	5	10
10 - 20	2	15	30
20 - 30	3	25	75
30 - 40	3	35	105
N = 10			$\Sigma fm = 220$

Arithmetic Mean $(\overline{X}) = \dfrac{\Sigma\, fm}{N}$

$= \dfrac{220}{10} = 22$

ii)   Assumed mean method :-

Here , first calculate the mid point (m) of each class - interval and find the assumed mean (A). For each class calculate the deviation (d) of mid - points from the assumed mean (d) = (m – A). Then multiply respective frequency with (d) and find Σfd .

Formula, Mean $(\overline{X}) = A + \dfrac{\Sigma\, fd}{N}$

Example :- Let the marks of students are.

Marks (x)	0-10	10 -20	20 - 30	30 - 40
No of students (f)	2	2	3	3

Solution :

Marks(X)	No of students	midle point	d = m – A	fd
0 - 10	2	5	–20	–40
10 - 20	2	15	–10	–20
20 - 30	3	㉕ –A	0	0
30 - 40	3	35	10	30
N = 10				Σfd = –30

$$\text{Mean (X)} = A + \frac{\Sigma \, fd}{N}$$

$$= 25 + \frac{(-30)}{10}$$

$$= 25 - 3 \quad = 22$$

the A.M of Marks obtained by students is 22 .

iii)   **Step deviation Method :-** In this method, deviation of mid point taken from assumed mean are divided by a common factor (c) to get step deviation.

$$\text{i.e} \qquad d' = \frac{d}{c} = \frac{m - A}{C} \left.\right\} \qquad A = \text{assumed mean}$$

$$m = \text{mid point}$$

$$c = \text{common factor}$$

$$\text{Formula}, \quad \text{Mean (} \overline{X} \text{)} = A + \frac{\Sigma \, fd'}{N} \times C$$

Example : Let the marks of students are .

Marks (X)	0 -10	10 -20	20 - 30	30 - 40
No of students (f)	2	2	3	3

Solution :-

Marks (X)	No of students (f)	Mid point (m)	d = m − A	$d' = \dfrac{d}{c}$	fd'
0 - 10	2	5	−10	− 5	−10
10 - 20	2	⑮− A	0	0	0
20 - 30	3	25	10	5	15
30 - 40	3	35	20	10	30
N = 10					Σ fd′ = 35

$$\text{Let } C = 2$$

$$A = 15$$

$$\text{Mean (} \overline{X}\text{)} = A + \frac{\Sigma \, fd'}{N} \times C$$

$$= 15 + \frac{35}{10} \times 2$$

$$= 15 + \frac{\cancel{70}}{\cancel{10}} \times 7$$

$$= 15 + 7 = 22$$

∴ Arithmetic Mean of Marks of the students is 22 .

* Weight Arithmetic Mean :- Weighted Mean refers to the average when different items of a series are given different weights according to their relative importance .

Let, W_1, W_2, W_3 .... Wn be the weights attached to variable values X_1, X_2, X_3 ......Xn

So weighted mean $(\overline{X}_w) = \dfrac{W_1 X_1 + W_2 X_2 + \ldots\ldots\ldots + W_n X_n}{W_1 + W_2 + \ldots\ldots\ldots + W_n}$

$$(\overline{X}_w) = \frac{\sum W_i X_i}{\sum W_i}$$

Calculate the weight mean of the following –

Items	10	12	20	17	22
Weight	5	9	5	6	7

Items (x)	Weights (w)	WX
10	5	50
12	9	108
20	5	100
17	6	102
22	7	154
	$\Sigma W = 32$	$\Sigma WX = 514$

$$\overline{X}_w = \frac{\sum WX}{\sum W} = \frac{554}{32} = 16.06$$

∴ Weight Mean = 16.06

2. Median :- The median is the 'middle' element when the data set is arranged in order of magnitude. i.e items are arranged either in ascending order or in descending order .

Median is, thus, called positional average, which divides the distribution into two equal parts.

i) **Calculation of Median in individual series :-**

Steps :- i) At first, arrange the data in ascending or descending order

ii) Apply the formula to know position of the median

$$\text{Position of Median} = \left(\frac{N+1}{2}\right) \text{th item} \quad \text{and} \quad \text{Me} = \text{size of} \left(\frac{N+1}{2}\right) \text{th item}$$

Me = Median

N = Number of items

Example :- Find the median value from the Marks of students 10, 11, 12, 15, 17, 16, 18

Solution :- Marks arrange in ascendings order 10, 11, 12, 15, 16, 17, 18.

Here, N = 7 (odd number)

$$\text{Me} = \text{Size of} \left(\frac{N+1}{2}\right) \text{th item}$$

$$= \text{size of} \left(\frac{7+1}{2}\right)$$

$$= \text{size of 4th item}, \quad \therefore \text{Median} = 15$$

** If N is even number, the median will be the average of two number.

Let Marks of students are 7, 8, 9, 10, 11, 13

Here N = 6 (even number)

$$\text{Me} = \text{size of} \left(\frac{N+1}{2}\right) \text{th item},$$

$$= \text{size of} \left(\frac{6+1}{2}\right) \text{th item},$$

$$= \text{size of 3.5th item}$$

3rd item   4th item

$$\text{Now, Median} = \frac{9+10}{2}$$

$$= \frac{19}{2} = 9.5$$

ii) Median in case of discrete series :-

Steps :- i) Arrange the data in a specific order (ascending order or descending order)

ii) Then calculate cummulative frequency (c.f)

iii) Find the median item using formula -

$$(Me) = \text{size of } \left(\frac{N+1}{2}\right) \text{th item}$$

$$Me = \text{Median}, \quad N = \text{Total frequency } (\Sigma f)$$

Find the corresponding value of c.f which is equal or next higher to it and then find the value corresponding to it . This value is the median .

Example :- Let the marks and frequency Number of students are

Marks (X) :-	10	12	14	16	17
No of students (f) :-	2	3	2	1	3

Solution :-

Marks(X)	No of student(f)	c.f
10	2	2
12	3	5
14	2	7
16	1	8
17	3	11
N = 11		

$$(Me) = \text{Size of } \left(\frac{N+1}{2}\right) \text{th item}$$

$$= \frac{(11+1)}{2} = \frac{12}{2} = 6 \text{ th .}$$

6th item come under the c f level 7.

So, Median = 14

iii) **Calculation of Median in case of continuous series :**

Steps :- i) Arrange the data in a specific order , i.e ascending or descending order .

ii) Calculate the cumulative frequency (c.f)

iii) Then find the median item (Median)= size of $\left(\dfrac{N}{2}\right)$ th item

Now find Median class where c.f is equal to (N/2) or c. f immediate greater than it.

iv) Apply the formula, (Median) $= l_1 + \dfrac{N/2 - f_1}{f} \times i$

l_1 = lower limit of Median class

i = difference between lower and upper limits of Median class .

f = frequency of Median class .

f_1 = c. f of the class preceding the median class .

Example :- find the median for the following data

Marks (X)	0 -10	10 -20	20 - 30	30 - 40
No of students (f)	2	3	3	2

Solution :

X	No of students	c.f	
0 - 10	2	2	
10 - 20	3	5	→Median class
20 - 30	3	8	
30 - 40	2	10	
N = 10			

Median = Size of (N / 2) th item

$$= \frac{N}{2} = \frac{10}{2} = 5th$$

$$Median = li + \frac{N/2 - f_1}{f} \times i$$

$$= 10 + \frac{5-2}{3} \times 10$$

$$= 10 + \frac{3}{3} \times 10$$

$$= 10 + 10 = 20$$

Median = 20

Quartile :- Quartiles are the measure which divide the data into four equal parts , each portion contains equal number of observation . There are three quartiles . First quartiles (Q_1) (Lower quartile) has 25 % of its items of the distribution . Second quartile (Q_2 or median) has 50 % of items below it. The third quartile (Q_3 or upper quartile) has 75% of items of the distribution . Second quartile is equal to Median. The value of Q_1 , Q_3 of and ordered series can be obtained by following formula–

$$Q_1 = \text{Size of} \left(\frac{N+1}{4}\right) \text{th items}$$

$$Q_3 = \text{Size of } 3 \left(\frac{N+1}{4}\right) \text{th items}$$

} Individual and Discrete series

$$Q_1 = \text{Size of} \left(\frac{N}{4}\right) \text{th items}$$

$$Q_3 = \text{Size of } 3 \left(\frac{N}{4}\right) \text{th item}$$

} Continuous series

Formula

$$Q_1 = l_1 + \frac{N/4 - cf_1}{f} \times i$$

$$Q_3 = l_1 + \frac{3(N/4) - cf_1}{f} \times i$$

} Continuous series

Decile :- Deciles divide whole distribution into ten equal parts . It is denoted by D_1 , D_2 ...... D_9 . There are Nine Deciles .

Percentiles :- It divides the whole distribution into hundred equal parts . It is denoted by P_1 , P_2 ...... P_{99} . P_{50} is the median value.

    Calculation of quartiles

i)   **In case of individual series :** From the following data calculate quartiles (Q_1 , Q_3)

    Marks of students → 62 , 38 , 45 , 43 , 50 , 55 , 57

Solution :- Here , N = 7 , / By ordering data , we get - 38 . 43 , 45 , 50 , 55 , 57 , 62

$$\therefore Q_1 = \text{Size of} \left(\frac{N+1}{4}\right) \text{th item}$$

$$= \text{Size of} \left(\frac{7+1}{4}\right) \text{th item}$$

$$= \text{Size of 2 th item}$$

} $\therefore Q_1 = 43$

$$Q_3 = \text{Size of } 3 \left(\frac{N+1}{4}\right) \text{th item}$$

$$= \text{Size of } 3 \left(\frac{7+1}{4}\right) \text{th item}$$

$$= \text{Size of } 6_{th} \text{ item .}$$

} $\therefore Q_3 = 57$

ii) **In case of discrete series–**

Complete Q_1 and Q_3

Marks	10	20	30	40	50	60	70
No of students	2	3	5	10	5	3	2

Solution : Here $N = \Sigma f = 30$

Marks(X)	f	c.f
10	2	2
20	3	5
30	5	10
40	10	20
50	5	25
60	3	28
70	2	30

$$Q_1 = \text{Size of} \left(\frac{N+1}{4} \right) \text{th item}$$

$$Q_1 = \text{Size of} \left(\frac{30+1}{4} \right) \text{th item}$$

$$= \text{Size of 7.7 th item}$$

$$\therefore \quad Q_1 = 30$$

$$Q_3 = \text{Size of } 3 \left(\frac{N+1}{4} \right) \text{th item}$$

$$= \text{Size of } 3 \left(\frac{30+1}{4} \right) \text{th item}$$

$$= \text{Size of 23.25 the item} \quad \therefore \quad Q_3 = 50$$

iii) **In case of continuous :-** Compute Q_1 and Q_3 from the following data

Marks	0 -5	5 -10	10 - 15	15-20	20-25	25-30	30-35	35-40
No of students	4	6	8	12	12	8	6	4

Solution :-

Marks	No of students (f)	C.f	
0 - 5	4	4	
5 - 10	6	10	
10 - 15	8	18	→ Q_1 class
15 - 20	12	30	
20 - 25	12	42	
25 - 30	8	50	→ Q_3 class
30 - 35	6	56	
35 - 40	4	60	
N = 60			

i) Q_1 = Size of (N/4) th item

= Size of (60/4) th item

= Size of 15th item

$\therefore$ $Q_1 = L_1 + \dfrac{(N/4) - cf_1}{f} \times i$

$= 10 + \dfrac{15 - 10}{8} \times 5$

$= 10 + \dfrac{5 \times 5}{8}$

$= 10 + \dfrac{25}{8}$

$= 10 + 3.12 = 13.12$

ii) Q_3 = Size of 3(N/4)th item

= Size of 3 (60/4) th item

= Size of 45th item

$\therefore$ $Q_3 = L_1 + \dfrac{3(N/4) - cf_1}{f} \times i$

$= 25 + \dfrac{45 - 42}{8} \times 5$

$= 25 + \dfrac{15}{8}$

$= 25 + 1.87 = 26.87$

Mode :- Mode is the most frequently observed data value of the series . It is denoted by Mo

i) **Calculation of Mode in case of individual series .**

Steps :- i) Convert the individual series into discrete series .

ii) Then on the basis of occurance of maximum number of items (frequency), Mode can be obtained

Example :- Let, the marks receive by the students are ,

5, 7, 2, 7, 6, 2, 3, 9, 9, 7, 2, 7 .

By converting individual series into discrete series we get

Marks	Frequency
2	3
3	1
5	1
6	1
7	4
9	2

Here Mode is 7 as it has maximum frequency .

ii) **Calculation of Mode in case of discrete series :-**

Calculation of Mode in discrete series is done on the basis of frequency numbers . Same as individual series.

iii) **Calculation of Mode in continuous series :-** In case of continuous series or frequency distribution, modal class is the class with highest frequency.

Mode is calculated by using the formula,

$$Mo = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

l_1 = Lower class limit of Modal class.

f_0 = frequency of the group preceding the Modal class.

f_1 = frequency of Modal class.

f_2 = frequency of the group succeeding the Modal class.

i = Class interval of the modal class.

Calculate the mode from following data .

Marks	0 -10	10 -20	20 - 30	30 - 40	40 - 50
No of students	2	4	9	3	2

Solution :-

Marks (X)	No of students (f)	
0 - 10	2	
10 - 20	4	$\rightarrow f_0$
20 - 30	⑨	$\rightarrow f_1$
30 - 40	3	$\rightarrow f_2$
40 -50	2	

Modal class → 20 - 30

$$M_0 = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$M_0 = 20 + \frac{9 - 4}{18 - 4 - 3} \times 10$$

$$= 20 + \frac{3}{11} \times 10$$

$$= 20 + \frac{30}{11}$$

$$= 20 + 2.72$$

$$= 22.72$$

Mode is 22 .72

(*) Merits and demerits of Arithmetic mean :-

Merits :-

    i)   It is simple to understand and easy to compute.

    ii)   It is based on all the values of variable.

    iii)   It is suitable for further algebric treatment.

    iv)   There is no arrangement requirment.

Demerits :-

    i)   It is affected by extreme values .

ii)	It can not be expressed through graph .

iii)	It can not be calculated if only one value is missing .

iv)	It can not be used in case of qualilative phenomenon .

**	**Merits and Demerits of Median :-**

Merits :-

i)	It is easy to calculate and easy to understand .

ii)	The value of median can also be determined graphically (ogive curve) .

iii)	It is not affected by extreme values .

iv)	It can be calculated in case of missing or incomplete data .

Demerits :-

i)	It is not suitable for further algebraic treatment .

ii)	In case of Median calculation, arranging of data is essential (ascending or descending order), which is time consuming .

iii)	It is affected by fluctuations of items .

**	**Merits and Demerits of Mode :-**

Merits :-

i)	It is easy to calculate.

ii)	It can be expressed graphically (histogram)

iii)	It is not affected by the extreme values .

iv)	It is useful for both quatitative and qualitative data .

Demerits :-

i)	It is not based on all observation of a variable .

ii)	Mode is not rigidly defined but sometimes ill- defined .

iii)	Mode is not capable of further Algebric treatment .

iv)	It is affected by fluctuation of sampling .

We have discussed the concepts of Arithmetic mean , median , and mode in brief . Different averages (due to their characteristics) are appropriate in different circumstances . Each of them has some strength and weakness . Arithmetic Mean and median are rigidly defined , whereas mode is not rigidly defined in all the situation .

Arithmetic Mean is based on all the observation but not median and mode . Mean is capable of further mathematical treatment properly.

Mean is affected by the extreme values but median and mode are not affected by it .

Exercise

1.1   State whether the following statements are True or False .

i)   Averages help in the formulation of economic policies .

ii)   There is least effect of extreme values on arithmetic mean .

iii)   Formula of Arithmetic mean is $\dfrac{EX}{N^2}$

iv)   Central tendency refers to a central value of a statistical series .

v)   The histogram is a measure of central tendency specially for Arithmetic mean .

vi)   The sum of deviation of items from median is zero . $\{\Sigma(X - M_d)= 0 \}$

vii)   The sum of deviation of item from mean is zero . $\{\Sigma (X - \overline{X})=0 \}$

viii)   Arithmetic mean is positional value .

ix)   Median is highly effected by extreme values .

x)   Mode of 3 , 4 , 4 , 5 , 5 , 2 , 1 , 4 number is 4 .

1.2   Multiple choice Questions :

i)   Median of these numbers 3 , 9 , 7 , 5 , 13 is–

  a) 3   b) 6   c) 7   d) 13 .

ii)   Median divides a series into how many parts ?

  a) Two   b) Three   c) Four   d) None of these .

iii)   For calculating median , all items of the series are arranged in –

  a) descending order   b) ascending order   c) Either (a) or (b)   d) None of these .

iv)   Formula of findings mean is

  a) $\overline{X} = \Sigma X$ b) $\overline{X} = \dfrac{\Sigma X}{N}$.   c) $\overline{X} = \Sigma X - N$ d) $\overline{X} = \dfrac{\Sigma X}{N - X}$

v)   Which of the following is not a measure of central tendency ?

  a) Mean   b) Mode   c) Standard deviation   d) Median

vi)   The value of all items are taken into consideration in the calculation of

  a) Median   b) Mode   c) Mean   d) None of these .

vii)   The algebric sum of deviations of 8 , 1 , 6 , 5 from the A . M viz 5 is–

  a) -1   b) 0   c) 1   d) None of these .

viii) Sum of square of the deviations about mean is–

a) Maximum   b) Minimum   c) Zero   d) None of these .

ix) _____ is not capable of algebric treatment

a) Arithmetic mean   b) Median   c) None   d) Both (a) and (b)

x) Arithmetic mean is of ----

a) two types   b) Eight types   c) Ten types   d) Seven types .

1.3 Fill in the blanks :-

i) _____ is the positional average .

ii) The median is the _____ of the series .

iii) $M_0 = l_1 + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$, Here 'l_i' means _____ class limit of the modal class .

iv) In case of discrete series the position of Median is _____ .

v) In case of continuous series the position of Median class is _____ .

1.4 Very short Questions :

1. Which arithmetic mean gives relative importance to each items ?

2. Give the meaning of Arithmetic mean.

3. What do you mean by measure of central tendency ?

4. What are the types of arithmetic mean ?

5. Give the formula of calculating arithmetic mean of a continuous series using short cut or assumed mean method .

6. Define Median .

7. Define Mode .

8. Which graph is used to locate median graphically ?

9. Which average divides the data series into two equal parts ?

10 . Calculate median :- 3, 4, 6, 9

2. Short answer type Question :   **(3/4 Marks)**

i) State two merits and demerits of Median .

ii) State two merits and demerits of Mode .

iii) State any three merits of Arithmetic mean .

iv) State any three demerits of Arithmetic mean .

v) Write an short note on weighted average .

3. **Anwer the following Questions** (6 Marks)

i) Discuss the objectives for measuring central tendency .

ii) What are the essential criteria of a good average ?

iii) Give a comparative idea of Mean , Median and Mode .

iv) Calculate Median and mean of the following data .

10 , 100 , 500 , 1000 , 1190

v) Find median and Mean from the following data .

10 , 12 , 17 , 16 , 14 , 11 , 15 , 17 .

vi) Calculate Mean , Median , and Mode from the following data .

33 , 20 , 35 , 50 , 35 , 52 , 35 , 39 .

vii) Find , Mean , Median and Mode from the following data .

X :	8	10	12	14	16	18	20
f:	2	8	12	28	11	9	5

viii) Calculate Mean , Median and Mode from the following data .

Marks	0 -10	10-20	20-30	30-40	40-50
No of students	3	13	18	12	5

ix) Compute mean marks from the data given below by–

i) Direct Method ii) Short cut method and iii) Step deviation method

Marks	5	15	25	35	45	55	65
Students	4	6	10	20	10	6	4

x) Find mean for the following data by using

i) Short cut Method ii) Step deviation method.

X	100 - 200	200 - 300	300 - 400	400 - 500	500 - 600
f	10	18	12	20	40

xi) Calculate the median and Mode from the following data .

Marks	0 - 10	10 -20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
No of students	2	18	30	45	35	20	6	3

xii) Calculate the value of Median and Mode from the following distribution .

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
frequency	4	15	10	7	3	1

Answer

1.1 Answer to the true or false questions :

i) True   ii) False   iii) False   iv) True   v) False   vi) False

vii) True   viii) False   ix) False   x) True

1.2 Answer to the MCQs

i) (c)   ii) (a)   iii) (c)   iv) (b)   v) (c)   vi) (c)

vii) (b)   viii) (b)   ix) (b)   x) (a)

1.3 Answer to the fill in the blanks :

i) Median   ii) Mid - point   iii) lower   iv) $\left(\dfrac{N+1}{2}\right)$ th item   v) $\left(\dfrac{N}{2}\right)$

1.4 Answer to the very short Questions :

1. Weighted Arithmetic mean .

2. Arithmetic mean is defined as the sum of the values of all observation divided by the number of observation .

3. Central tendency is the process to find a single value , which is used to represent an entire data .

4. There are two types of A . M .

5. $AM\,(\overline{X}) = A + \dfrac{\Sigma\,fd}{N}$, where ($d = m - A$)

6. The median is the 'middle' element when the data set is arranged in ascending order or descending order .

7. Mode is the most frequently observed data value of the series .

8. Ogive curve .

9. Median .

10 . Median is 5 .

Measures of Dispersion

In the earlier chapter we had learnt about measures of central tendency i,e , Mean , Median & Mode. But these averages are not sufficient to study or describe the characteristics of a statistical data . Let us analyse the wages of 5 workers in 3 factories given below :-

Factory A	Factory B	Factory C
4000	3500	500
4000	3800	750
4000	4000	4000
4000	4200	7250
4000	4500	7500

In all the 3 factories , the mean and median wages is same, i,e 4000. In factory 'A' there is no variation between mean wage & wages received by different labours. But in factory 'B' there is a small variation between average wage & actual wage of different labours. But in factory 'C' there is large gap between average wage & actual wages received by the laboures.

So it is necessary to define some additional summary measures to adequetly represent the charactertistics of a distribution . One such measure is known as "Measures of Dispersion".

6.1   Meaning of Dispersion :

Dispersion is the extent to which values in a distribution differ from the average of the distribution. It indicates lack of uniformity in the size of items. In other words, Dispersion is the degree to which numerical data tend to spread about an average value .

6.2   **Objectives related to the Measures of dispersion :**

Some specific objectives related to the measures of dispersion are -

a)   To know the variation of different values of the items from the average value of a series .

b)   To know about the composition of a series or the dispersal of values on either side of central tendency.

c)   To know the range of values <i.e the gap between the highest & lowest value> .

d)   To compare the disparity between two or more series in order to find out the degree of variation .

e)   To know whether the central tendency truly represents the series or not .

6.3 Absolute & Relative Measures of Dispersion :

When dispersion of the series is expressed in terms of the original unit of the series , it is called absolute measure of dispersion it is used when only one set of statistical distribution is under consideration . It can not be used when comparision is involved across two or more set of statistical series with different units of measurement < like 'rupees' in one case & 'kilogram' in another >. The methods of absolute measures of dispersion are–

a) Range .

b) Quartile deviation , inter quartile range .

c) Mean deviation .

d) Standard deviation .

e) Lorenz curve .

The relative measure of dispersion expresses the variability of data in terms of some relative value or percentage . It is used when one studies two or more series simultenously . Relative measures of dispersion is known as coefficient of dispersion.

The methods of relative measures of dispersion are –

a) Coefficient of range .

b) Coefficient of Quartile deviation .

c) Coefficient of mean deviation .

d) Coefficient of standard deviation .

e) Coefficient of variation .

6.3.1 Characteristics of a good Measures of Dispersion :

A measure of dispersion is the average of Second order.

Requirements for an ideal measure of dispersion are–

a) It should be based on all observation .

b) It should be rigidly defined .

c) It should be easy to calculate & easy to understand .

d) It is not unduly affected by the fluctuations of sampling and also by extreme observations.

e) It should be capable of further mathematical or algebraic treatment .

6.4 Range :

It is the simplest method of measuring dispersion of data . Range is the difference between the highest value and the lowest value in a series . So

Range = H - L   H = highest value in the series

L = lowest value in the series .

The relative measure of Range is coefficient of range . It is the ratio between the difference of highest & lowest value and the sum of highest & lowest value of the series .

So   Coefficient of Range $= \dfrac{H-L}{H+L}$

Solved Numerical example–

Find Range & Coeffiecient of range–

1.

Marks :	20	40	30	70	50

Answer : Range $= H - L = 70 - 20 = 50$

Coefficient of range $= \dfrac{H-L}{H+L} = \dfrac{70-20}{70+20} = \dfrac{50}{90} = 0.55$

2.

Marks :	10	20	30	40	50
Frequency :	7	10	12	8	3

Answer : Range $= H - L = 50 - 10 = 40$

Coefficient of Range $= \dfrac{H-L}{H+L} = \dfrac{50-10}{50+10} = \dfrac{40}{60} = 0.66$

3.

Class :	0 - 10	10 - 20	20 - 30	30 - 40
Frequency :	7	9	15	5

Answer : Range $= H - L = 40 - 0 = 40$

Coefficient of Range $= \dfrac{H-L}{H+L} = \dfrac{40-0}{40+0} = \dfrac{40}{40} = 1$

6.4.1 Merits & Demerits of Range :

Merits of Range :

a)   It is is simple to understand & easy to calculate .

b)   It provides a quick measure of variability .

c)   It provides the broad picture of data at a glance .

Demerits of Range :

a)   It is not based on all the observation . It is effected by extrem values.

b)   It is very much affected by fluctuations of sampling .

c)   It does not give any idea about the pattern of the distribution. (No use of central value)

d)   It can not be calculated in open - end distribution as there is no highest or lowest value

6.5 **Quartile Deviation** :- Quartile deviation is better than Range.

Inter Quartile Range : The difference between 3rd or upper Quartile (Q_3) and the 1st or lower Quartile (Q_1) is called interquartile range.

Inter Quartile Range = Q_3 - Q_1

Quartile deviation : Half of the difference between upper & lower Quartile is called Quartile deviation.

Quartile deviation (Q.D) = $\dfrac{Q_3 - Q_1}{2}$

Coefficient of Quartile deviation : The relative measure of Quartile deviation is called coeffiecient of Quartile deviation. Symbolically–

Coefficient of Quartile deviation = $\dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

6.5.1 Calculation of Quartile deviation :

(a)   Individual series.

*   Calculate InterQuartile range , Quartile deviation & coefficient of Quartile deviation from the following Table .

$$\boxed{200 , 160 , 210 , 208 , 220 , 250 , 300}$$

Solution : In ascending order -

$$\boxed{160 , 200 , 208 , 210 , 220 , 250 , 300}$$

Here N = 7

$\therefore$ 1st Quartile (Q_1) = Size of $\left(\dfrac{N+1}{4}\right)$ th item = Size of $\left(\dfrac{N+1}{4}\right)$ th item

= size of 2nd item = 200

$\therefore$ 3rd Quartile (Q_3) = Size of 3 $\left(\dfrac{N+1}{4}\right)$ th item = Size of 3 $\left(\dfrac{N+1}{4}\right)$ th item

= Size of 6th item = 250

Now,

Inter Quartile Range = Q_3 - Q_1 = 250 - 200 = 50

Quartile deviation = $\dfrac{Q_3 - Q_1}{2} = \dfrac{250 - 200}{2} = \dfrac{50}{2} = 25$

Coefficient of Quartile deviation = $\dfrac{Q_3 - Q_1}{Q_3 + Q_1} = \dfrac{250 - 200}{250 + 200} = \dfrac{50}{450} = 0.11$

b) Discrete Series :- Calculate Inter quartile range, Quartile deviation & Coefficient of quartile deviation from the following table –

Marks	2	4	6	8	10	12
Number of student	3	5	10	12	6	4

Solution :

Marks :	2	4	6	8	10	12
No of student :	3	5	10	12	6	4
Cumulative frequency (CF) :	3	8	18	30	36	40

$\therefore$ Q_1 = Size of $\left(\dfrac{N+1}{4}\right)^{th}$ item = Size of $\left(\dfrac{40+1}{4}\right)^{th}$ item

$$= \text{size of } 10.25^{th} \text{ item} = 6$$

$\therefore$ Q_3 = Size of $3\left(\dfrac{N+1}{4}\right)^{th}$ item = Size of $3\left(\dfrac{40+1}{4}\right)^{th}$ item

$$= \text{Size of } 30.75^{th} \text{ item} = 10$$

So, inter Quartile range = Q_3 - Q_1 = 10 -6 = 4

Quartile deviation $= \dfrac{Q_3 - Q_1}{2} = \dfrac{10-6}{2} = 2$

Coefficient of Quartile deviation $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1} = \dfrac{10-6}{10+6} = \dfrac{4}{16} = 0.25$

c) Continuous Series :- Calculate inter Quartile range , Quartile deviation & coefficient of Quartile deviation from the following data .

Size :	0-5	5-10	10-15	15-20	20-25	25-30
frequency :	3	9	15	23	30	20

Solution :

Size	frequency	Cumulative frequency (cf)
0 - 5	3	3
5 - 10	9	12
10 - 15	15	27
15 - 20	23	50
20 - 25	30	80
25 - 30	20	100
	N = 100	

$$Q_1 = \frac{N}{4} = \frac{100}{4} = 25^{\text{th}} \text{ item}$$

Here 25th item lies in the class (10 - 15) , which is the Q_1 class .

$$\therefore \quad Q_1 = l_1 + \frac{N\!/\!4 - CF}{F} \times h$$

$< $ here L_1 = lower boundary of Quartile class
F   = frequency of Quartile class
CF   = CF of pre- quartile class .
h   = width of Quartile class

$$= 10 + \frac{25 - 12}{15} \times 5$$

$$= 14.33$$

$$\therefore \quad Q_3 = 3\frac{N}{4}^{\text{th}} \text{ item} = \frac{3 \times 100}{4}^{\text{th}} \text{ item} = 75^{\text{th}} \text{ item}$$

Here 75th item lies in the class (20 - 25) which is Q_3 class .

$$\therefore \quad Q_3 = L_1 + \frac{N/4 - CF}{F} \times h$$

$$= 20 + \frac{75 - 50}{30} \times 5$$

$$= 24.17$$

$$\therefore \quad \text{Inter Quartile range} = Q_3 - Q_1 = 24.17 - 14.33 = 9.84$$

$$\therefore \quad \text{Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{24.17 - 14.33}{2} = \frac{9.84}{2} = 4.92$$

$$\text{Coefficient of Quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{24.17 - 14.33}{24.17 + 14.33}$$

$$= \frac{9.84}{38.50} = 0.25$$

6.5   Merits & Demerits of Quartile deviation :

Merits of Quartile deviation :

a)   It is quite easy to understand & Calculate .

b)   It is the only measure of dispersion which can be used to deal with a diistribution having open-end classes.

c)   It is less affected by extreme values as compared to range.

d)   It is suitable for skewed distribution of variaties.

Demetrits of Quartile deviation

a) It is not based on all the observations , and so it can not be regarded as a reliable measure of dispersion .

b) It is not capable of further algebric treatment .

c) It is considerably affected by the fluctuation in the sample . Change in the value of an item , may affect its value considerably.

d) It gives approximate idea of dispersion.

6.6 Mean Deviation

Range & Quartile deviation suffer from a common defect i.e . They are calculated by taking into account only either extreme values < Range> of a series or the value of the quartiles < Quartile deviation>. So it is better to have such a measure of dispersion which takes into account all the observations of a series and is calculated in relation to a central value .

Mean deviation is such a measure of dispersion .

6.6.1 Mean deviation : Mean deviation of a series is the arithmetic average of the deviations of various items from a measure of central tendency <Mean , Median & Mode >

Theoretically Mean deviation can be calculated by taking deviations from any of Mean , median & Mode . But mode is usually not considered as its value is indeterminate & it gives erroneous conclusions . And out of mean & median , the more appropriate one is median , because the sum of deviation form median is less than the sum of deviations form Mean . While calculating deviations , all the deviations are taken as positive , the signs (+ or –) are ignored .

For individual series ,

Mean Deviation from Mean $(MD_{\overline{X}}) = \dfrac{\Sigma |D|}{N}$ Where , $|D| = |X - \overline{X}|$

$$\overline{X} = \text{Mean}$$

Mean Deviation from Median $(MD_{Me}) = \dfrac{\Sigma |D|}{N}$ Where $|D| = |\overline{X} - \text{Median}|$

For Discrete series,

Mean Deviation from Mean $(MD_{\overline{X}}) = \dfrac{\Sigma f |D|}{N}$ $\{|D| = |X - \overline{X}|, \overline{X} = \text{Mean}\}$

$$f = \text{frequency}$$

Mean Deviation from Median $(MD_{Me}) = \dfrac{\Sigma f |D|}{N}$ $\{|D| = |X - \text{Median}|$

$$f = \text{frequency}$$

For Contineous series ,

Mean deviation from Mean ($MD_{\bar{x}}$) = $\dfrac{\sum f\ |D|}{N}$

[Where , $|D| = |M - \bar{X}|$, M = Midpoint , $\bar{X}$ = Mean]

Mean deviation from Median (MD_{Me}) = $\dfrac{\sum f\ |D|}{N}$

[Where , $|D| = |M - Me|$, M = Mid point, Me = Median]

6.2 Coefficient of Mean deviation :   The relative measure of Mean deviation is called coeffient of Mean deviation . It can be measured from both the mean & Median .

Coefficient of MD from Mean = $\dfrac{MD_{\bar{x}}}{\bar{X}}$ [$\bar{X}$ = Mean]

Coefficient of MD from Median = $\dfrac{MD_{Me}}{Me}$ [Me = Median]

6.6.3 Calculation of Mean deviation & its coefficient

a)   Individual Series

Calculate MD from Mean & Median and its coefficient from the following data

$$\boxed{5 , 8 , 11 , 12 , 14}$$

i)   MD from Mean & Coefficient of MD $\bar{X}$:

Value (X)	$|D| = |X - \bar{X}|$
5	5
8	2
11	1
12	2
14	4
$\sum X = 50$	$|\sum| D = 14$

∴   Mean ($\bar{X}$) = $\dfrac{\sum X}{N}$ = $\dfrac{50}{5}$ = 10

∴ $MD_{\bar{X}}$ = $\dfrac{\sum |D|}{N}$ = $\dfrac{14}{5}$ = 2.8

$$\therefore \quad \text{Coefficient of MD}_{\overline{X}} = \frac{MD_{\overline{x}}}{\overline{X}}$$

$$= \frac{2.8}{10} = 0.28$$

ii) MD from Median & Coefficient of MD_{Me} :

Value (X)	$\mid D \mid = \mid X \mid - Me$
5	6
8	3
11	0
12	1
14	3
N = 5	$\Sigma \mid D \mid = 13$

$$\text{Median (Me)} = \text{Size of } \left(\frac{N+1}{2}\right)^{th} \text{item}$$

$$= \text{Size of } \frac{5+1}{2}^{th} \text{item}$$

$$= \text{Size of 3rd item .}$$

$$= 11$$

$$\therefore \quad MD_{Me} = \frac{\Sigma \mid D\mid}{N} = \frac{13}{5} = 2.6$$

$$\therefore \quad \text{Coefficient of MD}_{Me} = \frac{MD_{Me}}{Median} = \frac{2.6}{11} = 0.23$$

b) Discrete Series : Calculate Mean deviation from Mean & Median and its Coefficient from the following table :

Values (X)	10	11	12	13
Frequency (f)	3	12	18	12

Solution :

i) Mean deviation from Mean :

Values (X)	Frequency (f)	(fX)	$\lvert D\rvert=\lvert X-\bar{X}\rvert$ $=\lvert X-11.87\rvert$	$f\,\lvert D\rvert$
10	3	30	1.87	5.61
11	12	132	0.87	10.44
12	18	216	0.13	2.34
13	12	156	1.13	13.56
	$N=\Sigma f=45$	$\Sigma f x=534$		$\Sigma f D=31.95$

$$\therefore \quad \text{Mean } (\bar{X}) = \frac{\Sigma\, fx}{N} = \frac{534}{45} = 11.87$$

$$\therefore \quad MD_{\bar{X}} = \frac{\Sigma f\,\lvert D\rvert}{N} = \frac{31.95}{45} = 0.71$$

$$\therefore \quad \text{Coeffiecient of } MD_{\bar{X}} = \frac{MD_{\bar{X}}}{Mean} = \frac{0.71}{11.87} = 0.056$$

ii) Mean deviation from Median :

Values (X)	Frequency (f)	Cumulative	$\lvert D\rvert = \lvert X - Me\rvert$	$f\,\lvert D\rvert=$ $\lvert X-12\rvert$
10	3	3	2	6
11	12	15	1	12
12	18	33	0	0
13	12	45	1	12
$N=\Sigma f=45$				$\Sigma f\,\lvert D\rvert=30$

$$\text{Median} = \text{Size of } \left(\frac{N+1}{2}\right)^{h} \text{item}$$

$$= \text{Size of } \frac{45+1}{2}^{th} \text{item}$$

$$= \text{Size of 23rd item} = 12$$

$\therefore$ Mean Deviation from Median (MD_{Me}) = $\dfrac{\Sigma f \ |D|}{N}$ = $\dfrac{30}{45}$ = 0.67

$\therefore$ Coeffiecient of MD_{Me} = $\dfrac{MD_{Me}}{Median}$ = $\dfrac{0.67}{12}$ = 0.05

c) Contineous Series : Calculate MD from Mean & Median and its Coeffiecient from the following table –

Class	2 - 4	4 - 6	6 - 8	8 - 10
frequency	3	4	2	1

Solution :

i) MD from Mean :

Class	Frequency (f)	Mid point (X)	fx	$\|D\| = \|X - \overline{X}\|$ $= \|X - 5.2\|$	$f \|D\|$
2 - 4	3	3	9	2.2	6.6
4 - 6	4	5	20	0.2	0.8
6 - 8	2	7	14	1.8	3.6
8 - 10	1	9	9	3.8	3.8
	$\Sigma fN = N = 10$		$\Sigma fX = 52$		$\Sigma f\|D\| = 14.8$

$$\text{Mean} \ (\overline{X}) = \dfrac{\Sigma X}{N} = \dfrac{52}{10} = 5.2$$

$$\therefore \ MD_{\overline{X}} = \dfrac{\Sigma f \ |D|}{N} = \dfrac{14.8}{10} = 14.8$$

$$\therefore \ \text{Coefficient of } MD_{\overline{X}} = \dfrac{MD_{\overline{x}}}{\overline{X}} = \dfrac{1.48}{5.2} = 0.28$$

ii) MD from Median :

Class	Frequency (f)	Cumulative frequency(cf)	Mid Point(X)	$\|D\| = \|X - Me\|$	$f\|D\|$
2 - 4	3	3	3	2	6
4 - 6	4	7	5	0	0
6 - 8	2	9	7	2	4
8 - 10	1	10	9	4	4
	N = 10				$\Sigma f\|D\| = 14$

$$\text{Median} = \frac{N}{2}^{\text{th}} \text{ item} = \frac{10}{2} \text{ th item} = 5^{\text{th}} \text{ item}$$

5th item lies in (4 - 6) class , so (4 - 6) is the median class .

$$\therefore \quad \text{Median (Me)} = L_1 + \frac{N/2 - CF}{F} \times h$$

$$= 4 + \frac{5-3}{4} \times 2 \quad = 5$$

$$\therefore \quad MD_{Me} = \frac{\Sigma f \,|D|}{N} = \frac{14}{10} = 1.4$$

$$\therefore \quad \text{Coefficient of MD}_{Me} = \frac{MD_{Me}}{Median} = \frac{1.4}{5} = 0.28$$

6.6.4 Merits & Demerits of Mean deviation :

Merits of Mean deviation are–

    a)   It is simple to calculate & easy to understand .

    b)   It is less effected by extreme observations .

    c)   It is rigidly defined and its value is precise & definite .

    d)   It is based on all observation of the series .

    e)   It provides a better means for comparison about formation of different distribution .

Demerits of Mean deviations are–

    a)   It is not capable of algebric treatment .

    b)   Mathematically it is not logical, if ignors negative sign.

    c)   It is not suitable for open end series .

    d)   It is less reliable when calculated from Mode because in many cases mode has no fixed value.

6.7 Standard Deviation

    Standard deviation is the most satisfactory scientific method of dispersion and so it is a widely used method in statistical analysis . Standard Deviation is the square root of the arithmetic average of the squares of the deviations measured from the mean . Symbolically it is denoted by the Greek letter $<\sigma$ sigma $>$.

    The relative measure of standard deviation $<$ SD $>$ is called coefficient of standard deviation .

$$\text{Coefficient of standard deviation} = \frac{SD}{Mean(\bar{X})}$$

$$\text{Coefficient of variation} = \frac{SD}{\bar{X}} \times 100$$

Standard deviation can be measured in the following four ways–

    a)   Actual Mean Method

    b)   Direct Method

    c)   Short cut Method or Assumed mean Method

    d)   Step division Method .

For different series of data & in different Methods of measuring SD , different formula are used .

6.7.1 SD in Individual series :-

Question : Calculate SD from the following data .

Marks :   5 , 8 , 7 , 11 , 14

Solution :-

a)   Actual Mean Method

Marks	$Y = X - \overline{X}$	Y^2
5	– 4	16
8	–1	1
7	–2	4
11	2	4
14	5	25
$\Sigma x = 45$		$\Sigma y^2 = 50$

$$\therefore \quad \overline{X} = \frac{\Sigma x}{N} = \frac{45}{5} = 9$$

$\therefore$ Standard deviation (σ)

$$= \sqrt{\frac{\Sigma y^2}{N}} = \sqrt{\frac{50}{5}} = \sqrt{10} = 3.16$$

$$\therefore \quad \text{Coefficient of SD} = \frac{SD}{\overline{X}} = \frac{3.46}{9} = 0.45$$

b)   Direct Method

Marks (X)	X^2
5	25
8	64
7	49
11	121
14	196
$\Sigma x = 45$	$\Sigma x^2 = 455$

$\therefore$ Standard deviation (σ)

$$= \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

$$= \sqrt{\frac{455}{5} - \left(\frac{45}{5}\right)^2}$$

$$= \sqrt{91 - 81} = \sqrt{10} = 3.16$$

c)   Short cut Method :

Marks	Y = X– A	Y²
5	– 2	4
8	1	1
7 (A)	0	1
11	4	16
14	7	49
N = 5	$\Sigma y = 10$	$\Sigma y^2 = 70$

$$\therefore \quad SD(\sigma) = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2} = \sqrt{\frac{70}{5} - \left(\frac{10}{5}\right)^2} = \sqrt{14 - 4} = \sqrt{10} = 3.16$$

6.7.2 SD in Discrete Series :

Calculate SD from the following table :

Size :	5	10	15	20
Frequency :	2	1	4	3

Solution :

a)   Actual Mean Method

Size(X)	frequency(f)	fx	y = X– $\overline{X}$	y²	fy²
5	2	10	– 9	81	162
10	1	10	4	16	16
15	4	60	1	1	4
20	3	60	6	36	108
	$\Sigma f = N = 10$	$\Sigma fx = 140$			$\Sigma fy^2 = 290$

$$\therefore \quad \overline{X} = \frac{\Sigma fx}{N} = \frac{140}{10} = 14$$

$$\therefore \quad SD = \sqrt{\frac{\Sigma y^2}{N}} = \sqrt{\frac{290}{10}} = \sqrt{29} = 5.36$$

b) Direct Method :

Size	frequency(f)	fx	x^2	fx^2
5	2	10	25	50
10	1	10	100	100
15	4	60	225	900
20	3	60	400	1200
	N = 10	$\Sigma fx = 140$		$\Sigma fx^2 = 2250$

$$\therefore SD\ (\sigma) = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{2250}{10} - \left(\frac{140}{10}\right)^2}$$

$$= \sqrt{225 - 14^2} = \sqrt{225 - 196}$$

$$= \sqrt{29} = 5.38$$

c) Short cut Method :

Size	frequency (f)	y = X– A	fy	fy^2
5	2	−5	−10	50
10 (A)	1	0	0	0
15	4	5	20	100
20	3	10	30	300
	N = 10		$\Sigma fy = 40$	$\Sigma fy^2 = 450$

$$\therefore \quad SD\ (\sigma) = \sqrt{\frac{\Sigma fy^2}{N} - \left(\frac{\Sigma fy}{N}\right)^2} = \sqrt{\frac{450}{10} - \left(\frac{40}{10}\right)^2} = \sqrt{45 - 16} = \sqrt{29} = 5.38$$

6.7.3 SD in continuous Series :

Calculate SD from the following table -

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	4	3	6	5	2

Solution :

a) Actual Mean Method :

Class	frequency (f)	Mid point (x)	fx	$y = X - \overline{X}$	y^2	fy^2
0 - 10	4	5	20	– 19	361	1444
10 - 20	3	15	45	– 9	81	243
20 - 30	6	25	150	1	1	6
30 - 40	5	35	175	11	121	605
40 - 50	2	45	90	21	441	882
	$\Sigma f = N = 20$		$\Sigma fx = 480$			$\Sigma fy^2 = 3180$

$$\therefore \quad \text{Mean } (\overline{X}) \frac{\Sigma fx}{N} = \frac{480}{20} = 24$$

$$\therefore \quad \text{SD } (\sigma) = \sqrt{\frac{\Sigma fy^2}{N}} = \sqrt{\frac{3180}{20}} = \sqrt{159} = 12.61$$

b) Direct Method :

Class	Frequency (f)	Mid point (x)	fx	$fx^2 \ (fx.x)$
0 - 10	4	5	20	100
10 - 20	3	15	45	645
20 - 30	6	25	150	3750
30 - 40	5	35	175	6125
40 - 50	2	45	90	4050
	N = 20		$\Sigma fx = 480$	$\Sigma fx^2 = 14700$

$$\therefore \quad \text{SD} = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2}$$

$$= \sqrt{\frac{14700}{20} - \left(\frac{480}{20}\right)^2} = \sqrt{735 - 576} = \sqrt{159} = 12.61$$

c)   Step-devision Method .

Class	Frequency (f)	Mid point (x)	$y = \dfrac{X - A}{d\,(1)}$	fy	fy^2
0 - 10	4	5	–2	–8	16
10 - 20	3	15	–1	–3	3
20 - 30	6	25(A)	0	0	0
30 - 40	5	35	1	5	5
40 - 50	2	45	2	4	8
	N=20		$\Sigma fy = (-)\,2$		$\Sigma fy^2 = 32$

$$\therefore\ SD = \sqrt{\frac{\Sigma fy^2}{N} - \left(\frac{\Sigma fy}{N}\right)^2} \times d$$

[here , d is the common difference = 10
A = Assumed Mean = 25]

$$= \sqrt{\frac{32}{20} - \left(\frac{-2}{20}\right)^2} \times 10$$

$$= \sqrt{1.6 - 0.01} \times 10 = \sqrt{1.59} \times 10 = 12.61$$

6.7.4 Varience :

Varience is another measure based on standard deviation. It means the square of the standard deviation. Symbolically–

Variance = (SD)^2 = σ^2

i , e   SD = $\sqrt{Variance}$

SD & Variance are measures of variability and they are closely related . Smaller the value of varience, lesser is variabillity or greater the consistency and vice-versa.

6.7.5 Properties of SD :

a)   SD is independent of change in origin and SD is affected by change in scale.

b)   If all values of a variable are same then SD = 0.

c)   SD can be compute from two or more groups having same unit– which is called combined. SD

d)   For a given set of observations, SD is never less than mean deviation from mean, i.e.

SD > MD$_{\overline{X}}$.

6.7.6 Merits & Demerits of SD :

Merits of SD are–

a)   It is based on all values of the series .

b)   It is widely used measures of Dispersion as it is definite measures of dispersion.

c)   It is less effected by fluctation of sampling.

d)   It is capable of further algebric treatment.

Demerits of SD are –

a)   It is more difficult to measure as compared to other measures of dispersion.

b)   It gives more weightage to extreme values & less to those which are nearer to mean.

c)   It can not be used to compare the dispersion of the distributions expressed in different units.

6.8   Lonenz Curve

It is the graphical method of measuring dispersion . This curve was used by Dr, Max o. Lorenz <a famous economists statistician > to measure the inequalities of income & wealth of a society . But now, it is also used to study the distribution of profit, wages, turnover etc.

Lorenz Curve is the measure of deviation of actual distribution from the line of equal distribution . This is cumulative percentage graph . Greater the distance of Lorenz curve from the line of equal distribu- tion more is the inequality or variability in its series and vice- versa .

6.8.1 Steps involved in Drawing a Lorenz curve :

Step 1   :   Calculate cumulative values of size of items < in case of discrete series > and mid-points (in case of continuous series)

Step 2   :   Calcualte percentage for these cumulative values . For this last cumulative total is considered as equal to 100 and then percentages are obtained .

Step 3   :   Determine cumulative frequencies

Step 4   :   Calculate percentage for each cumulative frequency . For this the last cumulative total is considered as equal to 100 and then percentages are obtained .

Step 5   :   On the X - axis , start from 0 to 100 and take , the percentage of cumulative frequencies .

Step 6   :   On the y - axis , start from 0 to 100 and take percentage of variable .

Step 7   :   Draw a diagram line joining 0 to 100. This line is known as " Equality Line" or line of equal distribution.

Step 8   :   Plot the various points corresponding to the values of the varibales X and Y and then join this points with a smooth free hand curve. The curve is obtained shows the actual distribution. This curve is known as Lorenz curve .

6.8.1 Numerical Example :

Draw a lorenz curve from the following table :

Income :	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No of person :	4	5	7	5	4

Solution :

Income	Mid value	Cumulative Mid value	% Cumulative Mid value	No of Person	Cumulative% frequency	Cumulative frequency
0 - 10	5	5	4	4	4	16
10 - 20	15	20	16	5	9	36
20 - 30	25	45	36	7	16	64
30 - 40	35	80	64	5	21	84
40 - 50	45	125	100	4	25	100

Figure shows : The curve drawn is farther from the line of equal distribution. So there is inequality in Income distribution.
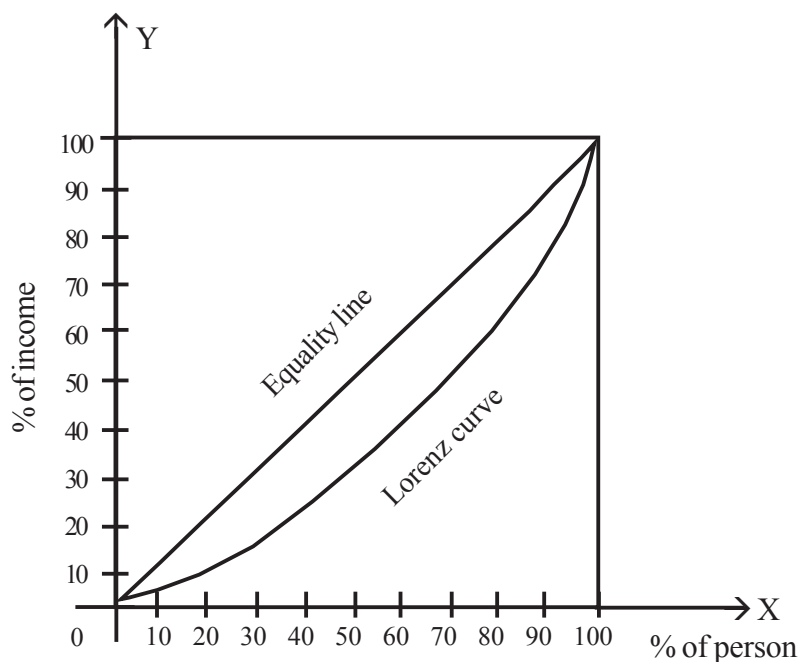
6.8.2 Merits & Demerits of Lorenz curve :

Merits of Lorenz curve are–

    a)   It is attractive & provide a rough idea of extent of dispersion.

    b)   It helps to compare two or more series.

Demerits of Lorenz curve are–

    a)   It does not provide an numerical value of the variability for the given distribution.

    b)   The method of drawing the curve is so difficult.

EXERCISE

Multiple choice Type of Question (1 Mark)

1.   Which is difficult to compute
   a) Relative Measure of dispersion   b) absolute measure of dispersion
   c) Both (a) & (b)   d) Range .

2.   The most commonly used measure of dispersion is
   a) Coeffiecient of variation   b) Standard deviation .
   c) Range   d) Quartile deviation

3.   Which one of the following is not a measure of dispersion ?
   a)   Variance   b) Mean deviation   c) standard deviation   d) Mode .

4.   Which one is the relative measure of dispersion ?
   a) Variance   b) Coefficient of variation   c) Mean deviation   d) all of the above .

5.   The best measure of dispersion to compare two different series–
   a) Mean deviation   b) Range   c) Standard deviation   d) Coefficient of variation

6.   The minimum value in a set is 9 and the range in 57. Find the maximum value–
   a) 35   b) 66   c) 48   d) None of these

7.   The measure of dispersion has a different unit other than the unit of measurement of values -
   a) Mean deviation   b) Range   c) Standard deviation   d) Variance .

8.   Which measure of dispersion is based on the absolute deviations only ?
   a) SD   b) Mean deviation   c) Range   d) Quartile deviation

9.   Which one is an absolute mesure of dispersion?
   a) Standard deviation   b) Mean deviation   c) Range   d) all of these.

10. Coefficient of variation is –
   a) absolute measure   b) Relative measure   c) both (a) & (b)   d) None of the these .

11. The appropriate measure of dispersion for open - end classificaton is–
   a) Mean deviation   b) Standard deviation   c) Quartile deviationi d) None of these.

12. The $Q_1 = 104$, Quartile deviation $= 8$, then $Q_3 = ?$
   a) 130   b) 120   c) 136   d) 146

Very short answer type Question :   **(1 Mark)**

1.   Define dispersion .

2.   What is absolute measure of dispersion ?

3.   What is relative measure of dispersion ?

4.   Define range .

5.   Which graphical method is used to measure dispersion ?

6.   What is varience .

7.   What is standard deviation ?

8.   Write the formula of inter quartile range .

9.   Which measure of dispersion covers middle 50 % of items ?

10.   What are the two mesures of dispersion ?

Short Answer type Question :   **(3/4 Marks)**

1.   Why is there a need for measure of dispersion ?

2.   State the characteristics of good measures of dispersion ?

3.   Write the merits & demerits of standard deviation .

4.   Write the steps involved to construct Lorenz curve .

5.   Write the merits & demerits of Mean deviation .

6.   Write the properties of standard deviation .

7.   What is the meaning of relative & absolute measures of dispersion ? Give example .

8.   Write the merits & demerits of Lorenz curve .

Numerical Question   (3/4 marks)

1.   Find range & its coefficient from the following :

a)
22 , 35 , 32 , 45 , 42 , 48 , 39

$<$ Ans - R $= 26$, coefficient Range$= 0.37>$

b)
Size :	10	20	30	40	50	60	70
Frequency :	8	12	7	30	10	5	2

$<$ Ans ; 60 , 0.75$>$

c)
Marks :	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency :	12	18	14	63	19

$<$ Ans : 50 , 0.71$>$

d)
Marks :	5 - 9	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34
Frequency :	4	6	3	2	6	4

$<$ Ans : 30 , 0.76 $>$

2.   Find Quartile deviation coeffiecient of Quartile deviation from the following :

a)   35 , 60 , 70 , 90 , 110 , 90 , 120 , 130 , 170 , 155 , 145 , 145

$<$ Ans : 35 , 0. 31$>$

b)

Marks	:	10	20	30	40	50	60
Frequency :		4	7	15	8	7	2

<Ans : 10 , 0.33>

c)

Height	:	153	155	157	159	161	163	165	167	169
No of student :		25	21	28	20	18	24	22	18	23

<Ans : 4 , 0.025>

d)

Classs	:	11 - 15	16 - 20	21 - 25	26 - 30	31 - 35	36 - 40	41 - 45
Frequency :		10	17	22	31	42	32	28

46 - 50	51 - 55
19	14

<Ans : 7.7 , 0.22>

e)

Class	:	Below 20	20 - 30	30 - 40	above 40
Frequency :		7	10	14	9

<Ans : 8.14 , 0.26 >

f)

Classs	:	5-10	10-15	15-20	20-25	25-30	30- 35	35-40	40-45	45-50
Frequency :		6	10	18	30	15	12	10	6	4

<Ans : 6.73 , 0.27>

g)

Size	:	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45
Frequency :		2	10	26	16	7

<Ans : 3.4 , 0.1>

3 . Find Mean deviation from mean & coefficient of Mean deviation

a)   210 , 220 , 225 , 225 , 225 , 235 , 235 , 240 , 250 , 270 , 280 .

<Ans : 17.6 , 0.07>

b)

Marks	:	5	10	15	20	25	30	35	40
frequency :		16	32	36	44	28	18	12	14

<Ans : 7.65 , 0.38 >

c)

Class	:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency :		3	5	7	2	9	4

<Ans : 14.3 , 0.44>

d)

Class	:	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30
Frequency :		4	7	8	2	6	3

<Ans : 6.33 , 0.506 >

e)

Class :	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency :	10	16	30	32	12

4. Find Mean deviation from Median & its Coefficient .

a) | 90 , 160 , 140 , 200 , 100 , 150 , 80 | < Ans : 34.28 , 0.24>

b)

Marks :	10	11	12	13	14
Frequency :	3	12	18	12	3

< Ans : 0.75>

c)

Class :	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency :	10	16	30	32	12

< Ans : 19.12 , 0. 34>

d)

Class :	140-150	150-160	160-170	170-180	180-190	190-200
Frequency :	4	6	10	18	9	3

< Ans : 10 .24 >

e)

Income :	80	100	150	180	200	250
No of person :	16	24	26	30	20	6

< Ans : 39.51 , 0.26 >

5. Find SD & coefficient of variation from the following -

a)

Variable :	10	20	30	40	50	60	70
frequency :	6	8	16	15	32	11	12

< Ans : 16.43 , 37 .34>

b)   Find SD :

Expenditure :	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
No of person :	6	10	12	10	8

< Ans : 6.41>

c)   Find SD :

Size :	0 - 10	10 - 20	20 - 30	30 - 40
frequency :	2	3	4	1

< Ans : σ = 9.16 >

d)   Find varience & coefficient of variation :

Value :	2	6	10	14
frequency :	4	8	2	1

< Ans : 10.66 , 54.4 % >

Page - 214

e) Find SD & coefficient of variation .

Class	:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	:	5	15	20	25	18	10	7

<Ans : 15.67 , 45.55 % >

f) Find SD.

Class	:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	:	2	4	6	8	6	4	2

<Ans : 15.81 >

g) Find SD & coefficient of variation .

Class	:	0-4	4-8	8-12	12-16	16-20	20-24
Frequency	:	10	15	20	25	20	10

<Ans : σ = 5.85 , cv = 47.19%>

h) Find SD : $\boxed{8 , 9 , 15 , 23 , 5 , 11 , 19 , 8 . 10 , 12}$ <Ans : σ = 5.23>

i) Find SD : $\boxed{3 , 5 , 7 , 10 , 12 , 15 , 18}$ <Ans : σ = 4.87 >

Answer

Multiple Choice Question :

1. a) Relative measure of dispersion .
2. b) Standard deviation .
3. d) Mode .
4. b) Coefficient of variation
5. d) Coefficient of deviation
6. b) 66
7. d) Varience
8. b) Mean deviation .
9. d) all of these .
10. c) both a & b
11. c) Quartile deviation .
12. b) 120 .

Very Short Answer type question :

1. Dispersion indicates the extent to which the individual values fall away from the central value.

2. The measures of dispersion which are expressed interms of the original units of a series are termed as absolute measures.

3. When the dispersion is measured as a percentage or ratio of the average , it is called relative measures of dispersion.

4. It is the difference between highest & lowest item in a distribution .

5. Lorenz curve.

6. Varience is the square of standard deviation.

7. SD is the square root of the arithmetic average of the deviations of various items from a measure of central tendency.

8. $Q_3 - Q_1$.

9. Quartile deviation .

10 . Absolute & relative measure .

Correlation

We have discussed so far the problems relating to one variable , but in reality there are a large number of events where two variables are interlinked . In this chapter we will learn how to examine the relationship between two variables . In statistics , correlation is such a analysis which deals with the association between two or more variables .

7.1 **Correlation** – Correlation studies and measures the direction and intensity of relationship among variables . It measures covariance , not the causation . It does not seek to explain the cause and effect relation .

If there is correlation between two variables (between X and Y) simply means that the change of the value of one variable in one direction casues the change of the value of another variable either in the same direction or in the opposite direction .

7.1.1 Significance of correlation studies

Significant areas :-

i)   It studies the relationship between variables. It shows the degree and direction of relationship of variables .

ii)   It helps us to understand economic behaviour . In Economics a large number of economic variables are interlinked . Say , price and demand , price and supply , Income and expenditure etc . Correlation helps us to understand their relationship .

iii)   It reduces the range of uncertainity . Forecasting without any prior correlation analysis may prove to be defective , less reliable and more uncertain .

7.1.2 Types of correlation

i)   Positive and Negative correlation :-The correlation is said to be positive when the variable s move together in the same direction . e.g sale of ice cream and temperature move in same direction

In case of negative or inverse correlation , the variables move in the opposite direction , e.g increasing price of a product and its demand for the repective prices .

ii)   Linear and Non - linear correlation :- In case of linear correlation , the ratio of change between two variables is uniform whether they increase or decrease . If the changes are plotted on a graph paper they will form a straight line . This is also called simple correlation .

In case of non- linear or curvilinear correlation the ratio of change between two variable is not uni-

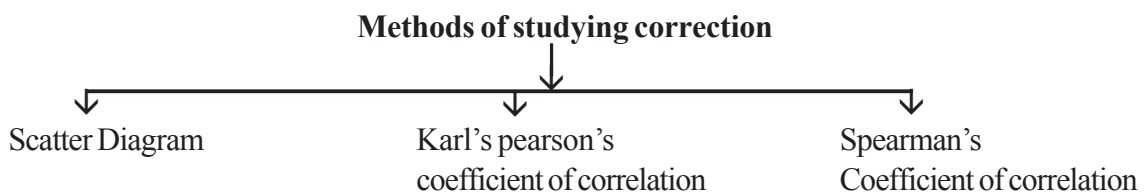form. If the changes are plotted on a graph paper they will form a curve line .

iii)   Simple and multiple correlation :- In case of only two variables are involved in correlation it is called simple correlation . When we study more than two variables , its called multiple correlation

iv)   Perfect positive and perfect negative correlation :-If the changes in the two variable are exactly proportional the relation is called perfect correlation . When the equal proportional change between two variables moves in same direction , it is perfect positive correlation . When it moves in opposite direction , it is called perfect negative correlation .

v)   No correlation :- In case there is no interdepencdence between the two variables, there is absence of correlation.

7.1.3 Degrees of correlation

Description	Positive (+)	Negative (−)
Perfect correlation	+ 1	−1
High correlation	Between 0.75 and 1	Between −0.75 and −1
Moderate correlation	Between 0.5 and 0.75	Between −0.5 and −0.75
Low Correlation	Less than 0.5	Less than −0.5
No Correlation	0	0

7.1.4 Techniques For Measuring correlation

Correlation can be studied by many methods . Some of the important methods are mentioned below:

Methods of studying correction

| Scatter Diagram | Karl's pearson's coefficient of correlation | Spearman's Coefficient of correlation |

7.2   Scatter Diagram or Dot Method

A scatter diagram can be defined as the graph in which two variables plotted on graph paper .

7.2.1 Application :- A scatter diagram is a visible method , which shows the presence of correlation, without calculating any numerical value. In this technique, the values of two variables are plotted as points on a graph paper. The degree of closeness of the scatter points and their overall direction enable us to examine the relationship. These relationships are expressed as linear, non- linear or zero correlation.

7.2.2 Merits :-

i)   It is easy to make and simple to understand .

ii)   It is a non - mathematical method of studying correlation .

iii)   Scatter diagram is not influenced by the extreme size of items .

7.2.3 Demerits:-

i)   It only gives us a rough idea about correlation , exact value of correlation is not provided .

ii)   It is not a method to obtain degree of correlation , we can simply get an idea to take direction of correlation .

7.2.4 Nature and Intensity of Scatter Diagram -

a)   Positive and Negative correlation :- when a scatter (points) around an upward rising line indicating the movement of the variables in the same direction , it is called positive correlation. On the other hand, if a scatter around an downward sloping line and the movement of variables in opposite direction , it is called negative correlation.
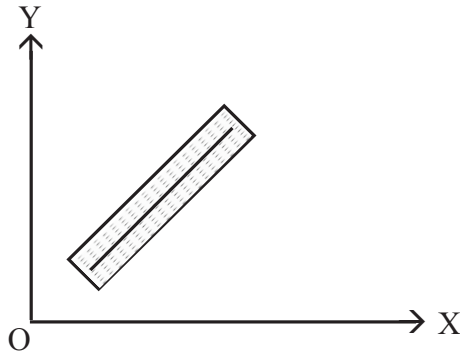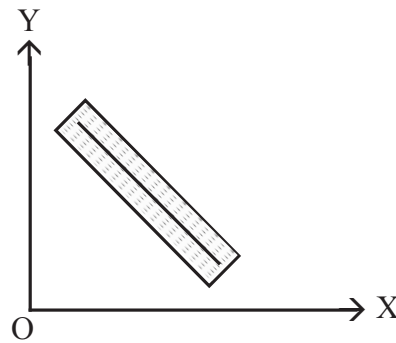
Positive correlation   Negative correlation

b)   Perfect positive and perfect negative correlation - In a scatter diagram when points themself are on the line only it is referred as perfect correlation . If points are on upwards rising line it is perfect positive correlation and if points are on downward falling line this is called perfect negative correlation .
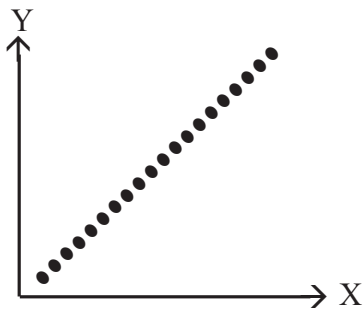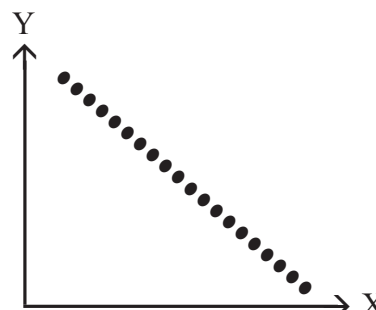
Perfect positive correlation   Perfect negative correlation

c)   Zero correlation or No correlation :- In a scatter diagram when the plotted points are no longer scattered around an upward rising or downward falling line , this is referred as zero correlation
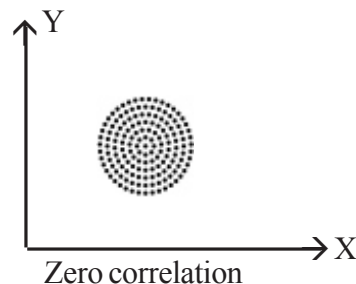
Zero correlation

7.3 Karl pearson's coefficient of correlation :

Karl pearson's coefficient of correlation is the process of determining a precise numerical value of the degree of linear relationship between two variables . This is also known as product moment correlation coefficient or simple correlation co - efficient .

7.3.1 Application – Karl pearson's coefficient correlation should be used only when there is a linear relationship between the variables . So it is better to first draw the scatter diagram of the variables and then find the value of co- efficient of correlations. Sign 'r' is used as the karl pearson's coefficient of correlation . The value of r may be positive , negative or zero .

7.3.2 Merits–
    i) Popular method of measuring relationship between two variables .

    ii) Gives summarised and precise quantitative figure of correlationship .

    iii) Indicates direction and degree of relationship .

7.3.3 Demerits–
    i) Greater influence of extreme items .

    ii) Time consuming process of calculating .

    iii) Assumption of linear relationship .

7.3.4 Formula :

i)   Actual Mean Method : karl pearson's coefficient of correlation between variable X and Y ,

$$r_{xy} = \frac{\sum xy}{\sum x^2 X \sum y^2} \qquad \text{here } x = (X - \overline{X})$$

$$y = (Y - \overline{Y})$$

ii)   Standard deviation or covariance Method :

$$r_{xy} = \frac{Cov(x, y)}{\sigma x . \sigma y} \text{ OR } \frac{\sum xy}{N \sigma x . \sigma y}$$

here , $x = (X - \overline{X})$

$y = (Y - \overline{Y})$

σx = standard deviation of X series .

σy = standard deviation of y series .

N = Number of observation .

iii)   Direct Method - when the values of X and Y variables are directly used ,

$$r_{xy} = \frac{\sum xy - \dfrac{\sum x \sum y}{N}}{\sqrt{\sum x^2 - \left(\dfrac{\sum x}{N}\right)^2} \sqrt{\sum x^2 - \left(\dfrac{\sum x}{N}\right)^2}}$$

iv)   Step - Deviation Method - When the values of X and Y are large , then using a property of r , their values are reduced.

$$r_{uv} = \frac{\sum uv - \dfrac{\sum u \sum v}{N}}{\sqrt{\sum u^2 - \left(\dfrac{\sum u}{N}\right)^2}\sqrt{\sqrt{v^2} - \left(\dfrac{\sum v}{N}\right)^2}}$$

where, $u = \dfrac{X-A}{C}$ [Here, A & B are assumed means of X and Y series

$v = \dfrac{X-B}{D}$ respectively C and D are common factors]

7.3.5 Properties of correlation coefficien (r)

i)   Correlation coefficient (r) has no unit .

ii)   A negative value of r indicates an inverse relation .

iii)   If r is positive then two variables move in the same direction .

iv)   The value of r lies between -1 and +1 i,e , $-1 \leq r \leq 1$

v)   If $r = 0$ then the two variables are uncorrelated . They have no linear relationship but may have other relation .

vi)   If $r = +1$ or $r = -1$, the correlation is perfect

vii)   A strong value of r (near to +1 or –1) indicates strong linear relationship and a low value of r (near to zero) indicates weak linear relationship .

viii) The value of r is unaffected by the change of origin and change of scale .

Given two variables x and y, let us define two new variables u and v where

$u = \dfrac{X-A}{B}$ & $v = \dfrac{Y-C}{D}$, then

$r_{xy} = r_{uv}$

7.3.6 Numerical problem

Calculate coefficient of correlation between the birth and death rate from the following data :

Year	Birth Rate	Death Rate
2002	26	20
2003	32	22
2004	33	24
2005	34	28
2006	30	26

Solution : Calculation of coefficient of correlation (Y)

Birth Rate (X)	Death Rate (Y)	$X - \bar{X} =$ (x)	$Y - \bar{Y} =$ (y)	x^2	y^2	xy
26	20	−5	−4	25	16	20
32	22	+ 1	−2	1	4	−2
33	24	+ 2	0	4	0	0
34	28	+ 3	+ 4	9	16	12
30	26	−1	+ 2	1	4	−2
ΣX= 155	ΣY = 120	Σx = 0	Σy = 0	Σx²=40	Σy²= 40	Σxy=28

$$\bar{X} = \frac{\Sigma x}{N} = \frac{155}{5} = 31 \qquad\qquad \bar{Y} = \frac{\Sigma y}{N} = \frac{120}{5} = 24$$

a) Actual Mean Method :-

$$r = \frac{\sqrt{\Sigma xy}}{\sqrt{\Sigma x^2 \times \Sigma y^2}} \qquad\qquad \text{Here , } \Sigma xy = 28$$

$$= \frac{28}{\sqrt{40 \times 40}}$$

$$= \frac{28}{40} = 0.7$$

b) Standard deviation Method :-

$$r = \frac{\Sigma xy}{N \sigma x \ \sigma y} \qquad\qquad \text{Here N=5}$$

$$= \frac{28}{5.\sqrt{8}.\sqrt{8}} \qquad\qquad \sigma_x = \sqrt{\frac{\Sigma x^2}{N}} = \sqrt{\frac{40}{5}} = \sqrt{8}$$

$$= \frac{28}{5 \times 8} = 0.7 \qquad\qquad \sigma_y = \sqrt{\frac{\Sigma y^2}{N}} = \sqrt{\frac{40}{5}} = \sqrt{8}$$

Therefore, $r = 0.7$, shows moderate positive correlation .

7.4 Sparman's Rank correlation :-

Sprearman's Rank correlation is the process of determining correlation between the series of two variables by obtaining ranks to variables of both the series subject to specific characteristics . This method is developed by British psychologist professor C.E. Spearman.

7.4.1 Application :– There are many situations where simple correaltion , coefficient is not applicable like,

i) In some cases the values of varibles are not given or not possible to obtain . Then we can use their rank depending on specific characteristics to get correlation coefficient.

ii) In case variables of qualitative nature like , intelligence , beauty , honesty etc , rank correlation method is the most suitable .

iii) In case presence of extreme values of data , rank correlation is more useful than karl Pearson's correlation coefficient .

iv) When relationship of variables is non - linear , the use of rank correlation is suitable .

7.4.2 Merits–

i) Spearman's rank correlation can be easily calculated and understood than Pearson's correlation coefficient .

ii) In case of rank of values in the series are given rank correlation is the only way to compute coefficient of correlation.

iii) It is useful for data of qualitative nature.

iv) The result are same in both the methods of coefficient of correlation ..

7.4.3 Demerits–

i) This method is unsuitable for grouped frequency distribution.

ii) It is also unsuitable if values of the series exceed 30.

iii) This method is based on rank . All the information relating to data is not used, so it lacks precision as compare to pearson's coefficient of correlation.

7.4.4 Formula -

a) In case of ranks are given and ranks are not given -

$$r_K(\rho) = 1 - \frac{6.\sum D^2}{N^3 - N}$$

Where , $r_K(\rho = rho)$ = Coefficient of correlation .

N = Number of observation

D = Difference of corresponding ranks .

b) In case of ranks are repeated–

$$r_K = 1 - \frac{6.\left[\sum D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + ...\right]}{N^3 - N}$$

Where , m = Number of times a rank is reapeated .

7.4.5 Numerical problem -

a)   If rank are given : There are nine students whose preference has been ranked by the two teachers as under , calculate coefficient of correlation by Rank method .

Students	:	A	B	C	D	E	F	G	H	I
Teacher I	:	1	2	3	4	5	6	7	8	9
Tacher II	:	3	2	1	5	6	4	7	8	6

Solution : Caculation of of coefficient of correlation (r)

Students	Rank by Teacher I (R$_1$)	Rank by Teacher II (R$_2$)	R$_1$ - R$_2$ (D)	D^2
A	1	3	−2	4
B	2	2	0	0
C	3	1	+2	4
D	4	5	−1	1
E	5	6	−1	1
F	6	4	+2	4
G	7	7	0	0
H	8	8	0	0
I	9	6	+3	9
N = 9				ΣD^2 = 23

$$\therefore \ r_K = 1 - \frac{6.\Sigma D^2}{N^3 - N} = 1 - \frac{6 \times 23}{9^3 - 9} = 1 - \frac{138}{720} = 0.81$$

It show highly positive correlation .

b)   If Ranks are not given : From the following data calculate r_K

X	30	50	14	59	36	27	38	47	9	54
Y	44	29	64	19	52	69	54	39	83	32

Solution : Calculation of rank coefficient of correlation

X	R_1	Y	R_2	$R_1 - R_2 = (D)$	D^2
30	4	44	5	−1	1
50	8	29	2	+6	36
14	2	64	8	−6	36
59	10	19	1	+9	81
36	5	52	6	−1	1
27	3	69	9	−6	36
38	6	54	7	−1	1
47	7	39	4	+ 3	9
9	1	83	10	− 9	81
54	9	32	3	+ 6	36
N=10					$\Sigma D^2 = 318$

$$\therefore \quad r_k = 1 - \frac{6.\Sigma D^2}{N^3 - N} = 1 - \frac{6 \times 318}{10^3 - 10} = 1 - \frac{1908}{990} = 1 - 1.927 \quad = -0.93$$

It shows high degree of negative correlation .

c) If marks are repeated calculate r_K

X	45	30	37	6	13	13	62	22	12	54
Y	10	10	21	3	12	1	17	6	3	16

Solution : Calculation of rank coefficient of correlation (r_K)

X	R_1	Y	R_2	$R_1 - R_2 = D$	D^2
45	4	10	5.5	+2.5	6.25
30	6	10	5.5	+0.5	0.25
37	7	21	10	−3.0	9.00
6	1	3	2.5	−1.5	2.25
13	3.5	12	7	−3.5	12.25
13	3.5	1	1	+2.5	6.25
62	10	17	9	+1.0	1.00
22	5	6	4	+1.0	1.00
12	2	3	2.5	− 0.5	0.25
54	9	16	8	+1.0	1.00
N=10					$\Sigma D^2 = 39.5$

$$\therefore \quad r_{\text{K}} = 1 - \cfrac{6\left[\sum D^2 + \cfrac{1}{12}(m^3 - m) + \cfrac{1}{12}(m^3 - m) + \dots\dots\right]}{N^3 - N}$$

$$= 1 - \cfrac{6.\left[39.5 + \cfrac{1}{12}(2^3 - 2) + \cfrac{1}{12}(2^3 - 2) + \cfrac{1}{12}(2^3 - 2)\right]}{10^3 - 10}$$

$$= 1 - \cfrac{6(39.5 + 1.5)}{990}$$

$$= 1 - \cfrac{6 \times 41}{990}$$

$$= 1 - 0.25$$

$$= 0.75$$

It shows high degree of positive correlation .

Exercise

Multiple Choice Question :

1) Simple correlation measures–

   i) Covarience, ii) Casuation , iii) both i) & ii), iv) None of these .

2) r_{xy} will be positive , when the relation between X & Y will be like–

   i) When Y increases , X also increases ii) When Y decreases , X increases .

   iii) When Y increaes , X is unchanged . iv) None of these .

3) When two varibles charge in the same direction with equal proportionate change means -

   i) Perfect positive correlation , ii) Perfect negative correlation .

   iii) Linear correlation iv) Positive correlation .

4) If $r_{xy} = 0$ the relation between X and Y is -

   i) Linear ii) Non - linear iii) Independent iv) None of these .

5. Which is not the value of 'r' -

   i) -1 ii) $+2$ iii) 0.75 iv) $+1$

6.   Which method can measure any relationship–
    i) Karl pearson's coefficient of correlation   ii) Spearman's Rank correlation .
    iii) Scatter Diagram   iv) All of these .
7.   Which method can not measure perfectly the degree of correlation -
    i) Karl Pearson's coefficient of correlation   ii) Spearman's Rank correlation .
    iii) Scatter Diagram ,   iv) All of these .
8.   The unit of the coefficient of correlation between height (in feet) and weight (in kg) –
    i) Kg/ ft   ii) Percentage   iii) ft / kg   iv) Non - existent .
9.   Which method of measuring correlation is suitable in case of qualitative data –
    i) Karl Pearson's coefficient of correlation ,   ii) Spearman's Rank correlation
    iii) Scatter Diagram   iv) None of these .
10. Karl Pearson's coefficient of correlation is suitable when the relation is -
    i) Non - linear   ii) Linear   iii) Zero   iv) All are applicable .

True / False

1.   The price of goods and its demand shows postitive correlation .
2.   Scatter diagram shows the direction and intensity of correlation .
3.   In case of zero correlation , scatter diagram is not linear .
4.   The value of 'r' changes with charge in origin and scale .
5.   In case of non - linear relationship , spearman's Rank correlation is a not applicable .

Fill up the blanks

1 . Correlation measures the _____ and _____ of the relationship between the varibales .
2.   The ratio of change between two variables is _____ when they are linearly related .
3.   _____ indicates the presence of correlation in graph paper .
4.   In a scatter diagram when point are _____ the upward rising line only, it is the situation of
     perfect positive correlation .
5.   The most popular method of measuring correlation is _____ .

Short Answer type Question   **1 mark Question**

1.   What is correlation ?
2.   What do you meant by covarience ?
3.   Give an example of a positive correlation ?
4.   What is perfect negative correlation ?
5.   What is the degree of high degree of negative correlation ?
6.   What is the shape of scatter diagram in case of positive correlation ?

7. What is the value of '*r*' ?

8. In which condition spearman's rank correlation is applied ?

Long Answer type Question   **3/4 Marks Question**

1. Correlation measures covariance , not the causation - Discuss .

2. Discuss the different types of scatter diagram .

3. Write the properties of coefficient of correlation (*r*)

4. When the value of *r* be +1 , −1 , and 0 ?

5. When is Spearman's Rank correlation more suitable than karl pearson's coefficient of correlation ?

6. Why does Rank correlation coefficient differ from Pearson's correlation coefficient ?

7. Calculate r_{xy} .

a)

X :	78	89	97	69	59	79	68	61
Y :	125	137	156	112	107	136	123	108

(Ans : 0.96)

b)

Price (X):	4	6	8	10	12	14	16	18
Supply(Y):	10	15	20	25	30	35	40	45

(Ans : 1)

c)

X :	10	12	11	13	12	14	9	12	14	13
Y :	7	9	12	9	13	8	10	12	7	13

(Ans : - 0. 115)

8. Calculate Rank correlation coefficient (r_k) :

a)

X :	90	37	42	82	50
Y :	21	73	68	52	56

b) In a competition , two judges accorded following ranks to 12 competitors . Calculate rank correlation coefficient .

Cormpetitor	1	2	3	4	5	6	7	8	9	10	11	12
Judge I	7	8	2	1	9	3	12	11	4	10	6	5
Judge II	6	4	1	3	11	2	12	10	5	9	7	8

(Ans :- 0.86)

c) Calculate Coefficient of correlation between the marks in English and Economics, as indicated by 8 answer books .

English	:	15	10	20	28	12	10	16	18
Economics	:	16	14	10	12	11	15	18	12

(Ans : -0.36)

Answer

Multiple Choice Question :

    1 (i)   2 (i)   3 (i)   4 (ii)   5 (ii)   6 (iii)   7 (iii)   8 (iv)   9 (iii)   10 (ii)

True / False : -

    1 . False   2. False   3 , True 4 . False   5 . False .

Fill up the blanks :

    1. Direction, intensity,   2. Uniform   3. Scatter diagram   4. on

    5. Karl Pearson's coefficient of correlation .

Short Answer type Question   **1 mark Question**

1.   Correlation is a method of measuring the direction and intensity of relationship between two variables .

2.   Covarience is a method of measuring the relationship between two varibales .

3.   Relationship between the price of a goods & its supply .

4.   When the equal proportional change between two variables moves in opposite direction , it is called perfect negative correlation .

5.   The value lies between –1 and –0.75

6.   A scatter (points) around an upward rising line - is the shape of a positive correlation .

7. $-1 \leq r \leq +1$

8.   When rank of the variables of two series are given .

Index Number

At present, Index number have become one of the most widely used statistical technique for judging the pulse of economy . Although in the begining they were constructed to gauge the effect of changes in prices. Today we use index number for cost of living , industrial production , agricultur production etc .

8.1 Index Number

An index Number is a statistical device , designed to measure changes in a variable or a group of related variables . It is a single ratio (or a percentage) which measures the combined change or relative change in the variables between different time , places or situations Broadly spreaking , there are two major types of index number : simple and composite . The simple index is computed for one variable whereas the composite index is calculated from two or more variables . Most index numbers are composite in nature .

8.2 Issues in the construction of an index number :

Following are some of the important criteria / problems which have to be faced in the construction of index numbers .

a) Determination and Definition of the purpose : It is necessary that the purpose and scope of the desired index number must be determined and clearly defined in specific term .

b) Selection of weights : It is necessary to give suitable weightage to different items depending upon the purpose of index number in case of composite index members .

c) Selection of price : Price of items change from market to market , place to place and even shop to shop. It is therefore , necessary that the price of representative market should be selected .

d) Selection of Base period : Index number studies the changes during the current year as compared to certain specified previous year . This specific past period for comparison is termed as base period . Base period should be normal , i.e in base period the price of items should be free from all abnormalities . Base period should neither be too near nor too far from the current year . The base year needs to be changed from time to time because many items from the basket of previous base period may be disappeared at present.

e. Selection of items : Items used for the construction of Index numbers must be representative .

f. Selection of sources of data : It is necessary to select reliable , accurate and representative sources of data. Data of poor reliability gives misleading result.

g. Choice of the methods : Another issue is the choice of the formula or method on the nature of question to be studied.

h.   Choice of Average : As index numbers are themselves specialize average , it has to be decided first the preferred average . The arithmetic mean , being easy to use and calculate , is preferred over other average (median , mode or geomatric mean) .

8.3   Characteristics of Index Number :

Following are some of the important characteristics of index numbers :

a)   Index Numbers are a special type of average that provides a measurement of relative changes in the level of certain phenomenon from time to time .

b)   Index numbers are expressed in terms of percentages to show the extent of relative change

c)   Index Number measures relative changes in a variable or a group of related variables .

d)   Index numbers can also measures changes which are not directly measurable . For example , the cost of living , the price level etc .

e)   It has no unit because it is a ratio of two relative changes .

8.4   Difference between price Index and Quantity Index .

Price Index	Quantity Index
1. Price Index measures the price change of certain goods .	1. Quantity index measures the change in the physical volume of production, construction or employment .
2. A price index affect the purchasing power of consumer .	2. A quantity index affect the production of producer .
3. Consumer price index is an example of a price index .	3. Producer price index is an example of a quantity index .

8.5   Uses of Index Numbers in Economics :

Index number are used in the following ways–

a)   Measuring the pulse of the economy : Index Number works as the barometer of the economy. It measures the level of business and economic activities and therefore helpful in gauging the economic status of the country. Indices of wages , prices, output , bank deposit , foreign exchange etc. throw light on the general economic conditions of the economy.

b)   Helping in formulation of suitable economic policies : Index numbers work as guidelines for formulation of business and economic policies .

c)   Index numbers are used in study trends and techniques.

d)   Index numbers measure relative changes between two set of variables , expressed in even different units. We can compare the increase in the agriculture and industrial output with the help of Index Number .

e)   Index numbers are used in forcasting future economic activities.

f) It gives us knowledge of real increase or decrease. For example, we can mesure real income, real wage etc.

g) It gives us knowledge of purchasing power of money through changes in consumer price index number.

8.6 Methods of construction of an Index Number.

Construction of an Index Number

A. Aggregative method

B. Average of price Relative method or relative method

1. Simple Aggregative method

2. Weighted Aggregative method

1. Simple

2. Weighted

Laspeyer's Method   Paasche's method   Fisher's method
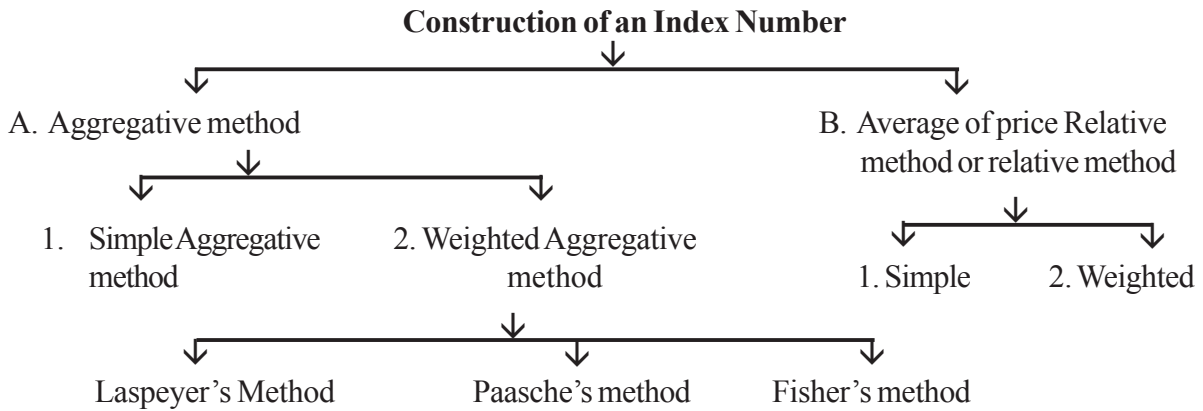
A. AGGREGATIVE METHOD

8.6.1 A. (1) Simple Aggregative Method

This is a method of un-weighted index number. This method is based on the assumption that various items and their prices are quoted in same units. Equal importance is given to all items.

Formula : $P_{01} = \dfrac{\sum P_1}{\sum P_0} \times 100$

Where, P_1 = Total of current year's price for the various items .

P_0 = Total of base year's price for the various items .

This method has limited use . Because in reality , the use and weight of various items are not same. Even, the different items may be expressed in different units . So weighted aggregative price index method is used.

A(2) Weighted Aggregated method.

In weighted aggregative price indices , the weights are assigned to each item in the basket. In most cases quantity of items is taken as weight of that item . Among the different formulas of weighted price index-Laspeyer's formula , paasche's formula and Fisher's formula are important .

i) Laspeyer's Formula : $P_{01} = \dfrac{\sum P_1 q_0}{\sum P_0 q_0} \times 100$

Here, quantities of base year (q_0) is taken weight of items .

ii)   Paasche's Formula :

$$P_{01} = \times \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100$$

Here , quantities of current year (q_1) is taken as weight of items .

iii)   Fisher's Formula   - $P_{01} = \sqrt{\frac{\sum P_1 q_o}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100$

This method uses both base period and current period quantities as weight

8.6.2 B.   Relative Method

B (1)   Simple Average of price Relative method

This is an un - weighted method , where price relatives of the commodities are quoted . So this method is not affected by unit . The price index number using price relatives is defined as follows :

$$P_{01} = \frac{1}{N} \sum \frac{p_1}{p_0} \times 100$$

Where , P_1 and P_0 indicate the price of the commodities in the current year and base year respectively.

$\frac{p_1}{p_0}$ indicates price relative and N   stands for the number of commodities .

Though this method is not influenced by the units , but it gives equal importance to all the items and thus neglects their relative importance in the group.

B (2)   Weighted Price Relative Method .

Here the price index is obtained by taking the average of all weighted price relatives . It is given by

$$P_{01} \ (\text{Weighted arithmetic mean}) = \frac{\sum \left(W \times \frac{P_1}{P_0} \right)}{\sum W} \times 100$$

Where , W = Weights

8.7   Some important Index Numbers

A.   Consumer price Index :   A consumer price index (CPI) measures changes in the price level of a basket of consumer goods and services purchased by households . CPI provides an idea about the effect of the change in general price level on the cost of living of different classes of people.

Special Features :-

i)   Consumer price index is also known as cost of living index or Retail price index .

ii)   CPI measures only changes in price .

iii) CPI is constructed separately for different classes of people or groups of society , such as govt employees , industrial workers , agricultural labourers etc . It may be different for different geopraphical areas , such as consumer for urban or rural or hilly areas etc .

iv) To construct a CPI , determining a particular class of people an enquiry is conducted on the family budget of the selected class of people . Mainly there are five main groups of goods & services in basket, looking into the consumption pattern of these families . These are -

    a) Food b) clothing   c) Fuel and lighting   d) House rent and   e) Miscellaneous .

v) Normally, CPI is calculated on weekly basis .

vi) CPI or cost of living Index $= \dfrac{\Sigma WP}{\Sigma W}$

    Where , $P = \dfrac{p_1}{p_0} \times 100$ and W = Weights

vii) CPI is helpful in wage negotiation , measuring the change in purchasing power, analysing market, determining government policies like wage policy, price policy , taxation policy , general economic policy etc .

viii) CPI is used in calculating–

    a) Purchasing power of Money $= \dfrac{1}{CPI} \times 100$ and   b) Real wage $= \dfrac{Money\ wage}{CPI} \times 100$

B . Wholesale price Index (WPI) :

The wholesale price index or WPI is the index of the price of a representative basket of wholesale goods . WPI number indicates the change in the general price level . Unlike the CPI , it does not have any reference consumer category .

Special features :

i) It represents general change in the wholesale prices of the commodities .

    WPI with 2011 as base year is 156 in March, 2014 means that the general price level has risen by 56% during this period .

ii) Some country use WPI charges as a central measure of inflation , in India new CPI is used .

iii) In India WPI is published by the Economic Adviser in the Ministry of commerce and industry .

iv) There are 3 groups for the constructions of WPI -

    a) Primary articles , number of items of this category is 98 and weightage is 22.62 .

    b) Fuel or power , number of items of this category is 19 with weigthage as 13.15

    c) Manufactured products , number of item of this category is 318 with weightage as 64.23.

v) Weighted geometric mean of price relatives is used to calculate WPI number .

vi) WPI is used to eliminate the effect of changes in prices on aggregate , such as, national income, capital formation etc .

C) Index of Industrial Production (IIP)

Index Number measuring the change in the level of industrial production in a given period as compared to some base period is known as Index of Industrial production . It tries to measure quantities. It uses weighted arithmetic mean of quantity relatives. The construction of this index number requires data of Industrial sectors and sub - sectors . The main branches of IIP are mining , manufacturing and electricity with weightage as 14.4 , 77.6 and 8.0 respectively . IIP is also available according to the 'use' of the product .

D) Sensex : Sensex is benchmark index of India's Bombay stock Exchange (IBSE), created in 1986 with 1978-79 as base. It consists of 30 stocks which represent 13 sectors. When the sensex rises, it indicates that the market is doing well and investors expect between earnings from companies

8.8 Limitations of Index Numbers :

The major Limitations of index Numbers are–

a) Limited coverage - Index numbers are usually based on sample items , not include all the commodities.

b) It ignors the qualitative changes of items selected.

c) Index numbers not able to show the true picture of changes in the consumption pattern .

d) It has limited applicability . An index number is constructed based on a particular prupose . These are no for all - purpose index numbers .

e) It may provide misleading results in the cases of wrong selection of base year , formula , weightage or sample of items .

Mathematical problems

a) Construct price index number for 2009 , taking 2004 as base , using (a) Simple Aggregative Method,

(b) Simple Average of Price Relative Method .

Commodities	Price in 2004	Price in 2009
A	50	70
B	40	60
C	80	90
D	110	120
E	40	60
F	70	70

Solution :- Let , P_0 = Base year's price .

P_1 = Current year's price

a) Simple Aggregative Method :-

$$\therefore \quad \Sigma P_0 = 390$$
$$\Sigma P_1 = 470$$

$$\therefore \quad P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100 = \frac{470}{390} \times 100 = 120.5$$

This indicates net increase in prices of commodities in 2009 as compared to 2004 has been extent to 20.5%.

b) Simple Average of prices Relative method :-

$$N = 6$$

$$\therefore \quad P_{01} = \frac{1}{N} \ \Sigma \frac{P_1}{P_0} \times 100$$

$$= \frac{1}{6} \ (140 + 150 + 112.5 + 109 + 150 + 100)$$

$$= \frac{1}{6} \times 761.5 = 126.9$$

This index number indicates a price rise by 26.91 in 2009 over the year 2004 .

2) Calculate weighted price index from the following -

Commodity	Base period		Current period	
	P_0	q_0	P_1	q_1
A	2	10	4	5
B	5	12	6	10
C	4	20	5	15
D	2	15	3	10

Solution : i) Laspeyer's Method : $P_{01} = \dfrac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$

$$= \frac{4 \times 10 + 6 \times 12 + 5 \times 20 + 3 \times 15}{2 \times 10 + 5 \times 12 + 4 \times 20 + 2 \times 15} \times 100$$

$$= \frac{257}{190} \times 100 \quad = 135.3$$

ii)   Paache's Method : $P_{01} = \dfrac{\sum P_1 q_1}{\sum P_0 q_1} \times 100$

$$= \dfrac{4 \times 5 + 6 \times 10 + 5 \times 15 + 3 \times 10}{2 \times 5 + 5 \times 10 + 4 \times 15 + 2 \times 10} \times 100$$

$$= \dfrac{185}{140} \times 100 = 132.1$$

iii)   Fisher's Method : $P_{01} = \sqrt{\dfrac{\sum P_1 q_o}{\sum P_0 q_0} \times \dfrac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100$

$$= \sqrt{\dfrac{257}{190} \times \dfrac{185}{140}} \times 100 = 178.74$$

3)   Calculate CPI or the weighted price Relatives index from following :

Commodity	Weight (in %)	Base year Price (in Rs)	Current year price (in Rs)
A	40	2	4
B	30	5	6
C	20	4	5
D	10	2	3

Solution : Weighted price Relatives index Method :

$$P_{01} = \dfrac{\sum \left(W \times \dfrac{P_1}{P_0} \right)}{\sum W} \times 100$$

$$= \dfrac{\left[\left(40 \times \dfrac{4}{2} \right) + \left(30 \times \dfrac{6}{5} \right) + \left(20 \times \dfrac{5}{4} \right) + 10 \times \dfrac{3}{2} \right]}{40 + 30 + 20 + 10} \times 100$$

$$= \dfrac{(80 + 36 + 25 + 15)}{100}$$

$$= 156 .$$

4)   If the salary of a person in the base year is Rs 8000 per annum and the current year salary is Rs 12000, by how much should his salary be raised to maintain the same standard of living if the CPI is 250.

Solution : Salary of current year $= \dfrac{Salary \text{ of Base year}}{CPI \text{ of Base year}} \times CPI$ of current year

$$= \dfrac{8000}{100} \times 250$$

$$= \text{Rs. } 20,000$$

Therefore , Salary should be increased by

$$= \text{Rs } 20,000 - \text{Rs } 12000$$

$$= \text{Rs } . \; 8000$$

5) CPI for june , 2012 was 120 . The food index was 100 and that of other items 130 . What is the percentage of the total weight given to food ?

Solution : Let , the weight given to food be X and so weight of other items is (100 –X)

$$\text{Since , CPI} = \dfrac{\Sigma WR}{\Sigma W}$$

$$120 = \dfrac{100X + 130(100 - X)}{100}$$

$$X = 33.33$$

So food item weight as 33.33 .

Exercise

Multiple Choice Question :

1. An index number is called a simple index when it is computed from–

   a) Single variable   b) Bi - variable   c) Multi - variables   d) None of these .

2. If all the variables are of equal importance, the index number is called–

   a) Weighted   B) Unweighted   c) composite   d) Constant prices .

3. Index for base period is always taken as–

   a) 1   b) 200   c) 100   d) 0

4. When index number is calculated for several variables , it is called–

   a) Whole sale price index   b) Simple index   c) Volume index   d) composite index .

5. Price Relatives are a percentage ratio of current year price and -

   a) Base year quantity .   b) Previous year quantity

   c) current year quatity   d) Base year price .

6. In Laspeyer's formula of index number–
   Year quantities are taken as weight of items
   a) Current   b) Base   c) Both (a) & (b)   d) None of these .

7. In the construction of CPI , a family budget survey is conducted to collect information regarding–
   a) Prices of commodities   b) Consumption pattern of the families
   c) Weights of the various commodities   d) All of these .

8. Rapid increase in price index number implies– rate of inflation
   a) Lower   b) constant   c) Higher   d) None of these .

9. For CPI , price quotations are collected from –
   a) Fair price shops   b) Govt Departments   c) Retailers   d) Wholesale Dealers .

10. Purchasing power of money of a country is determined by–
    a) CPI   b) WPI   C) IIP   D) Exchange Rate .

Fill up the blanks

1. CPI is prepared by the average of _____ .

2. A CPI is also known as _____ .

3. A _____ measures the price change of certain goods .

4. There are _____ main groups for WPI construction .

5. In IIP , the branch with highest weightage is _____ .

6. Dearness allowance is calculated with the help of _____ .

7. With an _____ in consumer price index purchasing power of money declines .

True / False

1. WPI is hepful in wage negotiation .

2. Sensex consists of 50 stocks .

3. Index of industrial production is a price index .

4. Index number gives us knowledge of real increase or decrease .

5. Generally WPI is used to measure inflation .

6. Index numbers are used to measure only measurable changes of a variable .

7. A change in CPI number indicates change in real wages of a consumer .

Short Question carrying   **1 mark** each

1. What is index number ?

2. What is Base year ?

3. Where can we get different Index Numbers like CPI , WPI , IIP etc in India ?

4. What will be purchasing power of money when CPI is 526 ?

5. What do you meant by HDI ?

6. What is IIP number ?

7. Which CPI number is used by RBI ?

8. Define the weighted index number .

9. Define cost of living index .

10. Name the consumer groups for which CPI number is computed .

3/4 marks

1. Explain the characteristics of index numbers .

2. Explain the importance of index numbers .

3. What are the problems or issues before the construction of index numbers ? Explain briefly

4. Write the features of base period in the construction of an index numbers .

5. What are the consideration underlying the selection of weights and commodities in the construction of a weighted index of price ?

6. Explain the significance of consumer price index number .

7. Why it is essential to have different CPI for different categories of consumers ?

8) Constant Index Number for 2004 assuming 1995 as the base year :

Commodities :	A	B	C	D
Price (1995) :	25	18	26	31
Price (2001) :	30	25	30	30

(Ans :- 115)

9) Construct Index Number by simple average of price relative method for 2004 , base 2001

Commodities :	A	B	C	D	E
Price (2001) :	40	60	100	90	25
Price (2004) :	60	75	130	135	25

(Ans :- 131)

10) Construct index number by different weighted aggregative formula :

	Base Year		Current Year	
Commodities	Price	Quantity	Price	Quantity
A	10	12	12	15
B	7	15	5	20
C	5	24	9	20
D	16	5	14	5

(Ans :- Laspeyer's method - 1.11

Paasche's method - 1.13

Fisher's method - 1.25)

11. Constant Index Number by simple Average of price Relative method :

Commodities :	A	B	C	D
Price (2012) :	1	2	3	4
Price (2018) :	5	4	3	2

(Ans :- 212.5)

12. Construct CPI for 2002 , base 2001 by -

   A) Aggregate Expenditure Method (Laspeyer's method)

   B) Family Budget method (Weighted price Relatives Method)

13.

Commodities :	Wheat	Rice	Pulses	Gee	Oil
Price (2001) :	50	20	2	5	3
Price (2002) :	70	25	3	5	3
Weight :	20	30	5	20	10

(Ans :- a) 131.89   b) 131.89)

14.   CPI for March , 2012 was 125 . Food index was 120 and that of other items 135 . What is the percentage of the total weight given to other items ?   (Ans :- 33.33)

Answer

Multiple Choice Question :

    1. a,   2. b,   3. c ,   4. d,   5. d,   6. b,   7. b,   8.c,   9. c,   10. a

Fill up the blanks

    1. Price Relatives   2. Cost of living index   3) Price Index   4) three

    5) Manufacturing   6) WPI   7) Increase .

True / False

    1. False   2. False   3. False   4, True   5. True   6. False   7. True .

1 mark

1. Index numbers are devices for measuring differences in the magnitude of a group of related variables .

2. In a index number the specific past period selected for comparison is termed as base period

3. Economic survey .

4. Purchasing power of money $= \dfrac{1}{CPI} \times 100 = \dfrac{100}{526} = 0.19$

5. The Human Development index (HDI) is a summary measure of average achievement in key dimensions of human development : a long & healthy life , being knowledgeable and have a decent standard of living .

6. Index of industrial production (IIP) number is an index number measuring the change in the level of industrial production in a given period compared to some base period .

7. All India combined consumer price index .

8. When all commodities are not of equal importance in the construction of an index number , we assign weight to each commodity relative to its importance and the index number computed from these weights is called weighted index number .

9. A cost of living index means consumer price index (CPI) is the measure of changes in the price level of a basket of consumer goods and services .

10. CPI numbers is computed for different groups like industrial workers , agricultural labourers Rural labourers , urban non - mannual employees etc .

Use of Statistical Tools : Project for Application of Statistics in Economics

You have already studied the statistical tools and methods in the previous chapters . Now we are discussing these methods and tools by taking some project works .

9.1 Statistical projects :

The statistical project is the systematised process to make an enquiry , verify the truth or test the real situation . Statistical project are undertaken with certain specified and predetermined purpose . As such as a good effective survey requires careful planning , methodical application and detailed analysis of results.

9.2 Utility of project Report :

Following are the few utilities of project work :

i) To provide information about the development and growth of institution , business , Government activities .

ii) To provide information about the products to the consumers .

iii) It is helpful to find preference of consumers .

iv) It is also helpful to identify a problem and the reasons of the problem .

v) It is also helpful in the policy formulation about the economic and social development of any country .

9.3 Steps forwards making project :

a) Objective : The objective of the project should clearly be specified .On the basis of the objectives the required data will be collected .

b) Population : The population or target group is the next step in developing a project report . Focusing on the object of the study and target group an appropriate questionnaire is to be framed. For example, the project regarding marketing of cars requires the middle and higher income group to be studied.

c) Collection of data : It is the objective of the survey , which determines the collection of primary data or secondary data or both . Primary data can be collected by personal interview or by making questionnaire. Secondary data must be used with great care.

d) Organisation and Presentation of data : The collected data has to pass through the process of organising and presenting with the help of tables , groups or diagrames.

e) Analysis and Interpretation : After presenting data in the form of tables or graphs , for their analysis statistical techniques like measures of central tendency , measure of dispersion etc . may be used as per our requirements for anylysis . Then we can interpret the results .

f) Conclusion : The final stage of the study will be to draw meaningful conclusions , after analysis and interpreting the results . If possible , we must try to predict the future prospects and offer any suggesion.

g) Bibliography : In this section , we need to mention the details of all secondary sources i.e. magazines newspaper, research Reports used for developing the project .

9.4 Suggested list of project :

To get few suggested projects you can follow the text book , chapter - 9 , pp- 125- 126 .

9.5 Sample project .

Depending on the subject of the study the method used will obviously be different from one study to another. For your guidance you can follow a sample project in your text book, chapter-9, P. 126- 131.