MATHEMATICS WORK BOOK

CLASS - X

State Council of Educational Research and Training
Govt. of Tripura

MAHEMATICS WORK BOOK

Class - X

First Edition
September, 2021

Cover Design
Asoke Deb, Teacher

Type & Setting : SCERT, Tripura

Printed by :
Satyajug Employees Co-operative Industrial Society Ltd.
13, Prafulla Sarkar Street, Kolkata-72

Publisher :
State Council of Educational Research and Training
Government of Tripura

রতন লাল নাথ
মন্ত্রী
শিক্ষা দপ্তর
ত্রিপুরা সরকার

বার্তা

শিক্ষার প্রকৃত বিকাশের জন্য, শিক্ষাকে যুগোপযোগী করে তোলার জন্য প্রয়োজন শিক্ষাসংক্রান্ত নিরন্তর গবেষণা। প্রয়োজন শিক্ষা সংশ্লিষ্ট সকলকে সময়ের সঙ্গে সঙ্গে প্রশিক্ষিত করা এবং প্রয়োজনীয় শিখন সামগ্রী, পাঠ্যক্রম ও পাঠ্যপুস্তকের বিকাশ সাধন করা। এস সি ই আর টি ত্রিপুরা রাজ্যের শিক্ষার বিকাশে এসব কাজ সুনামের সঙ্গে করে আসছে। শিক্ষার্থীর মানসিক, বৌদ্ধিক ও সামাজিক বিকাশের জন্য এস সি ই আর টি পাঠ্যক্রমকে আরো বিজ্ঞানসম্মত, নান্দনিক এবং কার্যকর করবার কাজ করে চলেছে। করা হচ্ছে সুনির্দিষ্ট পরিকল্পনার অধীনে।

এই পরিকল্পনার আওতায় পাঠ্যক্রম ও পাঠ্যপুস্তকের পাশাপাশি শিশুদের শিখন সক্ষমতা বৃদ্ধির জন্য তৈরি করা হয়েছে ওয়ার্ক বুক বা অনুশীলন পুস্তক। প্রসঙ্গত উল্লেখ্য, ছাত্র-ছাত্রীদের সমস্যার সমাধানকে সহজতর করার লক্ষ্যে এবং তাদের শিখনকে আরো সহজ ও সাবলীল করার জন্য রাজ্য সরকার একটি উদ্যোগ গ্রহণ করেছে, যার নাম 'প্রয়াস'। এই প্রকল্পের অধীনে এস সি ই আর টি এবং জেলা শিক্ষা আধিকারিকরা বিশিষ্ট শিক্ষকদের সহায়তা গ্রহণের মাধ্যমে প্রথম থেকে দ্বাদশ শ্রেণির ছাত্র-ছাত্রীদের জন্য ওয়ার্ক বুকগুলো সুচারুভাবে তৈরি করেছেন। ষষ্ঠ থেকে অষ্টম শ্রেণি পর্যন্ত বিজ্ঞান, গণিত, ইংরেজি, বাংলা ও সমাজবিদ্যার ওয়ার্ক বুক তৈরি হয়েছে। নবম দশম শ্রেণির জন্য হয়েছে গণিত, বিজ্ঞান, সমাজবিদ্যা, ইংরেজি ও বাংলা। একাদশ দ্বাদশ শ্রেণির ছাত্র-ছাত্রীদের জন্য ইংরেজি, বাংলা, হিসাবশাস্ত্র, পদার্থবিদ্যা, রসায়নবিদ্যা, অর্থনীতি এবং গণিত ইত্যাদি বিষয়ের জন্য তৈরি হয়েছে ওয়ার্ক বুক। এইসব ওয়ার্ক বুকের সাহায্যে ছাত্র-ছাত্রীরা জ্ঞানমূলক বিভিন্ন কার্য সম্পাদন করতে পারবে এবং তাদের চিন্তা প্রক্রিয়ার যে স্বাভাবিক ছন্দ রয়েছে, তাকে ব্যবহার করে বিভিন্ন সমস্যার সমাধান করতে পারবে। বাংলা ও ইংরেজি উভয় ভাষায় লিখিত এইসব অনুশীলন পুস্তক ছাত্র-ছাত্রীদের মধ্যে বিনামূল্যে বিতরণ করা হবে।

এই উদ্যোগে সকল শিক্ষার্থী অতিশয় উপকৃত হবে। আমার বিশ্বাস, আমাদের সকলের সক্রিয় এবং নিরলস অংশগ্রহণের মাধ্যমে ত্রিপুরার শিক্ষাজগতে একটি নতুন দিগন্তের উন্মেষ ঘটবে। ব্যক্তিগত ভাবে আমি চাই যথাযথ জ্ঞানের সঙ্গে সঙ্গে শিক্ষার্থীর সামগ্রিক বিকাশ ঘটুক এবং তার আলো রাজ্যের প্রতিটি কোণে ছড়িয়ে পড়ুক।

(রতন লাল নাথ)

MAHEMATICS WORK BOOK

Class - X

WRITTEN BY :

Dr. Manimoy Paul, Asociate Professor
Smt. Manidipa Sutradhar, Lecturer
Sri Mrinal Kanti Baidya , Teacher
Sri Joydeep Choudhury, Teacher
Sri Siddhartha Sutradhar, Teacher
Sri Pradip Debnath, Teacher
Sri Somen Debnath, Teacher
Sri Prasanta Sarkar, Teacher
Smt. Madhumita Choudhuri, Teacher
Sri. Arnab Kumar Roy, Teacher

PROOF CHECKING & EDITING :

Sri Mrinal Kanti Baidya , Teacher
Sri Joydeep Choudhury, Teacher
Sri Litan Datta, Teacher

MAHEMATICS WORK BOOK

Class - X

CONTENTS

Chapter - 1	Real numbers	**3-8**
Chapter - 2	Polynomials	**9-13**
Chapter - 3	Pair of linear equations in two variables	**14-24**
Chapter - 4	Quadratic equations	**25-31**
Chapter - 5	Arithmetic Progressions	**32-37**
Chapter - 6	Triangles	**38-51**
Chapter - 7	Coordinate geometry	**52-59**
Chapter -8	Introduction to trigonometry	**60-67**
Chapter - 9	Applications of trigonometry	**68-74**
Chapter - 10	Circles	**75-86**
Chapter - 11	Constructions	**87-88**
Chapter - 12	Areas related to circles	**89-97**
Chapter - 13	Surface areas and volumes	**98-106**
Chapter - 14	Statistics	**107-119**
Chapter -15	Probability	**120-124**

CHAPTER-1

REAL NUMBERS

Key points and formulae

- **Euclid's Division Lemma** : Given two positive integers a and b, there exist unique integers q and r satisfying $a = bq + r$, $0 \le r < b$.

Note : Let a and b be two positive integers. If $a = bq + r$, $0 \le r < b$, then HCF $(a,b) =$ HCF (b,r).

- Fundamental Theorem of Arithmetic : Every composite number can be expressed as a product of primes, and this expression (factorisation) is unique, apart from the order in which the prime factors occur.
- For any two positive integers a and b, HCF $(a,b) \times$ LCM $(a,b) = a \times b$.
- Let p be a prime number. If p divides a, then p divides a^2, where a is a positive integer.
- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ... are irrational numbers.
- The sum or difference of a rational and an irrational number is irrational.
- The product or quotient of a non-zero-zero rational number and an irrational number is irrational.
- Let $x = \frac{p}{q}$, p and q are co-prime, be a rational number whose decimal expansion terminates. Then the prime factorisation of q is of the form $2^m.5^n$ where m, n are non-negative integers.
- Let $x = \frac{p}{q}$ be *a* rational number such that the prime facorisation of q is not of the form $2^m.5^n$ where m,n are non-negative integers. Then x has a non-terminating repeating decimal expansion.

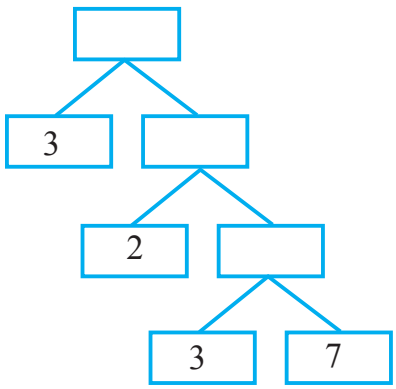
EXERCISE - 1

Group-A (1 mark each)

Very Short Answer type questions :

I. Fill in the blanks :

1. If q is any integer then the form of an odd integer is ——.
2. Fill in the box :



3. The product of a non-zero rational and an irrational number is ———.
4. If $p=a^2b^3c$ and $q=ab^2c^4$ then the LCM (p,q) is _____.
5. The decimal expansion of the rational number $\frac{23}{2^3 \times 5}$ will terminate after ——— decimal place.

II. Multiple Choice Questions :

1. Given positive integers a and 3 there exists unique whole numbers q and r such $a = 3q + r$, where
   (a) $0 < r < 3$ (b) $0 \le r < 3$ (c) $0 < r \le 3$ (d) $0 \le r \le 3$.
2. If the HCF of 46 and 69 is 23m – 115 then the value of m is.
   (a) 3 (b) 4 (c) 5 (d) 6
3. If $a = 2 \times 3^2 \times 5^2$ and $b = 2^2 \times 3 \times 5$ then the HCF of a and b is
   (a) 3 (b) 5 (c) 30 (d) 60
4. The largest number which divides 96 and 120 leaving remainders 8 and 10 respectively is
   (a) 20 (b) 22 (c) 24 (d) 26

5. The LCM and HCF of two numbers are 819 and 13 respectively. If one numbers is 91 then the other number will be

   (a) 117 (b) 26 (c) 52 (d) 130

6. Which of the following rational numbers have terminating decimal?

   (a) $\frac{16}{225}$ (b) $\frac{2}{21}$ (c) $\frac{7}{250}$ (d) $\frac{5}{18}$

7. The decimal expansion of the rational number $\frac{47}{1250}$ will terminate after :

   (a) Four decimal place (b) One decimal place (c) Three decimal place

   (d) Two decimal place.

8. The smallest number by which $\sqrt{8}$ should be multiplied so as to get a rational number is

   (a) $\sqrt{8}$ (b) $2\sqrt{2}$ c) $\sqrt{2}$ (d) 2

9. If n is a natural number, then $7^{2n}–4^{2n}$ is always divisible by

   (a) both 7 and 4 (b) 7 c) 4 (d) both 3 and 11

10. Which of the following is an irrational number?

    (a) 2.17347 (b) $1\cdot7\overline{32}$ c) $\frac{22}{7}$ (d) 1.3131131113...

III. Answer the following questions :

1. If a number is divided by 23 gives 21 as quotient and 17 as remainder, then find the number.
2. If $a = x^2y^3$ and $b = xy^4$, then find the HCF and LCM of a and b.
3. A positive integer n when divided by 7 gives 3 as remainder. What will be the remainder when $(3n–1)$ is divided by 7?
4. If HCF (26, 169) = 13, then LCM (26, 169) = ?
5. What is the least number that is divisible by all the natural numbers from 1 to 10?
6. If p and q are two prime numbers, then what is their HCF?
7. If p and q are two prime numbers, then what is their LCM?
8. Write the exponent of 2 in the prime factorization of 144.

9. Write whether rational number $\frac{7}{75}$ will have terminating decimal expansion or a non-terminating decimal.
10. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

IV. State whether the following statements are true or false :

1. π is an irrational number but $\frac{22}{7}$ is rational.
2. Any number of the form 4^n, where n is a natural number, can end with the digit 0.
3. $3 \times 5 \times 7 + 7$ is a prime number.
4. $4 + \sqrt{3}$ is an irrational number.
5. Every odd integer is of the form $2m - 1$, where m is an integer.

Group - B (2 marks each)

Short Answer Type Questions :

1. Show that $5 + \sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is irrational.
2. Prove that every positive integer is of the form $3q + 1$, q being a natural number.
3. Examine whether $\frac{17}{30}$ is a terminating decimal.
4. Give an example of two irrationals whose sum is rational.
5. The product of two consecutive positive integers is divisible by 2. Is this statement true? Give reasons.
6. The values of the remainder r, when a positive integer a is divided by 3 then the remainder r will be 0 and 1 only. Justify your answer.
7. The numbers 525 and 3000 are both divisible by 3, 5, 15, 25 and 75 only. What is HCF (525, 3000)? Justify your answer.
8. Can two numbers have 14 as their HCF and 230 as their LCM? Give reasons.
9. Show that any number of the form 12^n, where n is a natural number, can never end with the digit 0.

10. Give an example of two irrationals whose difference is rational.

Group-C (3/4 marks each)

Long Answer type questions :

1. Show that the square of an odd positive integer is of the form 8m + 1, for some whole number m.
2. Show that the square of an odd positive integer can be of the form $6q + 1$ or $6q + 3$ for some integer q.
3. Prove that $\sqrt{2} + \sqrt{5}$ is irrational.
4. Find the HCF of 441, 567 and 693.
5. Using Euclid's division algorithm, find the HCF of

   (i) 504 and 1188 (ii) 960 and 1575.
6. Using Euclid's algorithm find HCF of 1190 and 1445. Express the HCF in the form $1190m + 1445n$.
7. For any positive integer n, prove that $n^3 - n$ is divisible by 6.
8. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8
9. Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q.
10. Find the largest number which divides 320 and 457, leaving remainders 5 and 7 respectively.
11. If the HCF of 210 and 55 is expressible in the form $210 \times 5 + 55y$, find y.
12. Find the greatest number of four digits which is exactly divisible by 15, 24 and 36.
13. Find the least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3.
14. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

15. Write the denominator of the rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where *m, n* are non-negative integers. Hence, write its decimal expansion, without actual division.

Answer

Group-A

I. 1. $2q+1$ 2. 126, 42, 21 3. irrational 4. $a^2b^3c^4$ 5. 3

II. 1. (b) 2. (d) 3. (c) 4. (b) 5. (a) 6. (c) 7. (a) 8. (c) 9. (d) 10. (d)

III. 1. 500 2. xy^3; x^2y^4 3. 1 4. 338 5. 2520 6. 1 7. $p \times q$ 8. 4

9. Non-terminating decimal expansion. As denominator $75 = 3\times5^2$ is not of the form $2^m \times 5^n$.

10. 1.5 or 1.6 (Any rational number between 1.41 and 1.73)

IV. 1. True 2. False 3. False 4. True 5. True

Group-B

4. $(2+\sqrt{3})$ and $(2-\sqrt{3})$

5. True, because $n(n+1)$ will always be even, as one out of n or $(n+1)$ must be even.

6. No, because $a=3q+r$, where $0 \le r < 3$. $\therefore$ the value of r can be 0, 1 or 2.

7. HCF = 75, as HCF is the highest common factor.

8. No, because HCF (14) does not divide LCM (230).

10. $(4+\sqrt{3})$ and $(2+\sqrt{3})$.

Group-C

3. Hint : Take $\sqrt{2}+\sqrt{5}=a$, where a is rational. Then $\sqrt{2}=a-\sqrt{5}$ $\therefore 2=a^2+5-2a\sqrt{5} \Rightarrow \sqrt{5} = \frac{a^2+3}{2a}$, which is rational number while is irrational. Hence, $\sqrt{2}+\sqrt{5}$ is irrational.

4. 63. 5. (i) 36, (ii) 15 6. 85; $m=-6, n=5$, 10. 45, 11. –19, 12. 9720, 13. 23,

CHAPTER - 2

POLYNOMIALS

Key points and formulae

- **Polynomial** : An algebraic expression in which the variables involved have only non-negative integral power is called a polynomial.
- **Value of a polynomial at a given point** : If $p(x)$ is a polynomial in x and 'α' is a real number, then the value obtained by putting $x = \alpha$ in $p(x)$ i.e $p(\alpha)$ is called the value of $p(x)$ at $x = \alpha$
- **Zero of a polynomial** : A real number α is said to be zero of the polynomial $p(x)$, if $p(\alpha) = 0$
- **Relationship between the zeros and co-efficients of polynomials :**

i) If $ax+b$ is a given linear polynomial, then zeroes of linear polynomial $ax + b$ is

$$\frac{-b}{a} = \frac{-(\text{constant term})}{\text{co efficient of } x}$$

ii) If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

$$\alpha + \beta = \frac{-b}{a} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} \text{ and } \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

iii) If α , β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(\text{constant term})}{\text{coefficient of } x^3}$$

- **Geometrical meaning of zeroes of a polynomial** : The zeroes of a polynomial $p(x)$ are precisely the x - co-ordinates of the points where the graph of $y = p(x)$ intersects the x-axis.
- **Division Algorithm for Polynomials :** If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$, such that

  $$p(x) = g(x)\, q(x) + r(x)$$

  Where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$.

Note : (i) If $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.

(ii) Dividend = Divisor × Quotient + Remainder.

Exercise 2

Group -A (1 mark each)

Very Short Answer type questions :

I. Fill in the Blanks :

1. Sum of the zeroes of the quadratic polynomial $x^2 - 3x + 2$ is ———
2. If two zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, then the third zero is ——

II. Choose the correct answer from the given options in the following questions :

3. A quadratic polynomial, whose zeroes are –3 and 2, is

   (a) $x^2 - x + 6$ (b) $x^2 + x - 6$ (c) $x^2 - x - 6$ (d) $x^2 + x + 6$

4. The number of polynomials having zeroes 2 and –7 is

   (a) 1 (b) 2 (c) 3 (d) more than 3

5. If –1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then the other zero is

   (a) 1 (b) 9 (c) –9 (d) 7

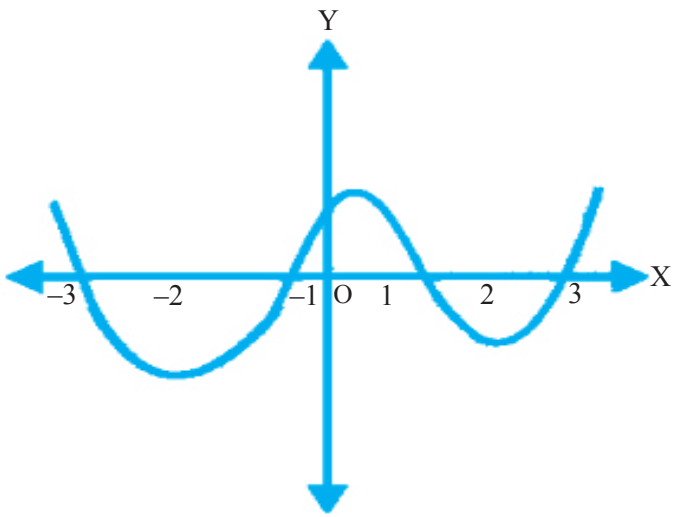
6. If the sum of the zeroes of the quadratic plynomial $3x^2 - kx + 6$ is 3, then the value of k is

   (a) 7 (b) 8 (c) 9 (d) 10

7. $2x^3 - x^2 + 5x + \frac{1}{x} - 3$ is a polynomial. State – True / False

8. $x = -1$ is a zero of $x^2+3x + 2$. State – True / False

9. Find the number of zeroes lying between –2 and 2 of polynomial $f(x)$ whoes graph is given below.



10. What is the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocl to each other?

Group-B (2 marks each)

Short Answer type questions :

1. Can $x + 2$ be the remainder on division of a polynomial $p(x)$ by $3x–2$? Justify your answer.
2. Find all the zeroes of $p(x) = x^2 – x – 2$.
3. If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$. Write the value of $\alpha + \beta + \alpha\beta$
4. If $p(x) = g(x) \cdot q(x) + r(x)$, where degree of $p(x) = 6$ and degree of $p(x) = 3$, then write the degree of $p(x)$.
5. Write the value of m so that $4x^2 – 6x – m$ is exactly divisible by $x–3$.
6. If $f(x)^2 = 5x^2 – 10$ is divided by $x - \sqrt{2}$, then find its remainder.
7. Find the number of real zeroes of a polynomial $p(x)$ whose graph intersects x axis exactly three points.
8. Find the remainder and quotient on dividing $p(x) = 4x^3 + 8x^2 + 8x + 7$ by $g(x) = 2x^2 – x + 1$.

9. Check whether the polynomial $g(x) = x^2 + 3x + 1$ is a factor of the polynomial $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$.
10. Find the values of a and b, if they are zeroes of the polynomial $x^2 + ax + b$.

Group-C (3/4 marks each)

Long Answer type questions :

1. Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$, and verify the relation between the co-efficient and the zeroes of the polynomial .
2. If α and β are the zeroes of the polynomial $x^2 + 8x + 6$, then form a quadratic polynomial $p(x)$, whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
3. Show that $x + 1$ and $2x - 3$ are factors of $p(x) = 2x^2 - 9x^2 + x + 12$.
4. What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x + 6$.
5. Find α and β if $x+1$ and $x + 2$ are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.
6. If 1 and -1 are zeroes of polynomial $px^4 + qx^3 + rx^2 + sx + t$, then show that $p + r + t = q + s = 0$.
7. What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that it is exactly divisible by $x^2 + x - 12$.
8. If $x + a$ is a factor of the polynomial $x^2 + px + q$ and $x^2 + mx + n$, then prove that $a = \frac{n - q}{m - p}$.
9. If α, β , γ are zeroes of the polynomial $kx^3 - 5x + 9$ and $\alpha^3 + \beta^3 + \gamma^3 = 27$, then find the value of k.
10. Using factor theorem factorise $p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$.
11. Verify division algorithm on dividing $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$ by $g(x) = 2x^2 - x + 1$.
12. Find all the zeroes of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if two of its zeroes are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$

13. If α, β are zeroes of quadratic polynomial $kx^2 + 4x + 4$, find the value of k such that $(\alpha+\beta)^2 - 2\alpha\beta = 24$.

14. Find m so that $x^2 + 2x + m$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$.

15. If the remainder on division $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

Answer

Group-A

1. 3 2. $\frac{-b}{a}$ 3. b 4. d 5. a 6. c 7. False 8. True 9. 2 10. $c = a$

Group-B

1. No. 2. –1 and 2 3. –1 4. 3 5. 18 6. 0 7. 3 8. Remainder = $11x + 2$, Quotient = $2x + 5$.
9. $g(x)$ is not a factor of $p(x)$. 10. $a = 1$, $b = -2$.

Group-C

1. Zeroes are $\frac{4}{3}$ and $\frac{-3}{2}$ 2. $p(x) = \frac{1}{6}(6x^2 + 8x + 1)$ 4. $2x + 3$.

5. $\alpha = -1$, $\beta = 0$ 7. $4x - 4$ 9. $k = -1$

10. $p(x) = (x-1)(x-3)(x+3)(2x-5)$ 12. $-\sqrt{\frac{3}{2}}$, $\sqrt{\frac{3}{2}}$, 2, –1

13. $k = -1, \frac{2}{3}$ 14. $m = -3$ 15. $k = -9$; –3, –2, 3.

CHAPTER - 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Key points and formulae

- Two linear equations in the same two variables are said to form a pair of linear equations in two variables.
- The most general form of a pair of linear equation is,

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Where, a_1, a_2, b_1, b_2, c_1, c_2 are real numbers, such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 = 0$

- A pair of linear equation is consistent if it has a solution-either a unique or infinitely many. In case of infinitely many solutions, the pair of linear equations is also said to be dependent.Thus, in this case, the pair of linear equations is dependent and inconsistent.
- A pair of linear equations is inconsistent, if it has no solution.
- Let a pair of linear equations in two variables be $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

I) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then, (i) the pair of linear equaions is consistent (ii) the graph will be a pair of lines intersecting at a unique point; Which is the solution of the pair of equations.

II) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then (i) the pair of linear equaions is inconsistent (ii) the graph will be a pair of parallal lines and so the pair of equations will have no solution.

III) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then, (i) the pair of linear equations is dependent and consistent. (ii) the graph will be a pair of coincident lines, Each point on the lines will be a solution and so the pair of equations will have infinitely many solutions.

- A Pair of linear equations can be solved algebraically by any of the following methods:-
  i) Substitution Method.
  ii) Elimination Method.
  iii) Cross multiplication Method.
- A pair of linear equation can also be solved geometrically/graphically.

IN BRIEF

Pair of lines $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation
$2x + 3y + 4 = 0$ $5x + 6y + 9 = 0$	$\frac{2}{5}$	$\frac{3}{6}$	$\frac{4}{9}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
$x + 2y + 5 = 0$ $3x + 6y + 15 = 0$	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{5}{15}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
$2x - 3y + 4 = 0$ $4x - 6y + 10 = 0$	$\frac{2}{4}$	$\frac{-3}{-6}$	$\frac{4}{10}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

Exercise-3

Group-A (1 mark each)

Very Short Answer type questions :

I. Fill in the Blanks :

1. The pair of equations $5x - 15y = 8$ and $3x - 9y = \frac{24}{5}$ has ____ number of solution(s).
2. The pair of equations $x + y = 7$, $2x - 3y = 9$ has ____ number of solution (s).
3. The pair of eqiations $3x - 5y + 4 = 0$ and $9x = 15y - 12$ has ____ number of solutions (s).
4. The pair of eqiations $x + 4y = 7$ and $2x - y = 5$ has ——— solutions.
5. The pair of eqiations $3x - 2y = 7$ and $2x + y = 7$ has ——— solutions.

6. The pair of eqiations $-x + 2y + 2=0$ and $\frac{1}{2}x-\frac{1}{4}y-1=0$ has ——— solutions.

7. Without drawing the graph, identify the lines representing the pair of linear equations $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$ are ———

8. The lines representing the pair of linear equations $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$ are ———

9. The lines representing the pair of linear equations $6x-3y+10 = 0$ and $2x-y+9 = 0$ are ———

10. The lines representing the pair of linear equations $a_1x+b_1y+c_1= 0$ and $a_2x+b_2y+c_2 = 0$ are intersecting lines if ———

11. The lines representing the pair of linear equations $a_1x+b_1y+c_1= 0$ and $a_2x+ b_2y+c_2 = 0$ are parallel if ———

12. The linear pair of equations $(k-1)x+3y = 7$ and $(k+1)x+6y = (5k-1)$ has infinitely many solutions, then the value of k = ———

13. The linear pair of equations $8x+5y = 9$ and $kx+10y = 8$ have no solutions. Find the value of k = ———

14. The eqiations $2x+3y = 7$ and $2\alpha x+(\alpha + \beta)y = 28$ have infinitely many solutions. The value of α & β are ——, —— respectively.

II. MCQ Type

1. The pair of equations $5x-15y = 8$ and $3x- 9y = \frac{24}{5}$ has ———

   (a) one solution (b) two solutions (c) infinitely many solutions (d) no solution

2. For what value of k, do the equation $2x+ ky = 1$ and $3x+5y = 7$ have a unique solution.

   (a) $k \neq -\frac{10}{5}$ (b) $k \neq -\frac{5}{3}$ (c) $k \neq \frac{-20}{3}$ (d) $k \neq -\frac{1}{3}$

3. For what value of k, do the equations $3x-y+ 8=0$ and $6x-ky = -16$ represent coincident lines ?

   (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) –2

4. If the lines given by $3x + 2\gamma x = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of γ is ———

(a) $\frac{-5}{4}$ (b) $\frac{2}{5}$ (c) $\frac{15}{4}$ (d) $\frac{3}{2}$

5. If the given equations $kx + 2y - 1 = 0$ and $5x - 3y + 2 = 0$ have no solution then, the value of k is–

(a) $k = -\frac{17}{3}$ (b) $k = -\frac{13}{3}$ (c) $k = -\frac{11}{3}$ (d) $k = -\frac{10}{3}$

6. $2x + 3x - 5 = 0$ and $kx - 6y - 8 = 0$ have unique solution then the value of k is –

(a) $k \neq -1$ (b) $k \neq -2$ (c) $k \neq -3$ (d) $k \neq -4$

7. The pair of equations $4x - 6 = 2y$ and $2x = y + 3$ have –

(a) a unique solution (b) exactly two solutions (c) infinitely many solutions

(d) no solution

8. The pair of equations $x - 2y = 2$ and $4x - 2y = 5$ have –

(a) one solution (b) two solutions (c) infinitely many solutions (d) no solution

9. The pair of equations $x = a$ and $y = b$ graphically represents lines which are–

(a) Parallel (b) intersecting at (b,a) (c) Coincident (d) intersecting at (a,b)

10. If a pair of linear equations is consistent, then the lines will be

(a) Parallel (b) always coincident (c) intersecting or coincident

(d) always intersecting

11. A pair of linear equations which has a unique solution $x = 2$, $y = -3$ is–

(a) $x + y = -1$ (b) $2x + 5y = -11$ (c) $2x - y = -1$ (d) $x - 4y - 14 = 0$

$2x - 3y = -5$ $4x + 10y = -22$ $3x + 2y = 0$ $5x - y + 13 = 0$

12. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is–

(a) $-5/4$ (b) $\frac{2}{5}$ (c) $\frac{15}{4}$ (d) $\frac{3}{2}$

13. The value of k for which the pair of equations $kx - y = 2$ and $6x - 2y = 4$ will have infinitely many solutions is–

(a) 3 (b) –3 (c) –12 (d) no value.

14. If $x = a, y = b$ is the solution of the equations $x–y = 2$ and $x+y = 4$, then the values of a and b , respectively.

(a) 3 and 5 (b) 5 and 3 (c) 3 and 1 (d) –1 and –3

15. For what value of k, the given linear pair of equations $2x+ky = 11$ and $5x–7y = 5$ has no solution.

(a) $k = \frac{-13}{5}$ (b) $k = \frac{-14}{5}$ (c) $k = -\frac{17}{5}$ (d) $k = -\frac{19}{5}$

16. The given equation $3x + 4y = 12$ and $(m+n)\,x + 2\,(m–n)\,y = 5m – 1$ have infinetely many solutions. Then m and n will be–

(a) $m = 4, n = 1$ (b) $m = 5, n = 1$ (c) $m = 1, n = 5$ (d) $m = 1, n = 4$

17. The given equations $(a–1)\,x + 3y = 2$ and $6x + (1–2b)\,y = 6$, find a and b, when they have infinetely many solutions–

(a) $a = 3, b = –4$ (b) $a = –4, b = 3$ (c) $a = 4, b = –3$ (d) $a = –4, b = –4$

18. The graphs of the equations $2x+3y = 2$ and $x–2y = 8$ are two lines which are–

(a) coincident (b) parallel (c) intersecting exactly at one point (d) none of these

19. For what value of k, do the equations $kx–2y = 3$ and $3x+y = 5$ represent two lines intersecting at a unique point?

(a) $k = 3$ (b) $k = –3$ (c) $k = 6$ (d) all real values except –6.

20. If $\frac{2}{x}+\frac{3}{y}=6$ and $\frac{1}{x}+\frac{1}{2y}=2$ then–

(a) $x=1, y=\frac{2}{3}$ (b) $x=\frac{2}{3}, y=1$ (c) $x=1, y=\frac{3}{2}$ (d) $x=\frac{3}{2}, y=1$

21. In a ΔABC, $\angle C=3\ \angle B=2\ (\angle A+\angle B)$, then $\angle B=?$

(a) 20° (b) 40° (c) 60° (d) 80°

22. The graphs of the equations $5x – 15y = 8$ and $3x – 9y = \frac{24}{5}$

(a) concident (b) parallel (c) intersecting exactly in one point (d) none of these

III. Answer the following questions :

1. What is the graphical representation of the equations $x+2y=3$ and $2x+4y+7=0$?

2. How many solutions are there for the pair of equations $2x+5y=5$, $3x+2y+8$?
3. For what value of k does the pair of linear equations $10x+5y-(k-5)=0$ and $20x+10y-k=0$ have infinitely many solutions?
4. Write the number of solutions of the pair of linear equations $x+3y-4=0$ and $2x+6y-7=0$?
5. Write the number of solutions of the equations $x+2y-8=0$ and $2x+4y=16$?
6. For what value of k, the linear equations $3x+ky=0$ and $2x-y=0$ has a unique solution?
7. Find the value of k for which the system $3x+5y=0$, $kx+10y=0$ has a non zero solution?
8. Find k for which the system $x+2y=3$ and $5x+ky+7=0$ is inconsistent?
9. Find k, for which $kx-y=2$ and $6x-2y=3$ has a unique solution?.
10. For what value of k, the equations $kx+3y=k-2$ and $12x+ky=k$ is inconsistent?

Group-B (2/3 marks each)

Short Answer type questions :

1. Show that the system of equations $-x+2y+2=0$ and $\frac{1}{2}x-\frac{1}{2}y-1=0$ has a unique solution.
2. Show that the equations $9x-10y=21$ and $\frac{3x}{2}-\frac{5y}{3}=\frac{7}{2}$ has infinitely many solutions?
3. Show that the paths represented by the equations $x-3y=2$ and $-2x+6y=5$ are parallal.
4. Show that the system $2x+3y=1$ and $4x+6y=4$ has no solution?
5. Find the condition that the system of equations $ax+by+c=0$, $lx+my+n=0$ has no solution?
6. Show that, $4x+y=5$, $12x+3y=10$ equation are in consistent?
7. Find the value of k for which the system of equations $4x+5y=0$, $kx+10y=0$ has infinitely many solutions?
8. For what value of k, the equations $x+2y=3$; $(k-1)x+(k+1)y=k+2$ has an infinite number of solutions?
9. Show that, $2x+4y=10$, $3x+6y=12$ has no solution.
10. Show that the system of equations $2x+7y=11$ and $5x+\frac{35}{2}y=25$ is inconsistent.

11. Two numbers differs by 3 and their product is 54. Find the numbers.

12. For what value of k will the following systems of equations have a unique solution–

    i) $x-2y=3$ and $3x+ky=1$ ii) $2x+5y=7$ and $3x-ky=5$

13. For what value of k will the following systems of equations have infinitely many solution–

    i) $3x+2y=1$ and $(2k+1)x+(k+2)y=k-1$

    ii) $x+(k+1)y=5$ and $(k+1)x+9y=8k-1$

14. Find the conditions so that the following systems of equations have infinitely many solution:

    i) $2x+3y=7$ and $(p+q)x+(2p-q)y=3(p+q+1)$ find p and q .

    ii) $3x+4y=12$ and $(m+n)x+2(m-n)y=5m-1$ find m and n .

    iii) $2x-(2a+5)y=5$ and $(2b+1)x-9y=15$, find a and b .

15. For what value of k the following systems of equations have no solution.

    i) $x-4y=6$ and $3x+ky=5$

    ii) $2x+ky=1$ and $3x-5y=7$

    iii) $4x+6y=11$ and $2x+ky=7$

16. Show that the following systems of equations are inconsistent.

    i) $3x-5y=20$, $6x-10y=-40$ (ii) $2y-x=9$, $6y-3x=21$ (iii) $4x+y=5$, $12x+3y=10$

Group-C (3/4 marks each)

Long Answer type questions :

1. Verify whether $x=2, y=1$ and $x=1, y=2$ are the solutions of the linear equation $2x+y=5$ Find two more solutions.
2. Draw the graph of the equation $3x+7y=10$. Find whether $x=1, y=1$ is the solution of this equation .
3. Solve : $6x+3y=6xy$, $2x+4y=5xy$.
4. Draw the graph of the equation $3x-2y+6=0$. Find, whether the points (2,6) and (1,2) lie on this graph or not.

5. Show that $x=3, y=2$ is not a solution of the system of linear equations $3x-2y=5$, $2x+y=7$.

6. Solve the system of equations graphically : $4x-5y=20$ and $3x+5y-15=0$

   Determine the vertices of the triangles formed by the lines representing the above equations and the y-axis.

7. Show graphically that the system of linear equations $x-y=8$, $3x-3y=16$ are inconsistent, ie it has no solution.

8. Solve graphically : $\left.\begin{aligned} 2x+3y=8 \\ x-2y+3=0 \end{aligned}\right\}$

9. Solve each of the following system of equations graphically and find the vertices and area of the triangle formed by these lines and x-axis–

   i) $2x-3y+4=0$ and $x+2y-5=0$

   ii) $x-y+1=0$, $3x+2y-12=0$

   iii) $4x-3y+4=0$, $4x+3y-20=0$

10. Show graphically that each of the following system of equations is inconsistent or has no solution.

    i) $x-2y=6$; $3x-6y=0$

    ii) $2x+y=6$; $6x+3y=20$

11. Solve for x and y; using substitution method

$$\frac{3x}{2}-\frac{5y}{3}=-2\,;\quad \frac{x}{3}+\frac{y}{2}=\frac{13}{6}$$

12. Solve for x and y of the following set of equations–

i) $\begin{aligned} x+y&=3 \\ 4x-3y&=26 \end{aligned}$

ii) $\begin{aligned} \frac{x}{3}+\frac{y}{4}&=11 \\ \frac{5x}{6}-\frac{y}{3}&=-7 \end{aligned}$

iii) $\begin{aligned} \frac{x}{2}-\frac{y}{9}&=6 \\ \frac{x}{7}+\frac{y}{3}&=5 \end{aligned}$

iv) $\begin{aligned} \frac{5}{x+y}-\frac{2}{x-y}&=-1 \\ \frac{15}{x+y}+\frac{7}{x-y}&=10 \end{aligned}$

v) $\begin{aligned} x+y&=a+b \\ ax-by&=a^2-b^2 \end{aligned}$

vi) $\begin{aligned} \frac{bx}{a}+\frac{ay}{b}&=a^2+b^2 \\ x+y&=2ab \end{aligned}$

vii) $0\cdot4x+0\cdot3y=1\cdot7$
$0\cdot7x-0\cdot2y=0\cdot8$

viii) $\frac{5}{x-1}+\frac{1}{y-2}=2$
$\frac{6}{x-1}-\frac{3}{y-2}=1$

ix) $px+qy=p-q$
$qx-py=p+q$

13. Solve each of the following system of equation by using cross multiplication Method :

i) $x+2y+1=0$
$2x-3y-12=0$

ii) $3x+2y+25=0$
$2x+y+10=0$

iii) $7x-2y=3$
$22x-3y=16$

iv) $\frac{x}{6}+\frac{y}{15}=4$
$\frac{x}{3}-\frac{y}{12}=\frac{19}{4}$

v) $\frac{1}{x}+\frac{1}{y}=7$
$\frac{2}{x}+\frac{3}{y}=17 \ (x\neq0, y\neq0)$

vi) $4x-7x+28=0$
$5y-7x+9=0$

14. Find the value of k for which the system of equations $5x–3y = 0$, $2x+ky = 0$ has a non zero solution.

15. For what value of k does the system of equations $x+2y = 3$, $5x+ky+7 = 0$ have

i) a unique solution ii) no solution

16. The monthly income of A and B are in the ratio 8:7 and their expenditure are in the ratio 19:16. If each saves R5000/- per months. Find the monthly income of each.

17. On selling a TV at 5% gain and a fridge at 10% gain, a shopkeper gains Rs. 3250/-. But if the sells the TV at 10% gain and the fridge of 5% loss, he gains Rs, 1500/-. Find the actual cost of TV and that of the fridge.

18. The sum of two numbers is 8 and the sum of their reciprocals is $8/15$. Find the numbers.

19. A two digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number?

20. A fraction becomes $\frac{1}{3}$ if 2 is added to both of its numerator and denominetor. If 3 is added to both of its numerater and denominator then it becomes $2/5$. Find the fraction.

21. Places A and B are 100 km a part on a highway. One Car starts from A and another from B at the same time. If the Cars travel in the same direction at different speeds they meet in 5 hours. If they travels towards each other, they meet in 1 hour. What are the speeds of the two cars?

22. Solve the following equations by the method of elimination by equating the Co-efficients–

i) $12x+5y=17,\ 7x-y=6.$

ii) $x+\frac{y}{2}=4,\ \frac{x}{3}+2y=5$

iii) $\frac{2x}{a}+\frac{y}{b}=2,\ \frac{x}{a}-\frac{y}{6}=4.\ (a\neq 0,\ b\neq 0)$

iv) $2^x+3^y=17,\ 2^{x+2}-3^{y+1}=5$

v) $6x+3y=8x+9y-5=10x+12y-8$

Answer

Group-A

I. 1) Infinitely many 2) Unique 3) Infinitely many 4) Unique 5) Unique

6) Unique 7) Intersecting 8) Coincide. 9) Parallel 10) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

11) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ 12) 3 13) 16 14) $a=3,\ b=-4$

II. 1) c 2) a 3) c 4) c 5) d 6) d 7) c 8) a 9) d 10) d 11) b 12) c
13) a 14) c 15) b 16) b 17) a 18) c 19) d 20) b 21) b 22) a

III. 1) Parallel lines 2) a uniue solution 3) $k=10$ 4) 0 5) infinitely many

6) $k\neq\frac{-3}{2}$ 7) $k=6$ 8) $k=\frac{2}{5}$ 9) $k\neq 3$ 10) $k=\pm 6$

Group-B

5. $\frac{a}{l}=\frac{b}{m}\neq\frac{c}{n}$ 7. $k=8$ 8. $k=3$ 11. (9,6) or (–9, –6) 12. (i) $k\neq -6$

(ii) $k \neq \frac{-15}{2}$ 13. (i) $k = 4$ (ii) $k = 2$ 14. (i) $p = 5, q = 1$ (ii) $m = 5, n = 1$

(iii) $a = -1, b = 5/2$ 15. (i) $k = -12$ (ii) $k = \frac{-70}{3}$ (iii) $k = 3$

Group-C

1. $x = 2, y = 1$ is the solution, but $x=1, y=2$ is not the solution. Other solutions are $x=3$, $y = -1$, and $x=1, y=3$ 2. Yes. 3. $x=1, y=2$ 4. Point (1,2) does not lie on the line, while (2,6) lie on it. 6. $x=5, y=0$ & the vertices are (0, –4), (0,3) & (5,0) 8. $x=1, y=2$ 9. (i) $x=1, y=2$; A (1,2), B (–2,0), C (5,0) Ar (ΔABC) = 7sq. units. (ii) $x=2, y=3$; A (2,3), B (–1, 0), C (4,0), Ar (ΔABC) = 7.5 sq. unit.

(iii) $x=2, y=4$; A (2,4), B (–1, 0) & C (5,0), Ar (ΔABC) = 12 sq. unit.

11. $x=2, y=3$ 12. (i) $x=5, y=-2$ (ii) $x=6, y=36$ (iii) $x=14, y=9$

(iv) $x=3, y=2$ (v) $x=a, y=b$ (vi) $x=ab, y=ab$ (vii) $x=2, y=3$

(viii) $x=4, y=5$ (ix) $x=1, y=-1$

13. (i) $x=3, y=-2$ (ii) $x=5, y=-20$ (iii) $x=1, y=2$ (iv) $x=18, y=15$

(v) $x = \frac{1}{4}, y = \frac{1}{3}$ (vi) $x=7, y=8$. 14. $k = -\frac{6}{5}$ 15. (i) $k \neq 10$ (ii) $k = 10$

16. A's income = Rs. 24,000; B's income = Rs. 21,000

17. The CP of a TV = Rs. 25,000 & the CP of a fridge = Rs. 20,000

18. 5 and 3. 19. 27 20. $1/7$ 21. Speed of the Car from A= 60km/hr.

Speed of the Car from B = 40 km / hr. 22. (i) $x=1, y=1$ (ii) $x=3, y=2$

(iii) $x=2a, y=-2b$ (iv) $x=3, y=2$ (v) $x = \frac{1}{2}, y = \frac{2}{3}$

CHAPTER-4

QUADRATIC EQUATIONS

Key points and formulae

- **Quadratic equation :** An equation of the form $ax^2 + bx + c = 0$, Where a, b, c are real numbers and, $a \neq 0$, is called a quadratic equation in x.
- **Roots of a quadratic equation :** A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$
- The roots of a quadratic equation $ax^2 + bx + c = 0$ are called the zeros of the polynomial $ax^2 + bx + c$.
- **Methods of solving quadratic equations :**

  i) Factorisaion Method (splitting the middle term)

  ii) Completing the square Method.

  iii) Quadratic Formula Method (Sridharacharya formula)

- **Finding the roots of a quadratic equation by the method of factorisation :**

  If we can factorise the quadratic polynomial $ax^2 + bx + c$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating to zero the linear factors of $ax^2 + bx + c$.

- **Finding the roots of a quadratic equation by the method of completing the square :** By adding and subtracting a suitable constant, we club the x^2 and x terms in the quadratic equation so that they become a complete square, and solve for x.
- **Finding the roots of a quadratic equation by Quadratic Formula :**

  If the Discriminant $D = b^2 - 4ac \geq 0$, then the real roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

- **Nature of Roots of Quadratic Equation :** A quadratic equation $ax^2+bx+c = 0$ has

i) real and unequal roots if $D > 0$ ie, $b^2–4ac > 0$

ii) real and equal roots if $D = 0$ ie, $b^2–4ac = 0$

iii) imaginary roots if $D < 0$ ie, $b^2–4ac < 0$.

Exercise 4

Group-A (1 mark each)

Very Short Answer questions :

I. Fill in the blanks :

1. For the equation $x^2 +10x + 24 = 0$, one rote is – 4. The other root is ——.
2. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has —— real roots.
3. If the discriminant of a quadratic equation is zero, then the roots are ____ and ____.
4. If the quadratic equation $x^2 –2x + k = 0$ has equal roots, then the value of k is ____.
5. If the roots of $px^2 +qx + 2 = 0$ are reciprocal to each other, then the value of p is____.
6. The value of p, for which one root of the quadratic equation $px^2 – 14x + 8 = 0$ is 6 times the other is ____.

II. Write True / False of the following statements :

1. $x^3 –x^2 = (x–1)^3$ is a quadratic equation.
2. A quadratic equation has always real roots.
3. The sum of two roots of the quadratic equation $4x^2 – 12x + 9 = 0$ is –3.
4. The discriminant of the quadratic equation $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$ is 32.
5. If the quadratic equation $3x^2 + 2x + k = 0$ has real roots, then the value of k is greater than equal to $\frac{1}{3}$.

III. Choose the correct answer from the given four options in the following questions:

1. Which of the following is a quadratic equation?

a) $2x^2 = (5+x)\left(2x - \frac{3}{5}\right)$ b) $x^3 - x^2 = (x-2)^3$

c) $x^2+2x=(4-x)^2+3$ d) $(k+1)x^2+\frac{3}{2}x=8$, where $k \neq -1$

2. Which of the following is not a quadratic equation?

a) $3(x-1)^2=4x^2-2x+1$ b) $2x-x^2=x^2+5$

c) $\left(\sqrt{2}x+\sqrt{3}\right)^2+x^2=3x^2-5x$ d) $\left(x^2+2x\right)^2=x^4+3+4x^3$

3. Which of the following equation has no real roots?

a) $3x^2+4\sqrt{3}x+4=0$ b) $x^2-4x-3\sqrt{2}=0$

c) $x^2+4x=3\sqrt{2}$ d) $x^2-4x+3\sqrt{2}=0$

4. $(x^2+1)^2-x^2=0$ has

a) two real roots b) no real roots

c) four real roots d) one real roots

5. If the sum of the roots of the equaion $kx^2+2x+3k=0$ is equal to their product then the value of k is.

a) $\frac{1}{3}$ b) $\frac{-1}{3}$ c) $\frac{2}{3}$ d) $\frac{-2}{3}$

6. The roots of quadratic equation $5x^2-4x+5=0$ are

a) Real and equal b) Real and unequal

c) Not real d) Non-real and equal.

7. Which constant should be added and subtracted to solve the quadratic equation $4x^2-\sqrt{3}x+5=0$ by the method of completing the square?

a) $\frac{9}{16}$ b) $\frac{3}{64}$ c) $\frac{3}{4}$ d) $\frac{\sqrt{3}}{4}$

8. If $\frac{1}{2}$ is a root of the equation $x^2+kx-\left(\frac{5}{4}\right)=0$ then the value of k is

a) 2 b) –2 c) 3 d) –3

9. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

   a) 3 b) 8 c) 4 d) 7

10. The value of $\sqrt{6+\sqrt{6+\sqrt{6+}}}$ ..... is

    a) 3.5 b) 4 c) 3 d) –3

11. If $x^2 (a^2+b^2) + 2x (ac + bd) + c^2+d^2=0$ has no real roots, then

    a) $ad = bc$ b) $ad < bc$ c) $ad > bc$ d) all of these

12. If the one root of the equation $4x^2 – 2x + p – 4 = 0$ be the reciprocal of other, then value of p is

    a) 8 b) –8 c) –4 d) 4

IV. Answer the following questions :

1. Find the number of real roots of the equation $(x + 2)^2 – x^2 = 0$
2. Find the value of k for which $x=1$ is a root of the equation $x^2 + kx + 3 = 0$.
3. Find the nature of the roots of the quadratic equation $4x^2 – 5x + 3 = 0$.
4. Is 0.1 a root of the equation $x^2 – 0.1= 0$? Justify.
5. State whether the following quadratic equations have two distinct real roots. Justify your answer.

   a) $x^2 – 6x + 6 = 0$ b) $3x^2 - 2\sqrt{6}x + 2 = 0$

   c) $x (1–x) – 2 = 0$ d) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

Group-B (2 marks each)

Short Answer type questions :

1. Solve the following quadratic equations by using factorisation method.

   a) $3x^2 – 243 = 0$ b) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

c) $3x^2+5\sqrt{5}x-10=0$ d) $\sqrt{7}x^2-6x-13\sqrt{7}=0$

2. Solve the following quadratic equations by using the quadratic formula :

a) $3x^2-2x=21$ b) $-x^2+7x-10=0$

c) $\frac{1}{2}x^2-\sqrt{11}x+1=0$ d) $10x-\frac{1}{x}=3$

Group-C (3/4 marks each)

Long Answer type questions :

1. Solve for x : $\frac{1}{x-3}-\frac{1}{x+5}=\frac{1}{6},\ x\neq 3,\ -5$

2. Solve for x : $\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)\ (2x+3)}=0,\ x\neq 3,\ -\frac{3}{2}$

3. Solve for x : $\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2};\ x\neq 1,\ 2,\ -2$

4. Solve for x : $x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$

5. Solve for x : $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3},\ x\neq 1,2,3$

6. Solve for x : $4x^2-4a^2x+a^4-b^4=0$

7. If the roots of the quadratic equation $x^2+2px+mn=0$ are real and equal, show that the roots of the quadratic equation $x^2-2(m+n)x+(m^2+n^2+2p^2)=0$ are also equal.

8. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

9. If a natural number is increased twice of it then it becomes 12 times of its reciprocal. Find the number.

10. A train, travelling of a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed for the train.

11. If Shely were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

12. At t minutes past 2 pm, the time needed by the minutes hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

13. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

14. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with larger diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which tap can fill the tank separately.

15. A two digit number is such that the product of the digits is 14. When 45 is added to the number, then the digits are reversed. Find the number.

16. In a class test, the sum of the marks obtained by X in Mathematics and Science is 28. Had he got 3 marks more in mathemtics and 4 marks less in science, the product of their marks would have been 180. Find his marks in the two subjects.

17. A shopkeeper buys a number of pens for Rs. 480. If he had bought 4 more pens for the same amount, each pen would have cost Rs. 4 less. How many pens did he buy.

18. The speed of a boat in still water is 8km/hr. It can go 15 km upstream and 22 km down stream in 5 hours. Find the speed of the stream.

19. The area and perimetes of a rectangular land are 2000 sq.m. and 180 m respectively. Find the length and breadth of the rectangular land.

20. Determine two consecutive multiples of 3 where product is 270.

Answer

Group-A

I. (1) -6 (2) 0 (3) real, equal (4) 1 (5) 2 (6) 3

II. (1) T (2) F (3) F (4) T (5) F

III. (1) b (2) c (3) b (4) b (5) d (6) c (7) b (8) a (9) b (10) c (11) d (12) a

IV. (1) 1 (2) $k = -4$ (3) No real roots. (4) No (5) a. No b. Yes c. No d. No.

Group-B

1. a) ± 9 b) $-\frac{1}{2}, 3$ c) $-2\sqrt{5}, \frac{\sqrt{5}}{3}$ d) $-\sqrt{7}, \frac{13}{\sqrt{7}}$

2. a) $-\frac{7}{3}, 3$ b) 5, 2 c) $\sqrt{11}-3, \sqrt{11}+3$ d) $\frac{1}{2}, -\frac{1}{5}$

Group-C

(1) $-9, 7$ (2) -1 (3) $-5, \frac{6}{5}$ (4) $\frac{-a}{a+b}, \frac{-(a+b)}{a}$ (5) 0, 4

(6) $\frac{a^2+b^2}{2}, \frac{a^2-b^2}{2}$ (8) 12 (9) 2 (10) 45 km/h (11) 14 years

(12) t = 14 minutes (13) 13 and 15 (14) Smaller tap 5 hours and larger tap 3 hours. (15) 27 (16) Mathematics 9 marks, Science 19 marks or Mathematics 12 marks, Science 16 marks (17) 20 nos. (18) 3km/hr (19) 50 metre and 40 metre

(20) 15, 18

CHAPTER - 5

ARITHMETIC PROGRESSIONS

Key points and formula

- **Arithmetic Progression (AP) :** It is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. This fixed number is called common difference, denoted by 'd'. It can be positive or negative.

  The general form of an AP is given by

  $a,\ \ a+d,\ \ a+2d,\ \ a+3d,$ .....

- The n^{th} term or general term of an AP is $a_n = a + (n-1)\,d$, where a is the first term and d is the common difference.

- The n^{th} term from the end of an AP is $a_n = l - (n-1)\,d$, where l is the last term of this AP.

- To an AP if we add, subtract, multiply or divide each term by the same number, then the resulting sequence would always be an AP.

- It is always convenient to make a choice of

  i) 3 numbers in AP as $(a-d),\ a,\ (a+d)$

  ii) 4 numbers in AP as $(a-3d),\ (a-d),\ (a+d),\ (a+3d)$

  iii) 5 numbers in AP as $(a-2d),\ (a-d),\ a,\ (a+d),\ \ (a+2d)$

- The sun S_n of the first n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

  If l is the last term of an AP of n terms, then sum of all the terms can also be given by

$$S_n = \frac{n}{2}(a + l)$$

- If s_n is the sum of the first n terms of an AP, then its n^{th} term, $a_n = s_n - s_{n-1}$.

Exercise 5

Group-A (1 mark each)

Very Short Answer type questions :

I. Fill in the blanks :

1. The 6^{th} term from the end of the AP 17, 14, 11, ... (–40). is ____.
2. The n^{th} term of two A.Ps 63, 65, 67, ... and 3, 10, 17..... are equal, then value of n is _____.
3. The common difference of A.P $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}$,..... ($a \neq 0$) equals to _____.
4. If s_n, the sum of first n terms of an A.P is given by $s_n = 3n^2 - 4n$, The common difference is ____.
5. Sum of first n natural number is _____.
6. The sun of first five multiple of 3 is _____.
7. The famous mathematician associated in with finding the sum of the first 100 natural numbers is ___.

II. Choose the correct answer from the given four options :

1. The 4th term from the end of the AP –11, –8, –5, ..., 49 is.

   i) 37 ii) 40 iii) 43 iv) 58
2. The first, second and last terms of an AP are respectively 4, 7 and 31. How many terms are there in the given AP?

   i) 10 ii) 12 iii) 8 iv) 13
3. The 4^{th} and 12^{th} term of an AP are 14 and 70. What is the first term?

   i) –10 ii) –7 iii) 7 iv) 10
4. In an AP, if $d = -4$, $n = 7$, $a_n = 4$, then a is

   i) 6 ii) 7 iii) 20 iv) 28
5. The 21st term of the AP whose first two terms are –3 and 4 is

   i) 17 ii) 137 iii) 143 iv) –143
6. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

   i) 5 ii) 20 iii) 25 iv) 30

7. If the numbers n–2, $4n$–1 and $5n + 2$ are in AP, then the value of n *is*

   i) 1 ii) 2 iii) –1 iv) –2

8. If the n^{th} term of an A.P –1, 4, 9, 14, ... is 129. Find the value of n.

   i) 25 ii) 24 iii) 22 iv) 27

9. Which term of the A.P 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$ .... is the first negative term :

   i) 27^{th} term ii) 28^{th} term iii) 26^{th} term iv) 25^{th} term

10. The list of numbers –10, –6, –2, 2 ..... is

    i) an AP with $d = -16$ ii) an AP with $d = 4$

    iii) an AP with $d = -4$ iv) not an A.P

III. Write True / False of the following statements :

1. The common differrence of an AP is always positive.

2. The common difference of the AP $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}$, ... is $-b$.

3. If 4, x_1, x_2, x_3, 28 are in AP then $x_3 = 22$

4. The sum of first n terms of an AP is $3n^2 + 6n$. Then the common difference of this AP is 9.

5. The number of two digit numbers divisible by 3 is 25.

IV. Answer the following questions :

1. Write the sequence with n^{th} term is $9–5n$.

2. Which term of the AP 21, 42, 63, 84, ... is 420 ?

3. Is 0 a term of the AP 31, 28, 25, ...?

4. Find the sum of first 16 terms of the AP 10, 6, 2, ...

5. Find the 12^{th} term from the end of the AP, –2, –4, –6, ... –100.

6. $a_1, a_2, a_3 \ldots\ldots a_n$ are in AP and $\frac{a_8}{a_6} = \frac{5}{4}$ then find $\frac{a_7}{a_4}$.

Group-B (2 marks each)

Short Answer type Questions :

1. Find a, b and c such that the following numbers are in AP : a, 7, b, 23, c.
2. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.
3. If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.
4. Find whether 55 is a term of the AP : 7, 10, 13, ... or not. If yes, find which term it is.
5. The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.
6. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.
7. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$, are there consecutive term of an AP.
8. The angles of triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.
9. Which term of the AP : 53, 48, 43, ... is the first negative term?
10. How many numbers lie betwen 10 and 300, which when divided by 4 leave a remainder 3?
11. In an AP, if $s_n = 4(4n+1)$, find the AP.
12. Find the sum of last ten terms of the AP : 8, 10, 12, ... 126.

Group-C (3/4 marks each)

Long Answer Type Questions :

1. How many terms of the AP : –15, –13, –11, ... are needed to make the sum –55? Explain the reason for double answer.
2. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first $2n$ terms of another AP whose first term is –30 and the common difference is 8. Find n.
3. Rehana was given her pocket money on March 1st, 2019. She puts Re 1 on Day 1, Rs. 2 on Day 2, Rs 3 on Day 3 and continued doing so fill the end of the month, from this

money into her piggy bank. She also spent Rs. 204 of her pocket money, and found that at the end of the month she still had Rs. 100 with her. How much was her pocket money for the month?

4. Suniti saves Rs. 32 during the first month, Rs. 36 in the second month and Rs. 40 in the third month. If she continues to save in this manner, in how many months will she save Rs. 2000?

5. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last terms to the product of the two middle terms is 7:15. Find the numbers.

6. Solve the equation

$$1+4+7+10 + ... + x = 287$$

7. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP in 235, find the sum of its first twenty terms.

8. Find the sum of those integers from 1 to 500 which are multiples of 2 or 5.

9. The ratio of the sums of first m and first n terms of an AP is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} term is $(2m–1) : (2n–1)$

10. If the p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$. Prove that the sum of first pq terms of the AP is

$$\left[\frac{pq+1}{2}\right]$$

11. If the ratio of the sums of first n terms of two AP's is $(7n+1) : (4n+27)$, find the ratio of their 9th terms.

12. In an AP, prove that the sum of the terms equidistant from the beginning and the end is always the same and equal to the sum of the first term and last term.

13. If the sum of first n, $2n$ and $3n$ terms of an AP be s_1, s_2 and s_3 and respectively, then prove that

$$s_3 = 3\ (s_2 – s_1)$$

14. The ratio of the 11^{th} term to the 18^{th} term of an AP is 2:3. Find the ratio of the 5^{th} term to the 21^{st} term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

15. Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to

$$\frac{(a+c)(b+c-2a)}{2(b-a)}$$

Answer

Group-A

I. 1) –25 2) 13 3) $-\frac{1}{3}$ 4) 6 5) $\frac{n(n+1)}{2}$ 6) 45 7) Gauss.

II. 1. (ii) 2. (i) 3. (ii) 4. (iv) 5. (ii) 6. (iii) 7. (i) 8. (iv) 9. (ii) 10. (ii)

III. 1. F 2. T 3. T 4. F 5. F

IV. 1. 4, –1, –6, –11, ... 2. 20th terms 3. No 4. –320 5. –78 6. $\frac{3}{2}$

Group-B

1. $a = -1$, $b = 15$, $c = 31$ 2. 3, 7, 11, 15, ... 4. yes, 17^{th} term 5. 1, 6, 11, 16, ...

6. 126 7. $k = 0$ 8. 40^o, 60^o, 80^o 9. 12^{th} term 10. 73

11. 5, 13, 21, ... 12. 1170

Group-C

1. $n = 5, 11$ 2. 11 3. Rs. 800 4. 25 month 5. 2, 6, 10, 14 6. 40

7. 970 8. 75250 11. 24:19 12. 16, $\frac{8}{3}$ 14. 1:3; 5: 49

CHAPTER-6

TRIANGLES

Key points and formulae

- Criteria for Similarity of Triangles : (i) AAA or AA (ii) SSS (iii) SAS.
- Two figures having the same shape but not necessarity the same size are called similar figures.
- All the congruent figure are similar but converse is not true.
- Two polygons of the same number of slides are similar if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio . (ie proportional).
- If a line is drawn paralled to one side of a triangle to interseet the other two sides, then these two sides are divided in the same ratio (Basic Proportionality Theorem) and its converse.
- Ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.
- Perpendicular drawn from the vertex of the right angle of a right triangle to its hypotenuse divides the triangle into two triangle which are similar to the whole triangle and to each other.
- In a right triangle, the square of the hypotenues is equal to the sum of the squares on the other two sides (Pythagoras Theorem) and its converse.

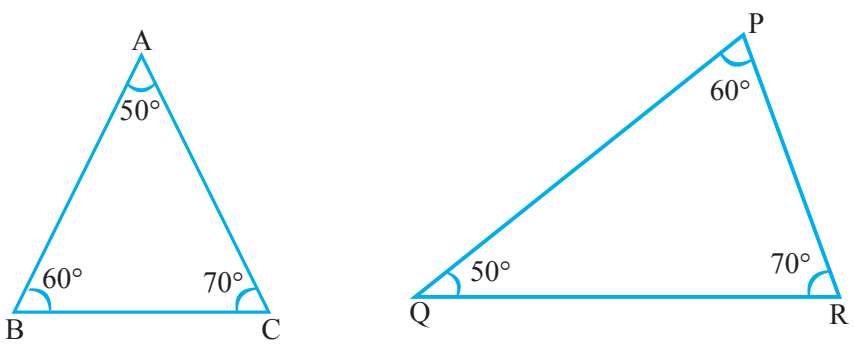
Exercise - 6

Group-A (1 mark each)

Vert Short Answer type questions :

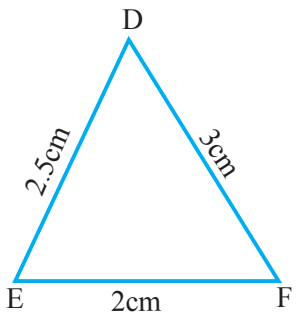
I. Fill in the blanks :

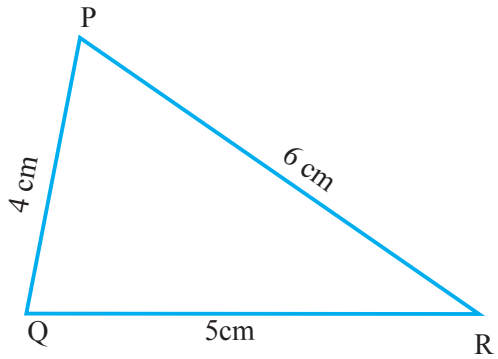
1.



The above figures are smilar. The similarity relation in symbolic form is ——

2.





If Δ DEF and Δ RQP are similar, then the similarity relation / RQP in symbolic form is_____.

3. Two Triangles are similar if their corresponding angles are———

4. All circles of same radii are ———.

5. All circles of different radii are ———

6. The reduced or enlarged photographs of an object made from the same "negative" are——

7. All equilateral triangles are ——

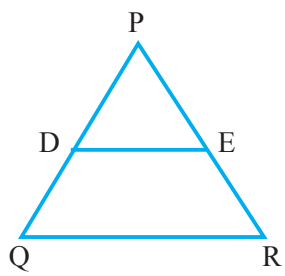
8. If the areas of two squares are in the ratio 4:1, then their sides are in the ratio——

9. The ratio of corresponding sides of similar triangles is 4:9. The ratio of their areas is ——

10. The corresponding sides of two similar triangles are in the ratio 5:6, then corresponding altitudes of the triangles must be in the ratio____.

11. Δ ABC ~ Δ DEF and their perimeters are 32 cm and 24 cm respectively. If AB = 10 cm, then DE = —— cm.

12. In a given figure, DE||QR.
If DE = 5 cm, QR=8cm and
PD = 3.5cm then PQ= —— cm.



II Choose the correct answer in each of the following questons :

1. A man goes 24m west and then 10m due north, How far is he from the starting point?

   (a) 34m (b) 17m (c) 26m (d) 28m.

2. Two poles of height 13m and 7m respectively stand vertically on a plane ground at a distance of 8m from each other. The distance between their top is ——

   (a) 9m (b) 10m (c) 11m (d) 12m

3. A verticle pole 6m long casts a shadow of length 3.6m on the ground. What is the height of a tower which casts a shadow of length 18m at the same time?

   (a) 10.8m (b) 28.8m (c) 32.4m (d) 30m.

4. A ladder 25m long just reaches the top of a building 24m high from the ground. What is the distance of the foot of the ladder from the building?

   (a) 7m (b) 14m (c) 21m (d) 24.5m

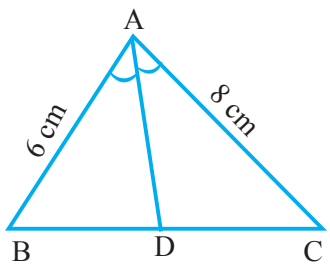
5. The hypotenuse of a right triangle is 25 cm. The other two sides are such that one is 5cm longer than the other. The length of these sides are

   (a) 10cm, 15cm (b) 15cm, 20cm (c) 12cm, 7cm (d) 13cm, 18cm.

6. Δ ABC is an isosceles triangle with AB=AC = 13cm and the length of altitude from A on BC is 5cm. Then BC = ?

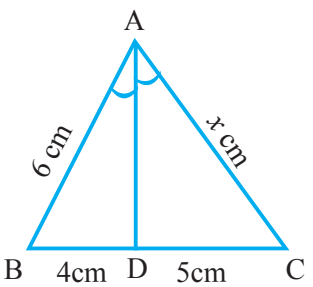
   (a) 12cm (b) 16cm (c) 18cm (d) 24cm.

7. In a triangle Δ ABC, it is given that AB = 6cm, AC=8cm and AD is the bisector of $\angle A$, then BD:DC=?



   (a) 3:4 (b) 9:19 (c) 4:3 (d) $\sqrt{3}$:2

8. In a ΔABC, it is given that AD is the internal bisector of $\angle A$. If BD=4cm, DC=5cm and AB=6cm, then AC = ?



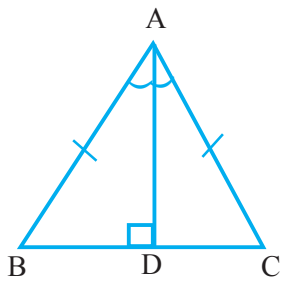
   (a) 4.5cm (b) 8cm (c) 9cm (d) 7.5 cm.

9. In an equilateteral triangle ABC, if AD$\perp$BC then which

of the following is true?

(a) $2AB^2 = 3AD^2$ (b) $4AB^2 = 3AD^2$

(c) $3AB^2 = 4AD^2$ (d) $3AB^2 = 2AD^2$

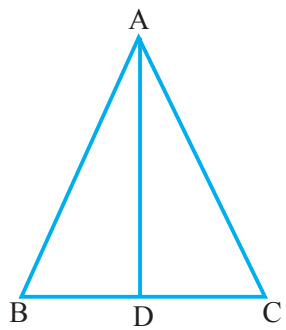


10. In a rhombus of side 10cm, one of the diagonals is 12cm long. The length of the second diagonal is–

(a) 20cm (b) 18cm (c) 16cm (d) 22cm

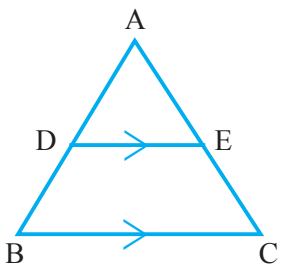
11. In ΔABC it is given that $\frac{AB}{AC} = \frac{BD}{DC}$. If $\angle B = 70°$ and $\angle C = 50°$, then $\angle BAD$ = ?

(a) 30° (b) 40° (c) 45° (d) 50°



12. In ΔABC, DE||BC so that AD=2.4cm, AE=3.2cm and EC=4.8cm, Then AB=?

(a) 3.6 cm (b) 6cm (c) 6.4cm (d) 7.2cm



13. If the diagonals of a quadrilateral divide each other proportionally then it is a

(a) parallelogram (b) trapezium (c) rectangle (d) square

14. ABC and BDE are two equilateral triangles such that D is the mid point of BC. Ratio of the areas of triangle ABC and BDE is–

(a) 1:2 (b) 2:1 (c) 1:4 (d) 4:1

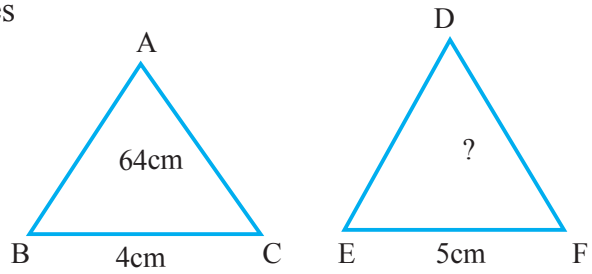
15. In ΔABC, $AB = 6\sqrt{3}$cm, AC=12cm and BC=6cm. Then $\angle B$ is–

(a) 45° (b) 60° (c) 90° (d) 120°

16. ABC and DEF are two similar triangles such that BC = 4 cm, EF = 5cm and the area of $\Delta ABC = 64cm^2$. Find the area of ΔDEF.

(a) $50cm^2$ (b) $100cm^2$

(c) $150cm^2$ (d) $200cm^2$



17. The areas of two similar triangle are $100cm^2$ and $49cm^2$ respectively. If the altitude of the bigger triangle is 5cm. Find the corresponding altitude of the other triangle is

(a) 3.5cm (b) 3cm (c) 2.5cm (d) 4.5cm

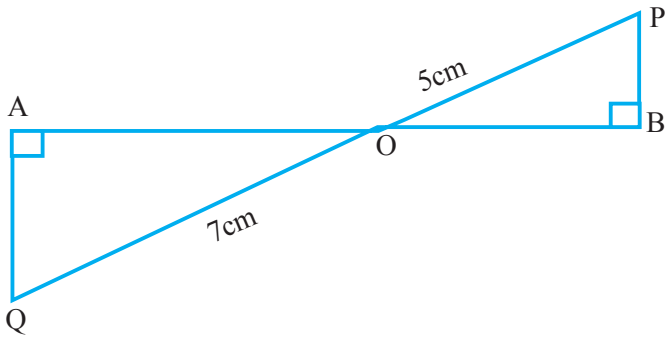
18. In the given figure PB and QA are perpendicular to segment AB. If PO=5cm QO=7cm and the area of ΔPOB = $150cm^2$. The area of ΔQOA is

(a) 299 sq.cm

(b) 394 sq.cm

(c) 289 sq.cm

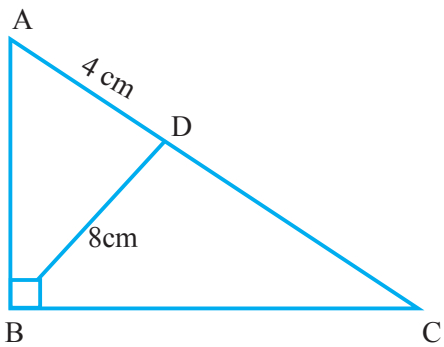
(d) 294 sq.cm



19. In the given figure, $\angle ABC = 90°$ and $BD \perp AC$. If BD=8cm and AD = 4 cm then CD = ?

(a) 14cm (b) 15cm

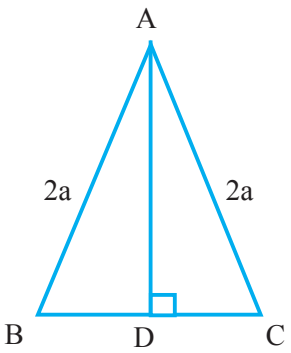
(c) 16cm (d) 17cm



20. The areas of two similar triangles ABC and DEF are $64cm^2$ and $121cm^2$ respectively. If EF=13.2cm. then BC = ?

(a) 9.8cm (b) 9.1cm (c) 9.2cm (d) 9.6cm

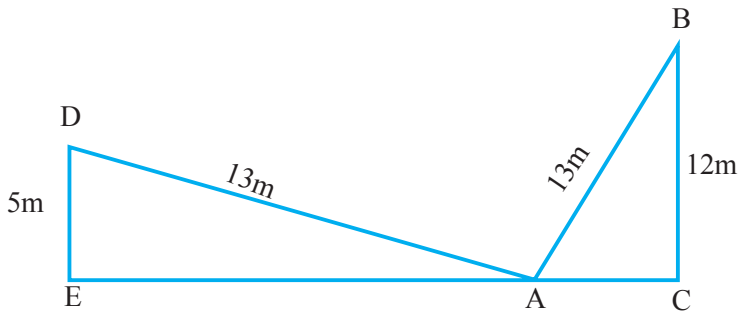
21. In figure, the length of the altitude AD of an isosceles triangle ABC of sides 2a, 2a and a is–



(a) $\frac{a\sqrt{15}}{2}$ unit (b) $\frac{15\sqrt{a}}{2}$ unit

(c) $\frac{\sqrt{15a}}{2}$ unit (d) $2a\sqrt{15}$ unit

22. A ladder 13m long reaches a window which is 12m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to other side of the street to reach a window 5m high. The widh of the stree is–



(a) 15m (b) 16m (c) 17m (d) 18m

23. In ΔABC and ΔDEF, it is given that ∠B=∠E, ∠F=∠C and AB = 3DE, then the two triangles are–

(a) Congruent but not similar (b) Similar but not congruent

(c) Neither congruent nor similar (d) similar as well as congruent

24. If ΔABC~Δ EDF then which of the following is not true?

(a) BC.EF = AC.FD (b) AB.EF = AC.DE

(c) BC.DE = AB.EF (d) BC.DE = AB.FD.

25. In ΔABC and ΔDEF, We have $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{5}{7}$, then

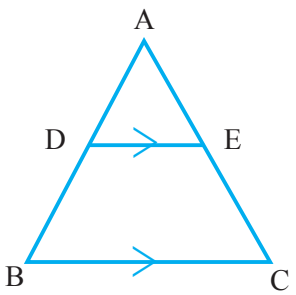
ar (ABC) : ar (DEF) = ?

(a) 5:7 (b) 25:49 (c) 49:25 (d) 125:343

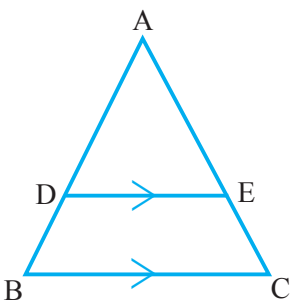
III. Answer the following questions :

1. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. What is the ratio of the areas of triangles ABC and BDE?
2. ΔABC and ΔDEF are similar triangles. The perimeters of ΔABC and ΔDEF are 30cm and 18cm respectively. If BC=9cm then EF= ?
3. In ΔABC, DE||BC such that

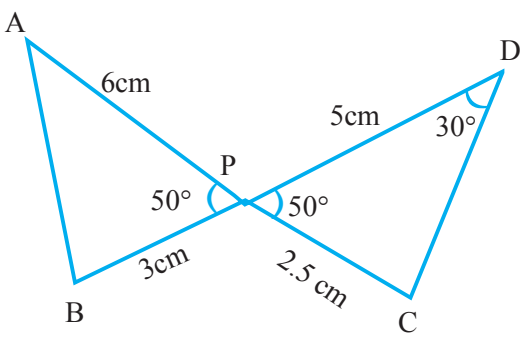
   $\frac{AD}{DB} = \frac{3}{5}$. If AC=5.6cm then AE=?



4. In ΔABC, DE||BC, So that AD = $(7x-4)$cm
   AE= $(5x-2)$cm, DB = $(3x+4)$ cm and
   EC = $3x$ cm. Find the value of x .

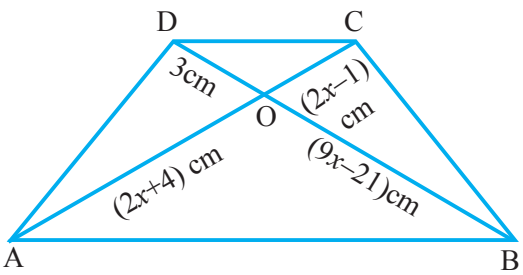


5. In the given figure, PA=6cm, PB=3cm, PC=2.5cm, PD=5cm, $\angle APB = 50°$ and $\angle CDP=30°$ then $\angle PBA$=?



6. Given $\Delta ABC \sim \Delta PQR$ and $\frac{BC}{QR} = \frac{2}{3}$ then $\frac{ar(\Delta PQR)}{ar(\Delta ABC)}$=?
7. Two isosceles triangles have corresponding angles equal and their areas are in the ratio 25:36. Find the ratio of their corresponding heights?
8. In an isosceles ΔABC, if AC=BC and $AB^2 = 2AC^2$ then $\angle C$=?

9. In the given figure, AB||CD and OA = (2x+4)cm, OB=(9x–21)cm, OC = (2x–1)cm and OD=3cm. Then x = ?

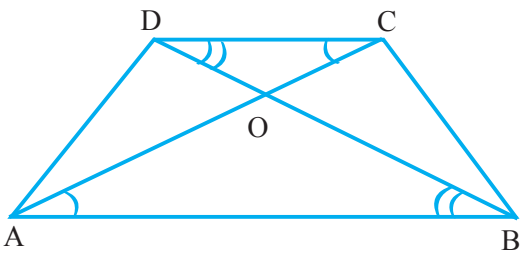


10. If $\Delta ABC \sim \Delta DEF$ such that 2AB = 3DF and BC = 6cm then EF = ?
11. A man goes 10m due east and then 20m due north. Find distance from the starting point .
12. What is the length of diagonal of a rectangle having length 8m and breadth 6m?
13. The lengths of the diagonals of a rhombus are 24cm and 10cm. What is the length of each side of the rhombus?
14. The shadow of a 5m long stick is 2m long. At the same time, what will be the length of the shadow of a 12.5m high tree?

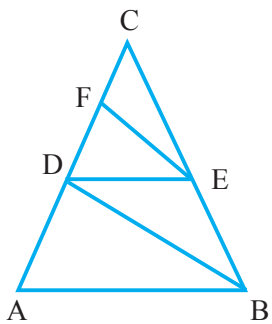
Group-B (2 mark each)

Short Answer type questions :

1. In a trapezium ABCD, O is the point of intersection of AC and BD, AB||CD and AB = 2×CD. If the area of $\Delta AOB = 84cm^2$. Find the area of ΔCOD?

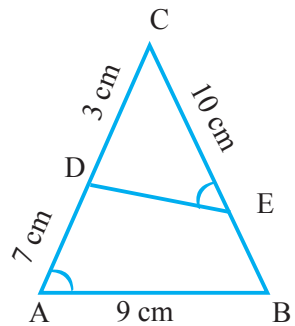


2. $\Delta ABC \sim \Delta DEF$ in which AX and DY are the bisectors of $\angle A$ and $\angle D$ respectively. If AX = 6.5 cm and DY= 5.2cm. Find the ratio of the areas of ΔABC and ΔDEF?
3. Find the length of the altitude of an equilateral triangle with side a?
4. In the given figure, AB||DE and BD||EF. Pove that $DC^2 = CF.AC$.

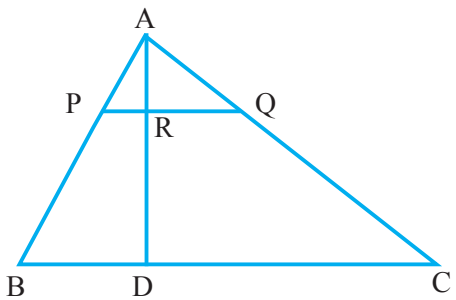


5. A ladder is placed in such a way that its foot is at a distance of 5m from a wall and its tip reaches a window 12m above the ground. Determine the length of the ladder?

6. In the figure, $\angle A = \angle CED$, AB=9cm, AD=7cm, CD=8cm and CE = 10cm. Find DE.

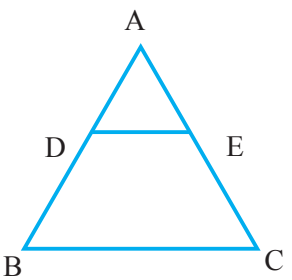


7. In the figure, AP=3cm, AR = 4.5cm, AQ=6cm, AB = 5cm and AC=10cm. Find the length of AD.

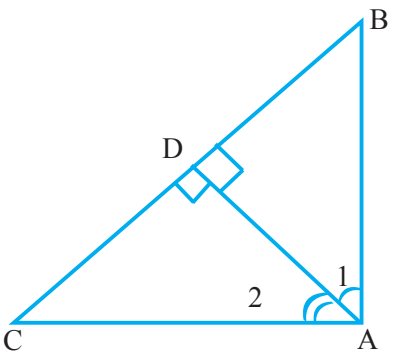


8. The perimeters of two similar triangles are 30cm and 20cm respectively. If one side of the first triangle is 15cm, find the length of the corresponding side of the seccond triangle.

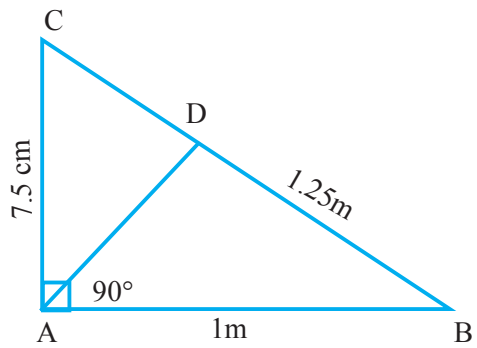
9. In fig, ABC is a triangle in which AB=AC. D and E are points on the sides AB and AC respectively, such that AD=AE. Show that the points B,C,E and D are concyclic.



10. In a ΔABC, if $\angle A = 90°$ and $AD \perp BC$. Prove that, $AD^2 = BD \times DC$



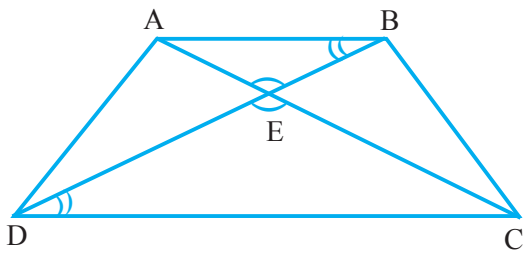
11. In the given figure $\angle CAB=90°$ and $AD \perp BC$. If AC=75cm, AB = 1m and BC = 1.25m. Find AD.



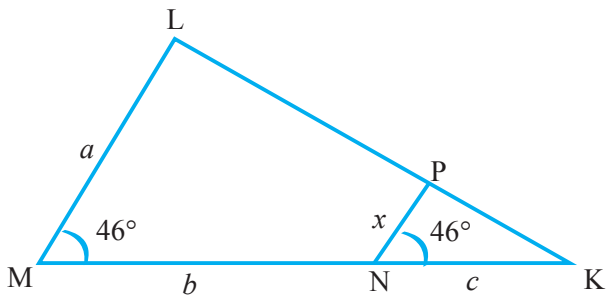
Group-C (3/4 mark each)

Long Answer type questions :

1. Rina of height 120cm is going away from the Lamp post at a speed of 1.5m/sec. If the Lamp post is 3.9m above the ground find the length of her shadow after 3 seconds?
2. ABCD is a trapezium with AB||DC.
   If ΔAED is similar to ΔBEC.
   Prove that, AD=BC



3. From the fig.
   Express x in terms of a, b and c

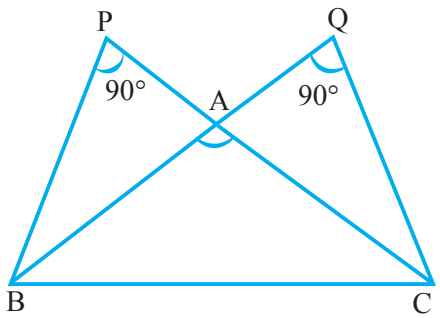


4. Two isosceles triangles have equal vertical angles and their areas are in the ratio 16:25. Find the ratio of their corresponding heights?
5. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

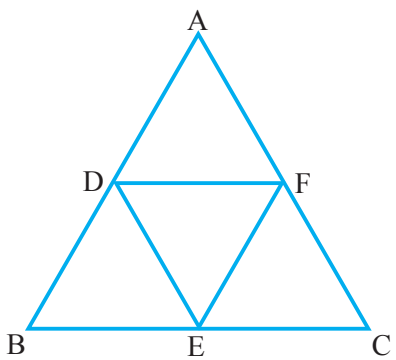
6. ABC is a right triangle, right angled at C. If p is the length of the perpendicular from C to AB and a,b,c have the usual meaning, then show that, $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

7. A point O in the interior of a rectengle ABCD is joined with each vertices A,B,C and D. Prove that, $OB^2 + OD^2 = OC^2 + OA^2$

8. In ΔABC, $\angle A$ is obtuse, $PB \perp AC$, $QC \perp AB$ prove that,
   (a) $AB \times AQ = AC \times AP$
   (b) $BC^2 = AC \times CP + AB \times BQ$



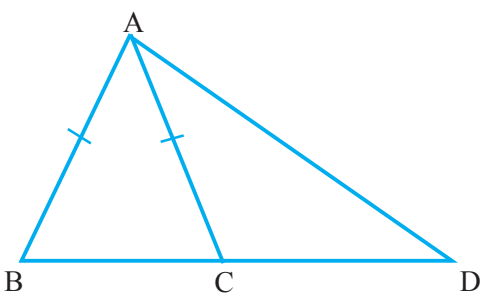
9. D,E,F are respectively the midpoints of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.



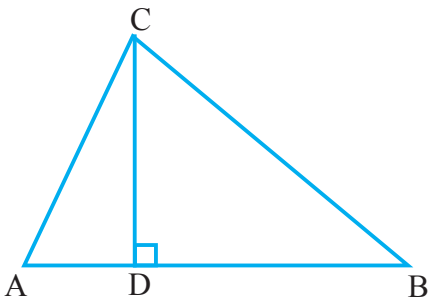
10. $\Delta ABC \sim \Delta DEF$ and their areas respectively $100cm^2$ and $49cm^2$. If the altitude of ΔABC is 5cm. Find the corresponding attitude of ΔDEF.

11. ΔABC is right angled at B and D is the mid point of BC.
    Prove that $AC^2 = (4AD^2 - 3AB^2)$

12. In ΔABC, AB=AC, side BC is produced to D. Prove that, $(AD^2 - AC^2) = BD.CD$

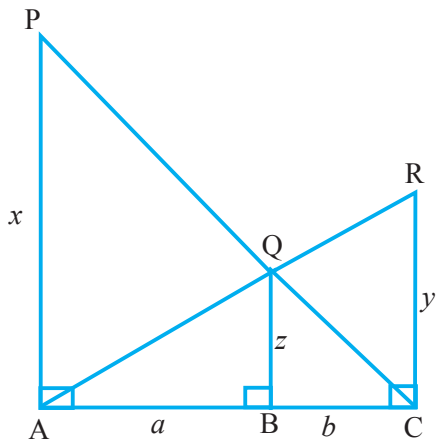


13. In the given figure, $\angle ACB=90°$ and CD$\perp$AB. Prove that, $\frac{BC^2}{AC^2}=\frac{BD}{AD}$



14. In the given figure, each one of PA, QB and RC is perpendicular to AC. If AP=x, QB = z, RC = y, AB=a and BC = b.

Show that, $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$



15. Given a right angled ΔABC. The lengths of the sides containing the right angle are 6cm and 8cm. A circle is inscribed in ΔABC. Find the radius of the circle.
16. Prove that the sum of the squares on the sides of a rhombus is equal to the sum of the squares on its diagonals.
17. In an equilateral triangle with side a, Prove that,

(i) altitude = $\frac{\sqrt{3}}{2}a$ (ii) area = $\frac{\sqrt{3}}{4}a^2$

18. In ΔABC, D is the mid point of BC and AE$\perp$BC. If AC>AB, Show that,

$$AB^2=AD^2-BC.DE+\frac{1}{4}BC^2$$

19. In ΔABC, AB=AC. Side BC is produced to D. Prove that, $(AD^2-AC^2) = BD.CD$.
20. ABC is an isosceles triangle, right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.

21. If the lengths of the sides BC, CA and AB of a ΔABC are a,b and c respectively and AD is the bisector of $\angle A$ then find the lengths of BD and DC?

22. The length of the diagonals of a rhombus are 40cm and 42cm. Find the length of each side of the rhombus.

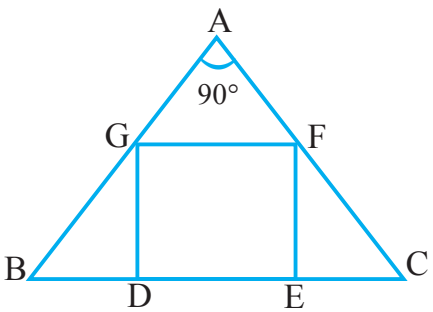
23. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD produced to E. Prove that EL=2BL.

24. In trapezium ABCD, AB||DC and DC=2AB. EF is drawn parallel to AB cuts AD in F and BC in E, Such that $\frac{BC}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove that 7FE=10AB.

25. In fig, DEFG is a square and $\angle BAC = 90°$. Prove that

    (i) $\Delta AGF \sim \Delta DBG$ (ii) $\Delta AGF \sim \Delta EFC$

    (iii) $\Delta DBG \sim \Delta EFC$ (iv) $DE^2 = BD \times EC$



26. In a quadrilateral ABCD, $\angle B=90°$ and $AD^2 = AB^2 + BC^2 + CD^2$.

    Prove that, $\angle ACD= 90°$.

27. In a rhombus ABCD, Prove that $AB^2+BC^2+CD^2+DA^2 = AC^2 + BD^2$.

28. BO and CO are respectively the bisectors of $\angle B$ and $\angle C$ of ΔABC. AO produced meets BC at P. Show that

    i) $\frac{AB}{BP} = \frac{AO}{OP}$ ii) $\frac{AC}{CP} = \frac{AO}{OP}$ iii) $\frac{AB}{AC} = \frac{BP}{PC}$ iv) AP is the bisector of $\angle BAC$

29. In a right triangle ABC, right angled at C. P and Q are the points on the sides CA and CB respectively, which divides these sides in the ratio 2:1. Prove that–

    (i) $9AQ^2 = 9AC^2 + 4BC^2$ (ii) $9BP^2 = 9BC^2 + 4AC^2$

    (iii) $9 (AQ^2 + BP^2) = 13AB^2$

30. In ΔABC, the bisector of $\angle B$ intersects the side AC at D. A line parallal to side AC intersects line segments AB, DB and CB at points, P, R and Q respectively. Prove that,

    (i) $AB \times CQ = BC \times AP$ ii) $PR \times BQ = QR \times BP$.

31. ABCD is a parallelogram, AB is divided at P and CD at Q, So that, AP:PB = 3:2 and CQ:QD = 4:1. If PQ meets AC at R, then prove that, AR = $\frac{3}{7}$ AC

32. P and Q are the mid points on the sides CA and CB respectively of ΔABC right angled at C. Prove that, $4(AQ^2 + BP^2) = 5AB^2$

Answer

Group-A

I. 1) AAA 2) SSS 3) equal 4) congrnent 5) Similar 6) Similar
7) Similar 8. 2:1 9. 16:81 10) 5:6 11) 7.5cm 12) 5.6 cm

II.

1) c	2) b	3) d	4) a	5) b	6) d
7) a	8) d	9) c	10) c	11) a	12) b
13) b	14) d	15) c	16) b	17) a	18) d
19) c	20) d	21) a	22) c	23) b	24) c 25) b

III. 1) 4:1 2) 5.4cm 3) 2.1cm 4) $x = 4$ 5) 100° 6) $\frac{9}{4}$ 7) 5:6 8) 90°
9) 3cm 10) 4 11) $10\sqrt{5}$ m 12) 10 m 13) 13cm 14) 5m

Group-B

1) $21cm^2$ 2) 25:16 3) $\frac{a\sqrt{3}}{2}$ unit 5) 13m 6) 6cm 7) 7.5cm
8) 10cm 11) 80cm.

Group-C

1) 2m 3) $\frac{ac}{b+c}$ 4) 4:5 9) 1:4 10) 3.5cm 15) r=2cm 20) 1:2

21) BD=$\frac{ac}{b+c}$, DC=$\frac{ab}{b+c}$ 22) 29cm

CHAPTER-7

COORDINATE GEOMETRY

Key points and formulae

Distance formula

1. The distance between two points P (x_1,y_1) and Q (x_2,y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ unit.

2. The distance of a point $p\,(x,y)$ from origin is $\sqrt{x^2+y^2}$ unit.

Section formula

3. The coordinates of the point P (x,y) which divides the line segment joining the points A (x_1,y_1) and B (x_2,y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\ \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

4. The coordinates of the mid point of the line segment joining points P (x_1,y_1) and Q (x_2, y_2) are $\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$.

Area of a Triangle

5. The area of triangle with vertices P (x_1,y_1), Q (x_2,y_2) & R (x_3,y_3) is

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right| \text{ sq. unit}$$

6. Condition for collinearity of three points :

i) Three points A,B and C will be collinear if AB+BC=AC

OR

ii) The points (x_1,y_1), (x_2,y_2) and (x_3,y_3) will be collinear if the area formed by the three points is zero i,e if $[x_1\,(y_2-y_3)+x_2\,(y_3-y_1)+x_3\,(y_1-y_2)]=0$

Exercise - 7

Group-A (1 mark each)

Very Short answer type questions :

Choose the correct answer :

1. The distance of the point (–3,4) from x axis is

   a) 3 b) –3 c) 4 d) 5

2. The distance between the points A (0,6) and B (0,–2) is

   a) 6 b) 8 c) 4 d) 2

3. The distance between the points P (–6,8) from the origin is–

   a) 8 b) $2\sqrt{7}$ c) 10 d) 6

4. The point on x-axis which is equidistant from points A (–1,0) and B (5,0) is

   a) (0,2) b) (2,0) c) (3,0) d) (0,3)

5. Abscissa of a point on the x axis is 3 and ordinate of a point on the y axis is –4. Find the distance of the two points.

   a) 5 b) 6 c) 15 d) –5

6. The coordinates of end points of a diameter of a circle are (7,9) and (–3,–5). The co-ordinates of its centre is–

   a) (3, 5) b) (4, 4) c) (2, 2) d) (2, 3)

7. The point which lies on the perpendicular bisector of the line segment joining the points A (–2, –5) and B (2,5) is

   a) (0,0) b) (0,2) c) (2,0) d) (–2,0)

8. Name the type of triangle formed by the points A (–5,6), B(–4,2) and C(7,5)

   a) right angled triangle b) equilateral triangle c) Isosceles triangle d) Scalene triangle.

9. AOBC is a rectangle whose three vertices are A(3,0), O (0,0) and B (0,5). The length of each of its diagonal is–

   a) 5 units b) 3 units c) 4 units d) $\sqrt{34}$ units

10. The values of x for which the distance between the points P (x,3) and Q (4,5) is $2\sqrt{2}$ unit–

    a) 6,4 b) 6,2 c) 4,6 d) –6, 2

11. The point A (–4,0), B(4,0) and C(0,3) are the vertices of a triangle, then the triangle is–

    a) isosceles triangle b) Equilaleral triangle c) Scalene triangle d) right angled triangle.

12. The point P which divides the line segment joning the points A (2,–5) and B (5,2) in the ratio 2:3 lies in the quadrant

    a) I quadrant b) II quadrant c) III quadrant d) IV quadrant.

13. If the co-ordinates of one end of a diameter of a circle are (2,3) and the coordinates of its centres are (–2,5) then the coordinates of the other end of the diameter are–

    a) (–6,7) b) (6,–7) c) (4,2) d) (5,3)

14. The point which divides the line segment joining the points (7,–6) and (3,4) in ratio 1:2 internally lies in the

    a) I quadrant b) III quadrant c) II quadrant d) IV quadrant.

15. The area of a triangle with vertices A (3,0) , B (7,0) and C (8,4) is

    a) 14 b) 28 c) 8 d) 6

16. If the points A (1,2), B (0,0) and C (a,b) are collinear, then

    a) a = b b) a = 2b c) 2a = b d) a = –b

17. If the points A (2,3), B (5,K) and C (6,7) are collinear, then

    a) K= 4 b) K= 6 c) $K = -\frac{3}{2}$ d) $K = \frac{11}{4}$

18. Find the area of the triangle whose vertices are (–8,4), (–6,6) and (–3,9)

    a) 2 b) 1 c) 0 d) 5

II. Fill in the blanks :

1. If the distance between the points (2,–2) and (–1, x) is 5 unit, then the values of x are–——.

2. If the distance between the points (4, P) and (1, 0) is 5 unit, then the value of P is——.

3. The distance between the points (1,0) and (2, cotθ) is ———.

4. If $P\left(\frac{a}{2}, 4\right)$ is the mid point of the line segment joining the points Q (–6, 5) and R (–2,3), then the value of *a* is ———.
5. Points A (–1, y) and B (5,7) lie on a circle with centre O (2, –3*y*), then the value of *y* is ——.
6. If the point P (*k*–1, 2) is equidistancce from the points A (3, *k)* and B(*k,* 5), then the values of *k* are ————.
7. If the points (*a*, 0), (0,b) and (1,1) are collinear, then $\frac{1}{a}+\frac{1}{b}=$ ——?
8. The centroid of the triangle formed by points (0,1), (1,2) & (0,2) is ——.
9. The area of a triangle with vertices (*a, b* + *c*), (*b, c* + *a*) and (*c, a* + *b*) is——
10. If the points (5,1), (–2, –3) and (8, 2m) are collinear, then the value of *m* is——.
11 If (1,2), (3,4) and (0,6) are the three vertices of a parallelogram taken in order, then the forth vertex is ———.

III. Answer the following questions :

1. What is the area of the triangle formed by the points O (0,0), A (6,0) and B (0,4)?
2. If square of the distance between the points (–2, *x*) and (*x*,–3) is 85 units, then find the value of *x*.
3. Write the condition of collinearity of points (x_1, y_1), (x_2, y_2) and (x_3, y_3).
4. If P (5, 6) is the mid point of the line segment joining A (6,5) and B (4, *y*). Find the value of *y*.
5. Find the perimeter of a triangle with vertices (0,4), (0,0) and (3,0).
6. If the point (*a,b*) is equidistant from the two points (–3, 4) and (3,6), then find the relation between *a* and *b.*
7. If A (1, 2), B (4, 3) and C (6, 6) are the three vertices of a parallelogram ABCD, find the fourth vertex D.
8. Find the ratio, in which the point $\left(0, \frac{21}{5}\right)$ divides the line segment joining the points A(6,3) and B (–4, 5).
9. Find the ratio in which the line segment joining the two points (2, –3) and (5,6) is divided by *x*-axis.

10. If the points A (2, 3), B (4, k) and C (6, 2) are collinear, find the value of k.
11. Find the coordinates of the point, which is equidistance from the three vertices A($2x$,0), B(0,0) and C (0, $2y$) of ΔAOB.
12. Find tthe coordinates of the point at which the line segment joining the points (8,9) and (–7,4) divides internally in the ratio 2:3.

IV. **State whether the following statements are true or false :**

1. Points (5,–6) and (5,6) are equidistant from x axis.
2. The point (8,0) lies on x axis.
3. Triangle, in a sum of two sides is less than the third side.
4. The distance of the point (–4,–3) from the origin is 5 unit.
5. The point P (–4, 2) lies on the line segment joining the points A (–4, 6) and B (–4,–6).
6. The points (0,5), (0, –9) and (3,6) are collinear.

Group-B

Short answer type questions : (2 marks each)

1. Find the value of a, if the distance between the points A (–3, –14) and B (a,–5) is 9 units.
2. Find the coordinate of the point R on the line segment joining the point P(–1,3) and Q (2,5), such that PR = $\frac{3}{5}$ PQ.
3. Prove that the points (3,0), (6,4) and (–1, 3) are vertices of a right angled isosceles triangle.
4. Prove that the points (7,9), (3,–7) and (–3, 3) are three vertices of a right angled isosceles triangle.
5. Find the co-ordinates of the points of trisection of the line segment joining the points A (5,–6) and B (–7,5).
6. The centre of a circle is ($2a$, a–7). Find the values of 'a', if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units.
7. Find the value of x, such that PQ = QR, where the co-ordinates of P, Q and R are (6,–1, (1, 3) and (x, 8) respectively.

8. An equilateral triangle has two vertices at the points (–4, 0) and (4, 0), find the coordinates of the third vertex.
9. If A (2, –2), B (3,4), C (7,2) are the mid points of the sides of PQ, QR and RP respectively of Δ PQR, then find its vertices.
10. Let A(–1,2) and D (3,4) be the ends points of the median AD of Δ ABC. Find the centroid of Δ ABC.
11. If the point P(–3,9), Q (a,b) and R (4,–5) are collinear and $a+b = 1$, find the values of a and b.
12. If the area of Δ ABC formed by A (x,y) B (1,2) and C (2,1) is 6 square units, then prove that $x+y= 15$ or $x+y+9 = 0$.
13. If P (x,y) is any point on the line joining the points A $(a,0)$ & B$(0,b)$, then show that $\frac{x}{a}+\frac{y}{b}=1$.
14. Find the lengths of the medians AD and BE of Δ ABC whose vertices are A (7,–3), B (5,3), C (3,–1).
15. If a ≠ b ≠ c, prove that (a,a^2), (b,b^2), (c,c^2) will not be cellinear.
16. Find the area of quadrilateral PQRS whose vertices are P(–5, –3), Q(–4,–6), R(2,–3) and S(1,2).
17. If two adjacent vertices of a parallelogram are (3,2) and (–1,0) and the diagonals intersect at (2, –5), then find the coordinates of the other two vertices.

Group-C

Long answer type questions : (3/4 mark each)

1. If the point A (1,–2), B (2,3), C (–3, 2) and D (–4, –3) are the vertices of parallelogram ABCD, then taking AB as base, find the height of the parallelogram.
2. If the coordinates of two points A&B are (3,4) and (5, –2) respectively, find the co-ordinates of any point P, where PA = PB and area of Δ PAB = 10.
3. The vertices of Δ ABC are A (4,6), B (1,5), C (7,2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$. Calculate the area of Δ ADE and compare it with the area of Δ ABC.

4. Prove that when points (2, –2) (8,4), (5,7) and (–1, 1) are join consecutively form a rectangle.
5. Find the circum centre and the circum radius of the triangle formed by the vertices (3,4), (3,–6) and (–1, 2).
6. The coordinates of the vertices of a quadrilateral (3,–2), (2,3), (–4, –2) and (–3, –5). Find the area of the quadrilateral.
7. If P (2,–1), Q (3,4), R (–2,3) and S (–3,2) be four points in a plane, show that PQRS is a rhombus but not a square. Find the area of the rhombus.
8. Two opposite vertices of a square are (–1,2) and (3,2). Find the coordinates of other two vertices.
9. The line segment joining the points P (3,3) and Q (6, –6) is trisected at the points A and B, such that A is nearer to P. If A also lies on the line given by $2x+y+k = 0$, find the value of k.
10. The mid point P of the line segment joining the points A (–10,4) and B(–2,0) lies on the line segment joining the points C (–9, –4) and D (–4, y). Find the ratio in which P divides CD. Also find the value of y.
11. If A (4,–6), B (3,–2) and C (5, 2) are the vertices of triangle Δ ABC, then verify the fact that a median of a triangle Δ ABC divides it into two triangles of equal areas.
12. If A and B are (3,–4) and (3,8) respectively, find the coordinates of P, such that AP= $\frac{4}{9}$ AB and P lies in the line segment AB.
13. Find the area of a rhombus, if its vertices are (–3,2), (–5,–5), (2, –5) and (4,4) taken in order.
14. Find the ratio in which the line $2x + 3y – 2=0$ divides the line segment joining the points (2,–1) and (5,7).

Answer

Group-A

I. 1) c 2) b 3) c 4) b 5) a 6) c 7) a 8) d 9) d 10) b 11) a
12) d 13) a 14) d 15) c 16) c 17) b 18) c

II. 1) 2,–6 2) ± 4 3) Cosec$c\theta$ 4) –8 5) $y = -1$
6) $k = 1$ or $k = 5$
7) 1 8) $\left(\frac{1}{3}, \frac{5}{3}\right)$ 9) 0 10) $\frac{19}{14}$ 11) (2, 4)

III. 1) 12 sq. unit 2. $x = -9, 4$ 3) $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
4) 7 units 5) 12 6) $3a + b = 5$ 7) (3,5) 8) 3:2 internally bisect 9) 1:2
10) $k = \frac{5}{2}$ 11) (x,y) 12) (2, 7)

IV. 1) True 2) True 3) False 4) True 5) True 6) False

Group-B

1) $a = -3$ 2) R $= \left(\frac{4}{5}, \frac{21}{5}\right)$ 5) $\left(1, -\frac{7}{3}\right), \left(-3, -\frac{4}{3}\right)$ 6) x=5,3 7) 5 or –3
8) $(0, 4\sqrt{3}), (0, -4\sqrt{3})$ 9) P (6, –4), Q (–2,0), R (8,8) 10) $\left(\frac{5}{3}, \frac{10}{3}\right)$
11) $a = 2, b = -1$ 14) AD= 5 units, BE = 5 units 16) 28 sq units
17) (1, –12) , (5, –10)

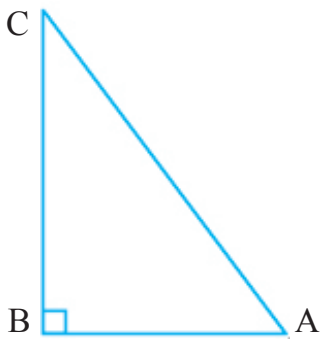
Group-C

1) $\frac{24}{\sqrt{6}}$ 2) (7,2) or (1,6) 3) Δ ADE $= \frac{15}{32}$ sq. units 5) (3,–1), 5 units
6) 28 sq. units 7) 24 sq units 8) (1,0) & (1,4) 9) k= – 8 10) 3:2, $y = 6$
12) $\left(3, \frac{4}{3}\right)$ 13) $9\sqrt{37}$ sq. units 14) 1:29

CHAPTER-8

INTRODUCTION TO TRIGONOMETRY

Key points and formulae



- Trigonometric ratios of the angle A in a triangle ABC right angled at B are defined as

- sine of $\angle A = \sin A = \dfrac{\text{side opposite to angle A}}{\text{hypotenuse}} = \dfrac{BC}{AC}$
- cosine of $\angle A = \cos A = \dfrac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \dfrac{AB}{AC}$
- tangent of $\angle A = \tan A = \dfrac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \dfrac{BC}{AB}$
- cosecant of $\angle A = \text{cosec } A = \dfrac{1}{\sin A} = \dfrac{AC}{BC}$
- secant of $\angle A = \sec A = \dfrac{1}{\cos A} = \dfrac{AC}{AB}$
- cotangent of $\angle A = \cot A = \dfrac{1}{\tan A} = \dfrac{AB}{BC}$
- $\tan A = \dfrac{\sin A}{\cos A}$
- $\cot A = \dfrac{\cos A}{\sin A}$
- The values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

- If one trigonometric ratio of an angle is given, the other trigonometric ratios of the angle can be determined.
- Trigonometric ratios of angle 0°, 30°, 45°, 60° and 90°.

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- The value of sin A and cos A never exceeds 1, where as the value of sec A or cosec A is always greater than or equal to 1.
- Trigonometric ratios of complementary angles :

  $\sin(90°-A) = \cos A$ $\quad\quad$ $\cos(90°-A) = \sin A$

  $\text{cosec}(90°-A) = \sec A$ $\quad\quad$ $\sec(90°-A) = \text{cosec}\, A$

  $\tan(90°-A) = \cot A$ $\quad\quad$ $\cot(90°-A) = \tan A$
- Trigonometric identities

  $\text{Cos}^2A + \sin^2A = 1$

  $1 + \tan^2A = \sec^2A$

  $\cot^2A + 1 = \text{cosec}^2A$

Exercise-8

Group-A (1 mark each)

Very Short answer type questions :

Choose the correct answer :

1. $\frac{\sec 30°}{\operatorname{cosec} 60°} = ?$

   a) $\frac{2}{\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) $\sqrt{3}$ d) 1

2. $\tan 5° \tan 25° \tan 30° \tan 65° \tan 85° = ?$

   a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) None of these

3. $\frac{2\sin^2 63° + 1 + 2\sin^2 27°}{3\cos^2 17° - 2 + 3\cos^2 73°} = ?$

   a) $\frac{3}{2}$ b) $\frac{2}{3}$ c) 2 d) 3

4. If sec 4A = cosec (A–10°) and 4A is acute, then $\angle A = ?$

   a) 20° b) 30° c) 40° d) 50°

5. $\frac{2\tan^2 30° \sec^2 52° \sin^2 38°}{\operatorname{cosec}^2 70° - \tan^2 20°} = ?$

   a) 2 b) $\frac{1}{2}$ c) $\frac{2}{3}$ d) $\frac{3}{2}$

6. If $2 \sin 2\theta = \sqrt{3}$, then $\theta = ?$

   a) 30° b) 45° c) 60° d) 90°

7. $(\cos 0° + \sin 30° + \sin 45°)(\sin 90° + \cos 60° - \cos 45°) = ?$

   a) $\frac{5}{6}$ b) $\frac{5}{8}$ c) $\frac{3}{5}$ d) $\frac{7}{4}$

8. If $\operatorname{cosec} \theta = \sqrt{10}$, then $\sec \theta = ?$

   a) $\frac{3}{\sqrt{10}}$ b) $\frac{\sqrt{10}}{3}$ c) $\frac{1}{\sqrt{10}}$ d) $\frac{2}{\sqrt{10}}$

9. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ = ?

a) $\frac{b}{\sqrt{b^2-a^2}}$ b) $\frac{\sqrt{b^2-a^2}}{b}$ c) $\frac{a}{\sqrt{b^2-a^2}}$ d) $\frac{b}{a}$

10. If $\tan\theta = \frac{a}{b}$, then $\frac{(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)}$ =?

a) $\frac{a+b}{a-b}$ b) $\frac{a-b}{a+b}$ c) $\frac{b+a}{b-a}$ d) $\frac{b-a}{b+a}$

11. (sec A + tan A) (1–sin A) = ?

a) sin A b) cos A c) sec A d) cosec A

12. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$, then the value of $\tan 5\alpha$ is–

a) $\frac{1}{\sqrt{3}}$ b) $\sqrt{3}$ c) 1 d) 0

II. Fill in the blanks :

13. The value of $(\sin 30° + \cos 30°)^2 - (\sin 60° - \cos 60°)^2$ is ———

14. Given that $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then $\alpha + \beta$ = ———

15. The value of $(\sin 45° + \cos 45°)$ = ———

16. If $\sin A = \frac{1}{2}$, then the value of cot A = ———.

17. If ΔABC is right angled at C, then the value of cos (A+B) = ———

18. $\sin(45° + \theta) - \cos(45° - \theta)$ = ———.

19. $\frac{\sin\theta}{\cos ec\theta}$ = ———.

20. If $\sqrt{3}\tan 2\theta - 3 = 0$, then θ = ———

21. The value of $(1 + \tan^2\theta)\cos^2\theta$ = ———

22. The value of $\left(\cot^2\theta - \frac{1}{\sin^2\theta}\right)$ = ———.

III. Answer the following questions either in one word or one sentence or as per requirement of the questions :

23. Find the value of $\tan 45° + 2 \cos 60° - \sec 60°$.

24. If $4 \tan\theta = 3$, then find the value of $\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}$.

25. If $\tan\theta + \cot\theta = 2$ and $0° < \theta < 90°$ then, find $\tan^{10}\theta + \cot^{10}\theta$.

26. If $A+B = 90°$, then what is the value of $\tan^2 A - \cot^2 B$.

27. If $\tan\theta = \frac{a}{b}$, then find $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

28. If $\cos A + \cos^2 A = 1$, then find the value of $\sin^2 A + \sin^4 A$.

29. If $\sqrt{3}\ \tan\theta = 3 \sin\theta$, find the value of $\sin^2\theta - \text{Cos}^2\theta$

30. Find the value : $\frac{5\sin^2 30° + 4\cos ec^2 60° - \tan^2 45°}{\sin^2 60° + 4\cos^2 60°}$

31. Find the value of : $\sqrt{\frac{1+\sin 30°}{1-\sin 30°}}$

32. If $\angle A = 45°$, then show that, $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

33. Given that $\sin(\alpha - \beta) = \frac{1}{2}$ and $\cos(\alpha + \beta) = \frac{1}{2}$, then find the value of α and β.

34. If ΔABC is an isosceles right triangle, right angled at B, then find the value of $\frac{\tan A + \cot C}{\cot A + \cot C}$

35. If $\frac{1+\sin\alpha}{1-\sin\alpha} = \frac{m^2}{n^2}$, then find the value of $\sin\alpha$.

Group-B (2/3 marks each)

Short answer type questions :

1. Find the value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \sin^2 85°$

2. Prove the following identity

$$\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\cos\theta+\sin\theta$$

3. If $\sec\theta + \tan\theta = p$, show that $\frac{p^2-1}{p^2+1}=\sin\theta$

4. If A, B and C are interior angles of triangle ABC, then show that

$$\tan\left(\frac{A+C}{2}\right)=\cot\frac{B}{2}$$

5. If $(\sin\theta + \cos\theta) = \sqrt{2}\cos\theta$, show that $\cot\theta = (\sqrt{2}+1)$

6. If $\cos\theta = \frac{7}{25}$, write the value of $(\tan\theta + \cot\theta)$

7. If $\tan\theta = \frac{1}{\sqrt{5}}$, write the value of $\frac{\cos ec^2\theta-\sec^2\theta}{\cos ec^2\theta+\sec^2\theta}$.

8. Find the value of $\frac{\cos 38^\circ \cos ec\ 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$

9. Find A, if $\tan 2A = \cot (A-24^\circ)$

10. If A & B are acute angles such that $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ and

$\tan (A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$, find A+B.

11. Prove that $(\sqrt{3}+1)(3-\cot 30^\circ) = \tan^3 60^\circ - 2\sin 60^\circ$.

12. If θ is an acute angle and $\sin\theta = \cos\theta$. Find the value of $2\tan^2\theta + \sin^2\theta - 1$.

13. Find the value of x.

$\sin 2x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$.

14. If $\sqrt{3}\tan\theta = 1$, find the value of $\sin^2\theta + \cos^2\theta$.

15. Simplify $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$.

16. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$.

17. Prove that $\tan\theta + \tan(90^\circ-\theta) = \sec\theta \sec(90^\circ-\theta)$.

18. Prove that $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha) = \sec\alpha + \operatorname{cosec}\alpha$

Group-C (3/4 marks each)

Long answer type questions :

1. If $1+\sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\frac{1}{2}$.

2. If $\sin\theta + 3\sin^2\theta + \sin^3\theta = 1$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

3. Prove that $(1+\cot A+\tan A)(\sin A-\cos A) = \dfrac{\sec A}{\operatorname{cosec}^2 A} - \dfrac{\operatorname{cosec} A}{\sec^2 A}$

   $= \sin A \tan A - \cot A \cos A$

4. Prove that :

   i) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

   ii) $\dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \dfrac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A}$

   iii) $2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = \cot^4\theta - \tan^4\theta$

   iv) $\tan^2 A - \tan^2 B = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \dfrac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A}$

   v) $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1+\sec\theta\ \operatorname{cosec}\theta)^2$

   vi) $\dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta$

   vii) $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = \dfrac{1}{\tan\theta + \cot\theta}$

   viii) $\dfrac{\tan A}{1-\cot A} + \dfrac{\cot A}{1-\tan A} = 1 + \tan A + \cot A$

   ix) $\dfrac{\cot A}{\operatorname{cosec} A + 1} + \dfrac{\operatorname{cosec} A + 1}{\cot A} = 2\sec A$

x) $\frac{1}{\sec A-\tan A}-\frac{1}{\cos A}=\frac{1}{\cos A}-\frac{1}{\sec A+\tan A}$

xi) $(1-\sin\theta+\cos\theta)^2=2(1+\cos\theta)(1-\sin\theta)$

xii) $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\cos ec\theta$

xiii) $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2}{2\sin^2\theta-1}$

xiv) $\frac{1-\cos\theta}{\sin\theta}+\frac{\sin\theta}{1-\cos\theta}=2\text{cosec }\theta$

Answer

Group-A

I. Choose the correct answer :

1) d 2) b 3) d 4) a 5) c 6) a 7) d 8) b 9) b 10) c 11) b 12) c

II. Fill in the blanks

13) $\sqrt{3}$ 14) 90° 15) $\sqrt{2}$ 16) $\sqrt{3}$ 17) 0 18) 0 19) $\sin^2\theta$ 20) 30° 21) 1 22) –1

III. 23) 0 24) $\frac{1}{2}$ 25) 2 26) 0 27) $\frac{a^2-b^2}{a^2+b^2}$ 28) 1 29) $\frac{1}{3}$ 30) $\frac{67}{21}$

31) $\sqrt{3}$ 33) $\alpha=45°$, $\beta=15°$ 34) 1 35) $\frac{m^2-n^2}{m^2+n^2}$

Group-B

1) $\frac{19}{2}$ 6) $\frac{625}{168}$ 7) $\frac{2}{3}$ 8) $\frac{1}{\sqrt{3}}$ 9) 38° 10) 45° 12) $\frac{3}{2}$ 13) 15°

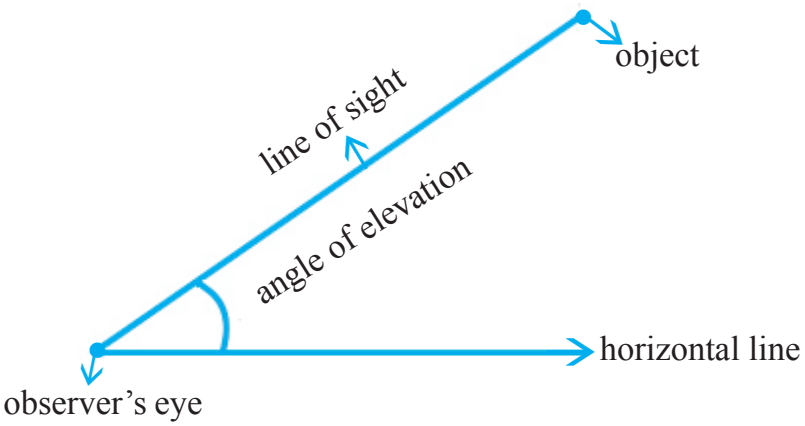
14) $-\frac{1}{2}$ 15) 1

CHAPTER-9

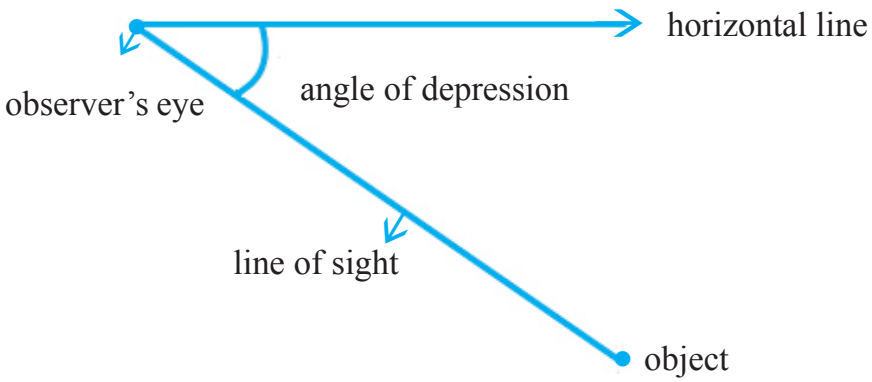
APPLICATIONS OF TRIGONOMETRY

Key points and formulae

- The 'line of sight' is the line from the eye of observer to the point in the object viewed by the observer.
- The 'angle of elevation' of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level.



- The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level.



- The height or length of an object or the distance between two distinct objects can be determined with the help of trigonometric ratios.

Exercise-9

Group-A

Very short answer type questions:

I. Choose the correct answer

1. The ratio of the length of a rod and its shadow is 1:$\sqrt{3}$. The angle of elevation of the sun is –
   (a) 30° (b) 45° (c) 60° (d) 90°
2. If the angles of elevation of the top of a tower from two points at a distance a and b from the base and in the same straight line with it are complementary, then the height of the tower is–
   (a) $\sqrt{a+b}$ (b) $\sqrt{ab}$ (c) $\sqrt{a-b}$ (d) $\sqrt{\frac{a}{b}}$
3. The tops of two poles of height 16m and 10m are connected by a wire of length l meters. If the wire makes an angle of 30°with the horizontal, then l =
   (a) 26 (b) 16 (c) 12 (d) 10
4. The angle of depression of a car, standing on the ground, from the top of a 75m tower is 30°. The distance of the car from the base of the tower (in meters) is–
   (a) $25\sqrt{3}$ (b) $50\sqrt{3}$ (c) $75\sqrt{3}$ (d) 150
5. If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground, then the angle of elevation of the sun at that time is–
   (a) 30° (b) 60° (c) 45° (d) 75°
6. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2m away from the wall, then the length of the ladder (in meters) is –
   (a) $\frac{4}{\sqrt{3}}$ (b) $4\sqrt{3}$ (c) $2\sqrt{2}$ (d) 4

II. Fill in the blanks :

1. If a pole 6 m high casts a shadow $2\sqrt{3}$m long on the ground, then the sun's elevation is———.
2. The angle of elevation of the sun when the shadow of a pole h meter high is $\sqrt{3}h$ meters long is ———.
3. If the height of a vertical pole is equal to the length of its shadow on the ground, then the angle of elevation of sun is ——.

4. A kite is flying at a height of 30m from the ground. The length of the string from the kite to the ground is 60m. Assumming that there is no slack in the string, the angle of elevation of the kite at the ground is ——.
5. In a rectangle, the angle between a diagonal and a side is 30° and the length of this diagonal is 8cm. The area of the rectangle is—— .
6. From the top of a hill, the angles of depression of two consecutive km stones due east are found to be 30° and 45°. The height of the hill is ——— km.
7. If the length of the shadow of a tower is increasing, then the angle of elevation of the sun will———.
8. The angle of elevation of the top of a tower from two points at a distance *s* and *t* from its foot are complementary. (Points are lying in the same side of tower). Then the height of the tower is———.

III. Answer the following questions :

1. A vertical pole stands on the level ground. From a point on ground, 25m away from the foot of the pole, the angle of elevation of its top is found to be 60°. Find the height of the pole. (Take $\sqrt{3}=1.732$)
2. The tops of two poles of height 20m and 14m are connected by a wire. If the wire makes an angle of 30° with horizontal, find the length of the wire.
3. From the top of a cliff 25m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower.
4. A ladder 15m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall. Find the height of the wall.
5. The angle of depression of a car parked on the road from the top of a 150m high tower is 30°. Find the distance of the car from the tower.

Group-B

Short answer type questions : (2marks each)

1. The angle of elevation of the top of a tower at a point on the ground 50m away from the foot of the tower is 45°. Find the height of the tower.
2. A tower is $100\sqrt{3}$ meters high. Find the angle of elevation of its top, from a point 100 metres away from its foot.

3. The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower is 30°. Find the height of the tower.
4. A kite is flying at a height of 60m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.
5. An observer 1.5m tall is 28.5m away from a tower. The angle of elevation of the top of the tower from her eyes is 45°. What is the height of the tower?
6. A tree 12m high, is broken by the wind, in such a way that its top touches the ground and makes an angle 60° with ground. At what height from the bottom the tree is broken by the wind?
7. A tree is broken by wind. The top stuck the ground at an angle of 30° and at distance of 30 meters from the root. Find the whole height of the tree.
8. The length of a shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. What is the angle of elevation of sun?
9. If the ratio of the height of tower and the length of its shadow is $\sqrt{3}:1$, what is the angle of elevation of the sun?
10. What is the angle of elevation of the sun, when the length of the shadow of a vertical pole is equal to its height?
11. AB is a pole of height 6m standing at a point B and CD is a ladder inclined at an angle of 60° to the horizontal and reaches upto a point D of pole. If AD=2.54m, find the length of the ladder. (use $\sqrt{3}=1.73$)
12. The length of a string between a kite and a point on the ground is 85m. If the string makes an angle θ with the ground level such that $\tan\theta = \frac{15}{8}$, then find the height of the kite from the ground. Assume that there is no slack in the string.

Group-C

Long answer type questions : **(3/4marks each)**

1. An aeroplane at an altitude of 1200 metres finds that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two ships.
2. Two pillars of equal height and on either side of a road, which is 100m wide. The angles of elevation of the top of the pillars are 60°and 30° at a point on the road between the pillars. Find the position of the point between the pillars and the height of each pillar.

3. As observed from the top of a light house, 100m above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 45°. Determine the distance travelled by the ship during the period of observation.
4. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40m vertically above X, the angle of elevation is 45°. Find the height of the tower PQ and the distance XQ.
5. From a window 15 metres high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45° respectively. Show that the height of the opposite house is 23.66 metres (take $\sqrt{3}=1.732$)
6. From the top of a building 60m high, the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.
7. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $3600\sqrt{3}$m. Find the speed of the jet plane.
8. A man standing on the deck of a ship, which is 10m above water level. He observes the angle of elevation of the top of a hill 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.
9. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α & β respectively. Prove that the height of the tower is $\frac{h\tan\alpha}{\tan\beta-\tan\alpha}$.
10. Two stations due south of a leaning tower which leans towards the north are at distances a and b from its foot. It α, β be the elevations of the top of the tower from these stations, prove that its inclination θ to the horizontal is given by $\cot\theta=\frac{b\cot\alpha-a\cot\beta}{b-a}$.
11. If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the clould is $\frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}$.
12. The angle of elevation of a cloud from a point 60m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud.
13. The angle of elevation of a cliff from a fixed point is θ. After going up a distance of k metres towards the top of the cliff at angle ϕ, it is found that the angle of elevation is α. Show that the height of the cliff is $\frac{k(\cos\phi-\sin\phi\cot\alpha)}{\cot\theta-\cot\alpha}$ metres.

14. The angle of elevation of the top of a tower from a point A due south of the tower is α and from B due east of the tower is β. If AB=d, show that the height of the tower is $\frac{d}{\sqrt{\cot^2\alpha+\cot^2\beta}}$.

15 A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching to the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the further time taken by the car to reach the foot of the tower.

16. The angles of depressions of two ships from the top of a light house are 45° and 30° towards east. If the ships are 100m apart, find the height of the light house.

17. Two persons are *a* meters apart and the height of one is double that of other. If from the middle point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, then find the height of the smaller person.

18. From a light house the angles of depression of two ships on opposite sides of the light house are observed to be 30° and 45°. If the height of the light house is *h* meters, what is the distance between two ships?

19. If the angle of elevation of a cloud from a point 200m above a lake is 30° and the angle of depression of its reflection in the lake is 60°, then find the height of the cloud above the lake.

20. The height of a tower is 100m. When the angle of elevation of the sun changes from 30° to 45°, the shadow of the tower becomes *x* meters less. Find the value of *x*.

21. If a 1.5m tall girl stands at a distance of 3m from a lamp post and casts a shadow of length 4.5m on the ground, then find the height of the lamp post.

Answers

Group-A

I. 1) a 2) b 3) c 4) c 5) b 6) d

II. 1) 60°, 2) 30° 3) 45° 4) 30° 5) $16\sqrt{3}$ cm^2 6) $\frac{1}{2}(\sqrt{3}+1)$ km

7) decreasing 8) $\sqrt{st}$

III. 1) 43.3 m 2) 12 m 3) 50m 4) $\frac{15\sqrt{3}}{2}$ m 5) $150\sqrt{3}$ m

Group-B

1) 50m 2) 60° 3) $10\sqrt{3}$m 4) $40\sqrt{3}$ m 5) 30m 6) 5.569 m 7) 51.96 m
8) 30° 9) 60° 10) 45° 11) 4 m. 12. 75 m.

Group-C

1) 1385.6 m
2) The required point is at a distance of 25 meters from the first pillar. Height of pillars is 43.3 m.
3) 73.2 m
4) PQ = 94.64m, XQ = 109.3m
6) 40 meters.
7) 864 km/hr.
8) distance $=10\sqrt{3}$ m, height of hill = 40m.
12) 120 m (from the surface of lake)
15) 3 seconds.

16) $50\left(\sqrt{3}+1\right)$m 17) $\frac{a}{2\sqrt{2}}$ 18) $\left(\sqrt{3}+1\right)h$ meters

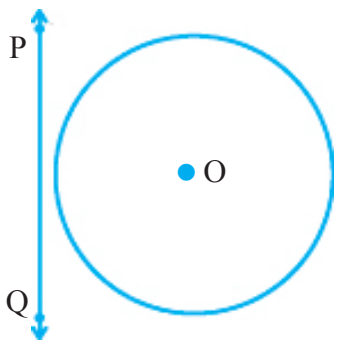
19) 400 m 20) $100\left(\sqrt{3}-1\right)$ m 21) 2.5 m.

CHAPTER-10

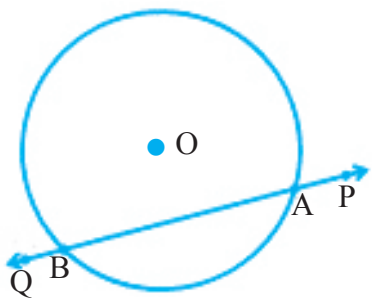
CIRCLES

Key points and formulae

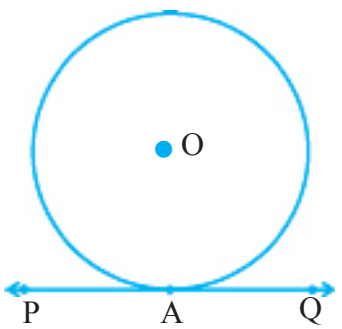
- If a circle and a line have no common point, then the line is called a non-intersecting line with respect to the circle.



- If a circle and a line have two common points or a line intersect a circle in two distinct points, then the line is called secant to the circle.



- If a line and a circle have only one point common, or a line intersect the circle in only one point, then it is called tangent to the circle.



- There is only one tangent at a point of the circle.

- Tangent is perpendicular to the radius through the point of contact.
- Only two tangents can be drawn to a circle from an external point.
- Lengths of tangents from an external point to a circle are equal.

Exercise-10

Group-A (1 mark each)

Very short answer type questions :

I. Fill in the blanks :

1. If angle between two radii of a circle is 120°, the angle between the tangents at the ends of the radii is _____.
2. If radii of two concentric circles are 4cm and 5cm, then the length of the chord of one circle which is tangent to the other circle is——
3. In fig 10.1, AB is a chord of the circle and AOC is diameter such that ∠ACB=40°. If AT is the tangent to the circle at the point A, then ∠BAT is equal to_____.

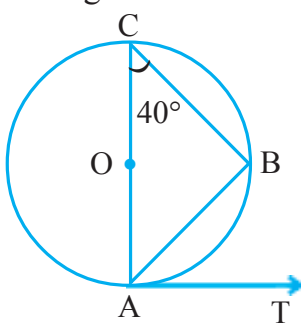


fig 10.1

4. In fig. 10.2, PA and PB are two tangents to the circle with centre O. If ∠APB = 70°, then ∠OAB is equal to_____.

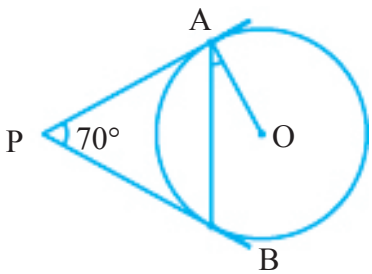


fig 10.2

5. In fig 10.3, PT is a tangent to the circle with centre O. If OT = 6 cm and SP = 4cm, then the length of tangent is _____.

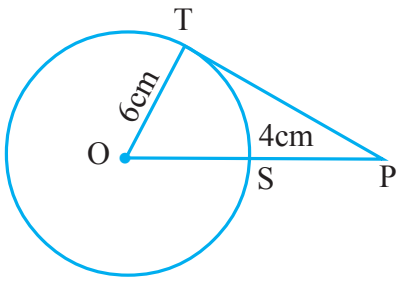


fig 10.3

II. Multiple choice questions :

Choose the correct answer from the given four options :

1. In the given fig 10.4, AD and AE are the tangants to a circle with centre O and BC touches the circle at F. If AE = 5cm, then perimeter of ΔABC is

   (a) 15cm (b) 10cm

   (c) 20cm (d) 22.5cm

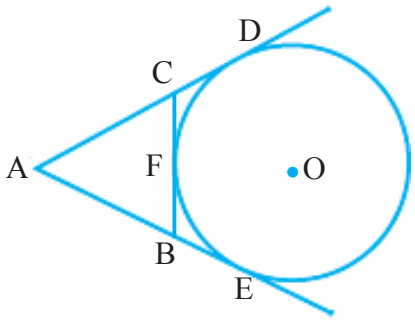


fig 10.4

2. At one end A of a diameter AB of a circle of radius 5cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8cm from A is

   (a) 6cm (b) 4cm (c) 5cm (d) 8cm

3. In fig 10.5, if ∠POS = 120°
   then ∠QOR is equal to

   (a) 80° (b) 70° (c) 60°

   (d) 50°

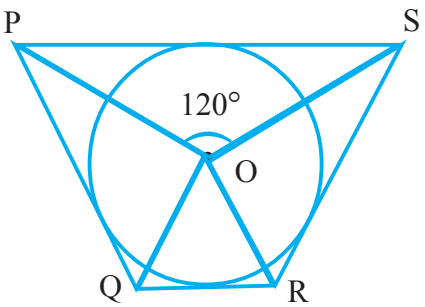


fig 10.5

4. In the given fig. 10.6, if O is the centre of a circle, AB is a chord and the tangent AC at A makes an angle of 45° with AB, then ∠AOB is equal to–

   (a) 80° (b) 90° (c) 100° (d) 85°

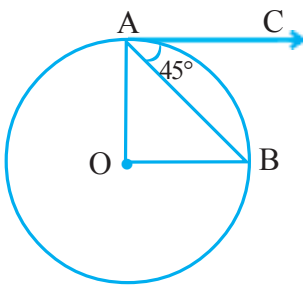


fig 10.6

5. In the given fig 10.7, if AF=4cm, BF= 3cm and AC = 11cm, then the length of BC is

(a) 10cm (b) 9cm (c) 11cm

(d) 12cm

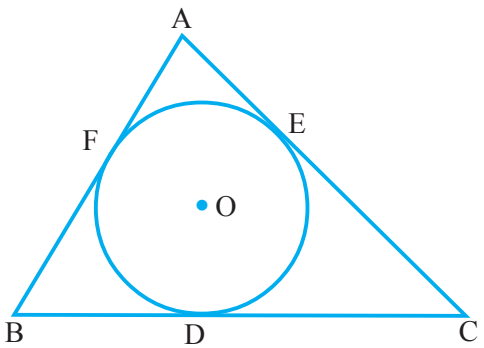


fig 10.7

6. PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then ∠OQP is equal to

(a) 60° (b) 30° (c) 45° (d) 90°

7. In the given fig 10.8, O is the centre of the circle. AB is the tangent to the circle at the point P. If ∠APQ = 57°, then the measure of ∠PQB is

(a) 57° (b) 123°

(c) 113° (d) 33°

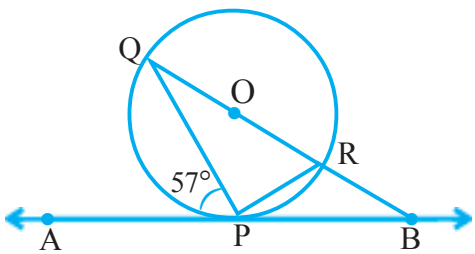


fig 10.8

8. In the given fig. 10.9, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, then the measure of ∠TRQ is

(a) 55° (b) 20°

(c) 45° (d) 65°

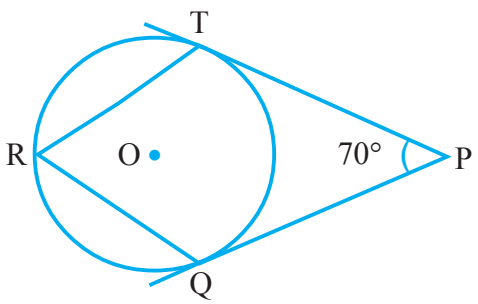


fig 10.9

9. In the given fig 10.10, two circles touch each other at C and AB is a tangent to both the circles, The measure of ∠ACB is

(a) 60° (b) 45°

(c) 90° (d) 120°

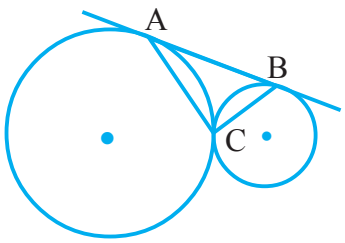


fig 10.10

10. In the given fig 10.11, PA and PB are tangents to the given circle such that PA=5cm and ∠APB=60°. The length of chord AB is

(a) 5√2cm (b) 5√3cm

(c) 7.5cm (d) 5cm

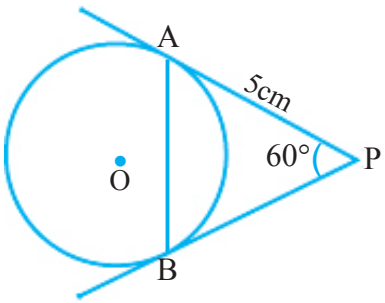


fig 10.11

III. Answer the following questions :

1. A tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q, so that OQ=13cm. Find the length of PQ.

2. In the given fig 10.12, AB is the diameter of a circle with centre O and AT is a tangent. If ∠AOQ=58°, find ∠ATQ.

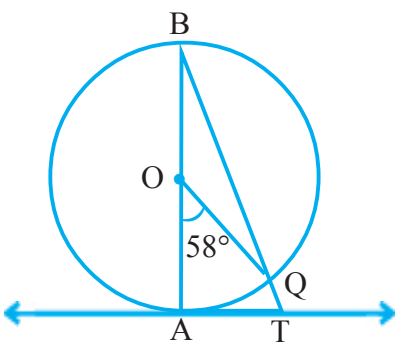


fig 10.12

3. In the given fig 10.13, O is the centre of the circle. AB is the tangent to the circle at the point P. If ∠PAO=30°, then find the measure of ∠CPB + ∠ACP.

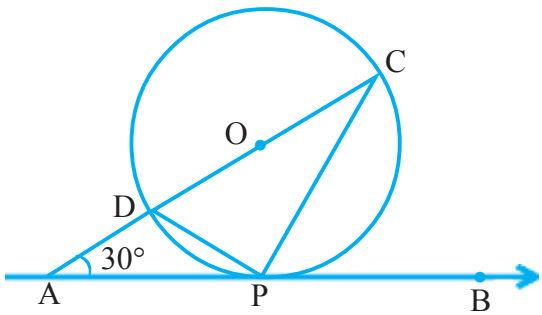


fig 10.13

4. In the given fig 10.14, O is the centre of a circle, BOA is its diameter and the tangent at the point P meets BA extended at T. If ∠PBO = 30°, then what is the measure of ∠PTA?

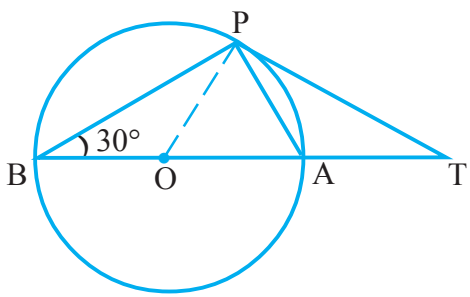


fig 10.14

5. In the given fig 10.15, PQR is a tangent to the circle at Q, whose centre is O and AB is a chord parallel to PR, such that ∠BQR = 70°. Then, ∠AQB = ?

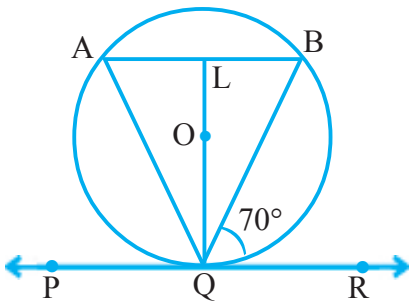


fig 10.15

IV. State whether the following statements are true or false :

1. A circle can have more than two parallel tangents, parallel to a given line.
2. If a point P lies on the circle, then one and only one tangent can be drawn to the circle at P.
3. If angle between two tangents drawn from a point P to a circle of radius "a" and centre O is 90°, then OP= a.

4. The length of tangent from an external point on a circle is always greater than the radius of the circle.
5. If a number of circles pass through the end point P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

Group -B (2 marks each)

Short answer type questions :

1. In fig. 10.16, find the perimeter of Δ ABC, if AP=12cm.

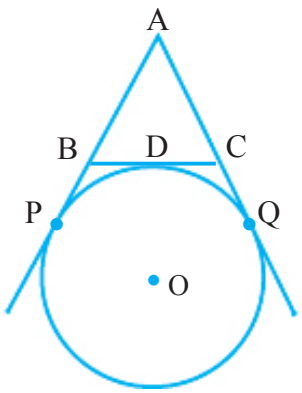


fig 10.16

2. Prove that the rectangle circumscribing a circle is a square.

3. In the fig. 10.17, AB and CD are common tangents to two circles of unequal radii. Prove that AB=CD.

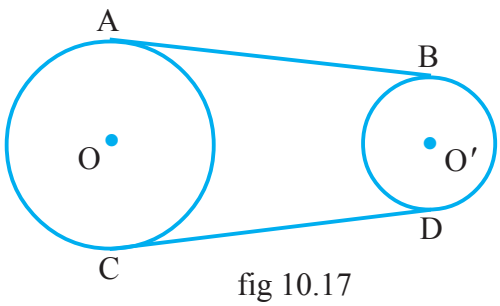


fig 10.17

4. In the given fig. 10.18, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT= 3.8cm, then find the length of QR.

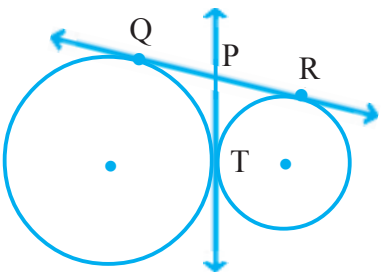


fig 10.18

5. In the given fig. 10.19, PA and PB are tangents to a circle from an external point P, such that PA=4cm and ∠BAC=135°. Find the length of chord AB.

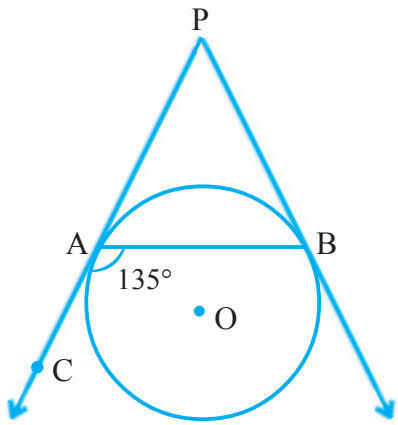


fig 10.19

6. In fig 10.20, O is the centre of the circle. PA and PB are two tangents to the circle from an external point P such that ∠APB=50°, find ∠ACB.

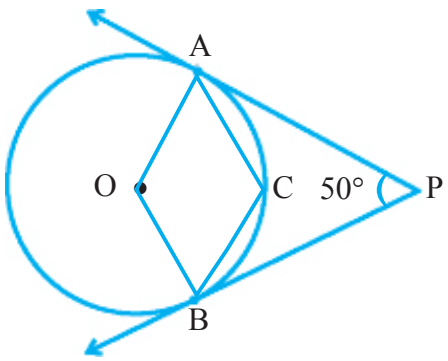


fig 10.20

7. In fig 10.21, common tangents AB and CD to two circles intersect at E, Prove that AB=CD.

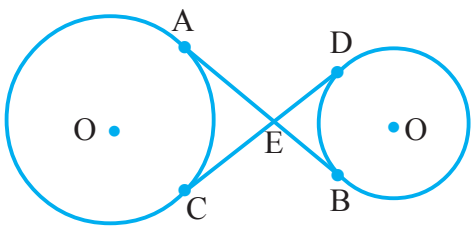


fig 10.21

8. In fig 10.22, O is the centre of a circle; PQL and PRM are the tangents at the points Q and R respectively and S is a point on the circle, such that $\angle SQL=50°$ and $\angle SRM=60°$, then find the measure of $\angle QSR$.

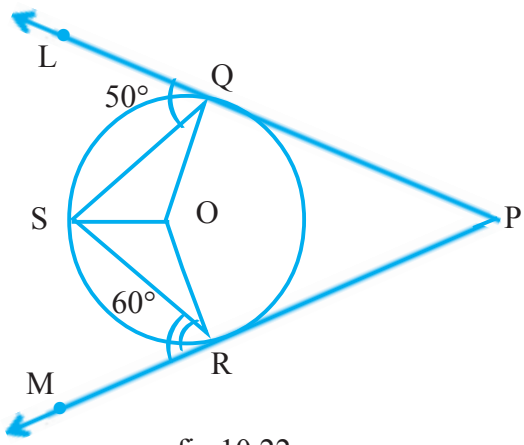


fig 10.22

9. In the given fig. 10.23, PT is a tangent at T and PAB is a secant. If PT=12cm and AB =7cm, then find PA.

[Hint : $PT^2 = PA \times PB$]

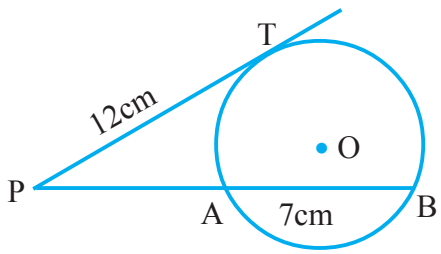


fig 10.23

10. Two circles with radii 9cm and 4cm touch each other externally. Then, what is the length of the direct common tangent?

Group -C (3/4 marks each)

Long answer type questions :

1. Prove that the tangent drawn at the midpoint of an arc of a circle is parallel to the chord joining the end points of the arc.
2. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.
3. If a, b, *c* are the sides of a right triangle where c is the hypotenuse, prove that the radius *r* of the circle which touches the sides of the triangle is given by

$$r = \frac{a+b-c}{2}$$

4. In fig. 10.25, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that:

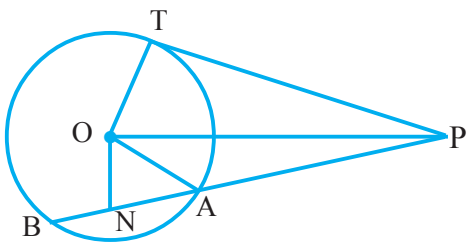


fig 10.25

(i) $PA.PB = PN^2 - AN^2$,

(ii) $PN^2 - AN^2 = OP^2 - OT^2$ (iii) $PA.PB = PT^2$

5. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

6. If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that $\angle QPR = 120°$. Prove that $2PQ = PO$.

7. If a circle touches the side BC of a triangle ABC at P and extended side AB and AC at Q and R respectively, prove that $AQ = \frac{1}{2}(BC+CA+AB)$.

8. If a hexagon ABCDEF circumscribe a circle, prove that AB+CD+EF=BC+DE+FA.

9. In fig. 10.26, AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A.Prove that $\angle BAT = \angle ACB$.

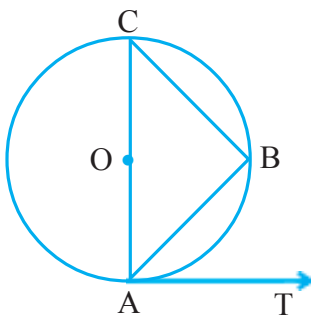


fig 10.26

10. In the given fig. 10.27, from a point P, two tangents PA and PB are drawn to a circle with centre O and radius *r*. If OP = 2*r*, show that ΔAPB is equilateral.

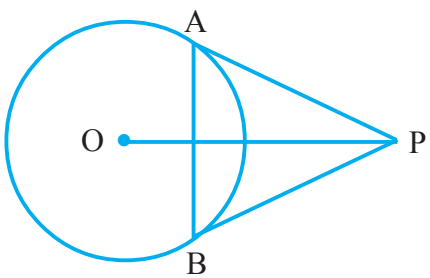


fig 10.27

11. AB is a diameter and AC is a chord of a circle with centre O such that ∠BAC = 30°. The tangent at C intersects extended AB at a point D. Prove that BC = BD.

12. If an isosceles triangle ABC, in which AB=AC=6cm, is inscribed in a circle of radius 9cm, find the area of the triangle.

13. In fig. 10.28, tangents PQ and PR are drawn to a circle such that ∠RPQ=30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS.

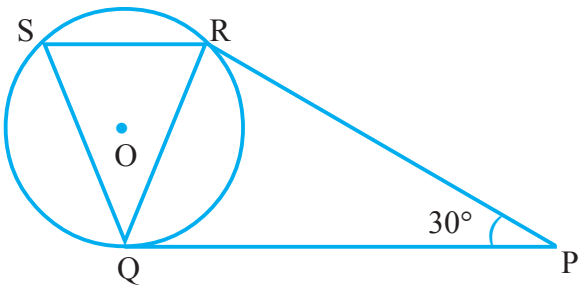


fig 10.28

14. The tangent at a paint C of a circle and a diameter AB when extended intersect at D. If ∠DCA = 110° as shown in fig. 10.29, find ∠CBA.

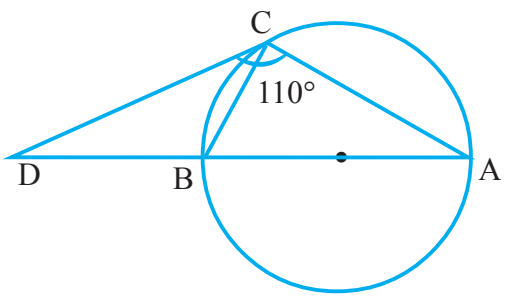


fig 10.29

15. AB is a chord of circle with centre O. At B, *a* tangent PB is drawn such that its length is 24cm, as shown in fig. 10.30. The distance of P from the centre is 26cm. If the chord AB is 16cm, find its distance from the centre.

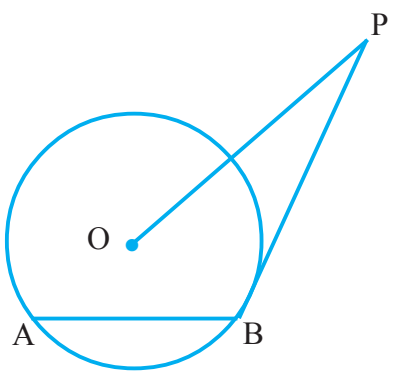


fig 10.30

Answer

Group-A

I. 1) 60° 2) 6cm 3) 40° 4) 35° 5) 8cm

II. 1) b 2) d 3) c 4) b 5) a 6) c 7) d 8) a 9) c 10) d

III. 1) 12cm 2) 61° 3) 90° 4) 30° 5) 40°

IV. 1) False 2) True 3) False 4) False 5) True.

Group-B

1) 24cm 4) 7.6cm 5) $4\sqrt{2}$cm 6) 115° 8) 70° 9) 9cm 10) 12cm

Group-C

12) $8\sqrt{2}cm^2$ 13) 30° 14) 70° 15) 6cm

CHAPTER-11

CONSTRUCTIONS

Key points and formulae

- Geometrical construction means using only a ruler and a pair of compasses.
- To bisect a given angle.
- To draw the perpendicular bisector of a line segment.
- To construct angles of 15°, 30°, 45°, 60°, 90° etc.
- To construct a triangle given its base, a base angle and the sum of other two sides.
- To construct a triangle given its base, a base angle and the difference of other two sides.
- To construct a triangle given its perimeter and the two base angles.

Exercise - 11

Group-A

1. Choose the correct answer : (1 mark each)

i) With the help of a ruler and a compass, it is possible to construct an angle of–

(a) 36° (b) 40° (c) 37.5° (d) 47.5°

ii) With the help of a ruler and a compass, it is not possible to construct an angle of–

a) 14.5° b) 15° c) 135° d) 22.5°

iii) If a, b and c are the lengths of the three sides of a triangle, then which of the following is true?

a) 44.5 m b) 89m c) 46.5 m d) None of these.

iv) To construct a bisector of a given angle, we need–

a) A ruler b) A compass c) A protector d) both ruler and compass.

v) If we want to construct a triangle, given its perimeter, then we need to know–

a) Sum of two sides b) One base angle c) Two base angle

d) Difference between two sides of triangle.

vi) The construction of a triangle ABC in which AB=4 cm, $\angle A=60°$ is not possible, when difference of BC and AC is equal to :

(a) 3.5 cm (b) 4.5 cm (c) 3 cm (d) 2.5 cm

Group -B

2. Short answer type questions : (2 marks each)

i) Construct an angle of 112.5° and bisect it.

ii) Draw a line segment AB of 6cm in length. Draw a line perpendicular to AB through A and B respectively. Are these lines parallel?

iii) Construct a square of side 4cm.

iv) Construct a rectangle whose length is 5cm and breadth is 3.5cm.

v) Construct a triangle whose sides are 3.7 cm, 4.2 cm and 4.8 cm and measure all the angles.

vi) Construct a rhombus whose side is of length 4cm and one of its angle is 60°.

vii) Construct a right angled triangle whose hypotenuse measures 8cm and one side 6cm.

Group-C

3. Long answer type questions : (3/4 marks each)

i) Construct an equilateral triangle if its altitude is 3.6 cm.

ii) Construct a Δ ABC in which BC=4.5 cm, $\angle B=60°$ and AB+AC=8cm.

iii) Construct Δ ABC in which AB+BC+AC=12cm, $\angle B=45°$ and $\angle C=60°$.

iv) Construct Δ PQR in which QR=3cm, $\angle PQR=45°$ and QP–PR = 2cm.

v) Construct a rhombus whose diagonals are 4cm and 6cm.

Answer

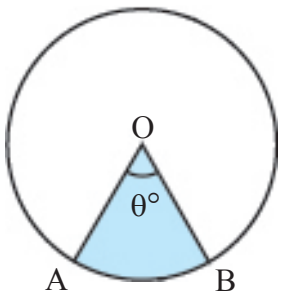
1. (i) c (ii) a (iii) b (iv) d (v) c (vi) b

CHAPTER-12

AREAS RELATED TO CIRCLES

Key points and formulae

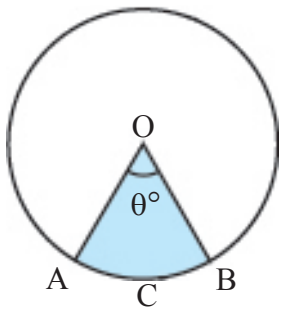
- Area of a circle = πr^2
- Circumference of a circle = $2\pi r$
- Length of an arc subtending an angle $\theta°$ at the centre = $\frac{2\pi r\theta}{360} = \frac{\pi r\theta}{180}$



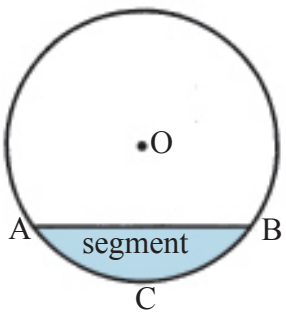
- Central Angle : Angle subtended by an arc at the centre of the circle is called central angle i.e $\angle AOB$
- Sector : Sector of a circle is a region enclosed by an arc of a circle and its two bounding radii.

Area of sector OACBO = $\frac{\pi r^2\theta}{360}$

Perimeter of sector OACBO = $\left[2r + \frac{2\pi r\theta}{360}\right]$



- Segment : A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.
- Area of minor segment ACBA = $\left[\frac{\pi r^2\theta}{360} - \frac{1}{2}r^2 sin\theta\right]$



Exercise - 12

Group-A (1 mark each)

Very short answer type questions :

I. Fill in the blanks :

i) Diameter is the _____ chord of a circle.

ii) A continuous piece of a circle is called the _____ of the circle.

iii) A segment of a circle is the region between an arc and a _____ of the circle.

iv) The radii of two concentric circles are 19cm and 16cm respectively. The area of the ring enclosed by these circle is _____.

v) Two circles having the same centre and different radii arc called _____ circles.

vi) Area of the sector of angle $\theta = \frac{\theta}{360} \times$ _____.

vii) The radius of a wheel is 0.25m. To covering 11km of distance, the wheel will have _____ no. of revolutions.

viii) The length of an arc, making a sector angle of $x°$ at the centre is _____

ix) If R and r be the outer and inner radii of a ring, then the area of the ring is _____.

x) A square ABCD is inscribed in a circle of radius r. The area of the square is _____.

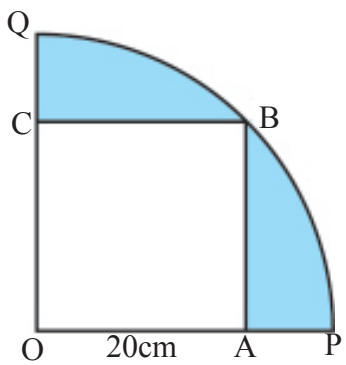
II. Multiple choice questions : (1 mark)

i) The diametre of a wheel is 84 cm. How many revolutions will it make to cover 792m?

(a) 200 (b) 250

(c) 300 (d) 350

ii) In the given figure, a square OABC has been incribed in the quadrant OPBQ. If OA=20cm then the area of the shaded region is (take π=3.14)



(a) 214cm^2 (b) 228cm^2

(c) 242cm^2 (d) 248cm^2

iii) If the perimeter of a square is equal to the circumference of a circle then the ratio of their areas is–

a) $4 : \pi$ b) $\pi : 4$ c) $\pi : 7$ d) $7 : \pi$

iv) The area of a circle is 49π cm^2. Its circumference is

a) 7π cm b) 14π cm c) 21π cm d) 28π cm

v) The circumferences of two circles are in the ratio 3:4. The ratio of their areas is–

a) 3:4 b) 4:3 c) 9:16 d) 16:9

vi) The length of the minute hand of a clock is 21cm. The area swept by he minute hand in 10 minutes is –

(a) 231cm^2 (b) 210cm^2 (c) 126cm^2 (d) 252cm^2

vii) The perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm. The area of the sector is–

a) 48.4cm^2 b) 84.4cm^2 c) 44.4cm^2 d) 44.8cm^2

viii) Area of a sector of angle ϕ of a circle with radious R is–

(a) $\frac{\phi}{180}\times 2\pi R$ (b) $\frac{\phi}{180}\times \pi R^2$ (c) $\frac{\phi}{360}\times 2\pi R$ (d) $\frac{\phi}{720}\times 2\pi R^2$

ix) A chord of a circle of radius 14cm makes a right angle at the centre. The area of the sector is–

(a) 154cm^2 (b) 164cm^2 (c) 145cm^2 (d) 146cm^2

x) The area of a square is the same as the area of a circle. Their perimeters are in the ratio.

a) 1:1 b) $2 : \pi$ c) $\pi : 2$ d) $\sqrt{\pi} : 2$

xi) On decreasing the radius of a circle by 30% its area is decreased by–

a) 30% b) 60% c) 45% d) None of these

xii) If the sum of the areas of two circle with radii R_1 and R_2 is equal to the area of a circle of radius R then.

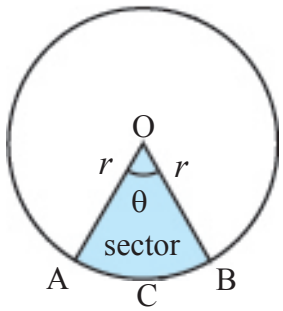
a) $R_1 + R_2 = R$ b) $R_1 + R_2 < R$ c) $R_1^2 + R_2^2 < R^2$ d) $R_1^2 + R_2^2 = R^2$

III. **Answer the following questions :**

i) Find the area of a ring whose outer and inner radii are respectively 20cm and 15cm?

ii) Find the area of a circle whose circumference is 66 cm?

iii) Find the perimeter of sector OACBO?



iv) If the perimeter and area of a circle are numerically equal, then find the radius of the circle?

v) The area of two circles are in the ratio 9:4. Find the ratio of their circumferences.

vi) In making 1000 revolutions, a wheel covers 88km. What is the diameter of the wheel?

vii) What is the measure of angle described by minute hand in 60 minutes.

viii) Write the name of an angle subtended by an arc at the centre of a circle?

ix) The area of incircle of an equilateral triangle is 154 cm^2 . Find the perimeter of the triangle.

x) Write the area of the sector of a circle whose radius is r and length of the arc is l units.

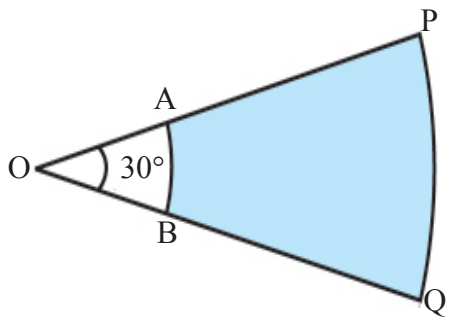
Group-B (2 marks each)

Short answer type questions :

1) The difference between the circumferencce and radius of a circle is 37cm. Using $\left(\pi = \frac{22}{7}\right)$ find the circumference of the circle.

2) Find the area of the sector of a circle having radius 6cm and of angle 30° (take π=3.14)

3) In the given figure, PQ & AB are the arcs of two concentric circles of radii 7cm and 3.5cm with centre O. If $\angle POQ=30°$, find the area of the shaded region?



4) Find the area of a quadrant of a circle whose circumference is 44cm.

5) The circumference of a circle is 8cm. Find the area of the sector whose central angle is 72°.

6) In a circle, the major arc is 3 times the minor arc. Find the corresponding central angles and the degree measures of two arcs?

7) A path of 8m width runs around the outside of a circular park whose radius is 17m. Find the area of the path?

8) A copper wire when bent in the form of a square encloses an area of $484cm^2$. The same wire is now bent in form of a circle. Find the area of this circle?

9) A chord of a circle of radius 30cm makes an angle of 60° at the centre of the circle. Find the area of the minor segment. (Take $\pi=3.1$, $\sqrt{3}=1.732$)

10) The radius of a circle is 5cm. A chord of length $\sqrt{50}$ cm is drawn in the circle. Find the area of the major segment.

11) The perimeter of the quadrant of a circle is 25cm. Find its area.

12) The minute hand of a clock is 12cm long. Find the area on the face of the clock described by the minute hand between 8 AM and 8.35 AM.

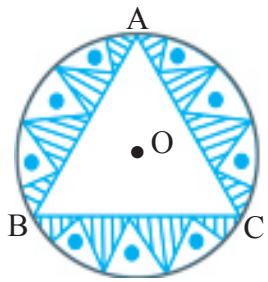
Group-C (3/4 marks each)

Long answer type questions :

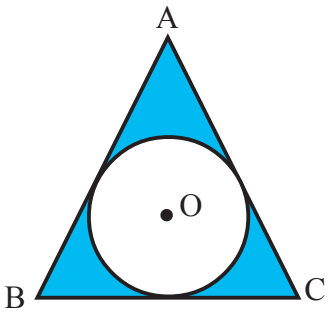
1) Rakhal has a cart whose wheels are making 4 revolutions per second. If the diameter of the wheel is 77 cm, find the speed of the cart.

2) Anup walks around a circular park of area 88704 sq.m. How long will he take to walk 10 rounds at the speed of 4.5 km per hour?

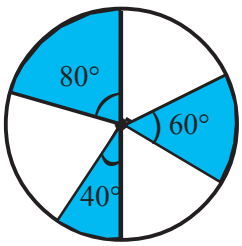
3) In a circular table cover of radius 42cm, a design is formed, leaving an equilateral triangle ABC in the middle, as shown in the figure. Find the area of the design [use $\sqrt{3}=1.73$].



4) In an equilateral triangle of sides 12cm, a circle is inscribed touching its sides. Find the area of the portion of the triangle not included in the circle. (Take $\sqrt{3}=1.73$, $\pi=3.14$]



5) In the given figure, three sectors of a circle of radius 7cm, making angle of 60°, 80° and 40° at the centre are shaded. Find the area of the shaded region.



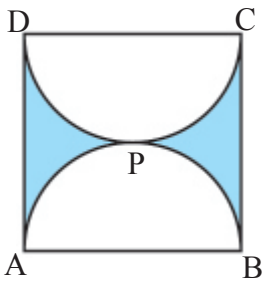
6) The difference between the area of a circle and the square of its radius is 16.8m^2. Find the radius of the circle.

7) A pendulum swings through an angle 60° and describes an arc 8.8cm in length. Find the length of the pendulum.

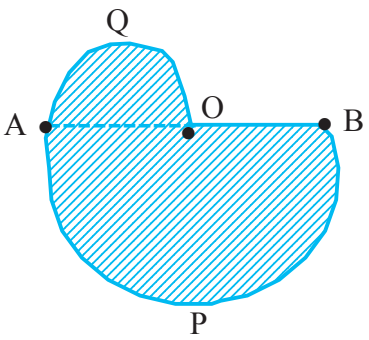
8) An umbrella has 8 ribs which are equaly spaced. Assuming umbrella to be a flat circle of radius 45cm. Find the area between the two consecutive ribs of the umbrella.

9) A square of side 4cm is inscribed in a circle. Find the area enclosed between the circle and the square.

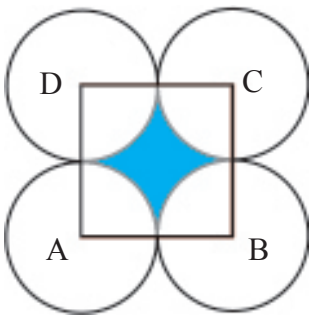
10) Find the perimeter of the shaded region in the figure, if ABCD is a square of side 14cm and APB and CPD are semicircles.



11) In the given figure, APB and AQO are semi circles and AO=OB. If the perimeter of the figure is 40 cm. Find the area of the shaded region.



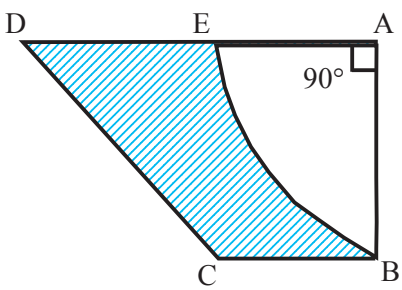
12) In the given figure, ABCD is a square, each of whose sides measures 28cm. Find the area of the shaded Region (Take $\pi = \frac{22}{7}$)



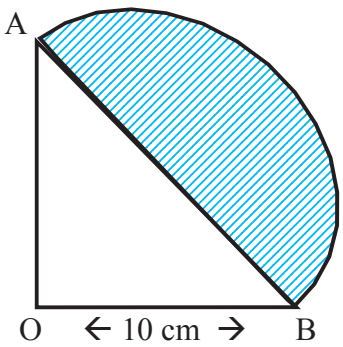
13) A chord of a circle of radius 30 cm makes an angle of 60° at the centre of the circle. Find of areas of the minor and major segments [take $\sqrt{3}$ =1.732, π = 3.14]

14) The radius of a circular garden is 100m. There is a road 10m wide, running all round it. Find the area of the road and the cost off levelling it at ₹ 20 per m^2 [use π =3.14]

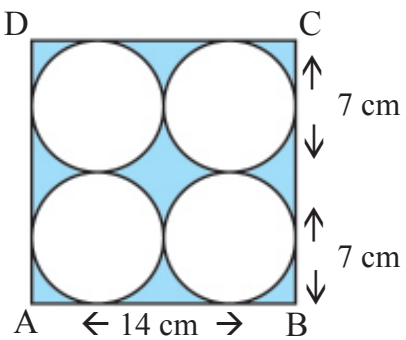
15) In the given figure, ABCD is a trapezium of area 24.5cm^2. If AD||BC, $\angle DAB = 90°$, AD = 10cm, BC = 4cm and ABE is quadrant of a circle then find the area of the shaded region.



16) Find the area of the first quadrant of shaded portion of the given figure.



17) Find area of the shaded region in the given figure where ABCD is a square of side 14 cm.



18) The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its area equal to the sum of the areas of the two circles.

Answers

Group-A

I. (i) Longest (ii) arc (iii) chord (iv) 330 cm^2 (v) concentric (vi) πr^2

vii) 7000 (viii) $\left(\frac{x}{360}\right)2\pi r$ (ix) $\pi (R^2 - r^2)$ (x) $2r^2$ square units

II. (i) c (ii) b (iii) b (iv) b (v) c (vi) a (vii) d (viii) d (ix) a (x) d (xi) d (xii) d

III. (i) 550cm^2 (ii) 346.5 cm^2 (iii) $\left(2r+\frac{\pi r\theta°}{180°}\right)$ (iv) 2 (v) 3:2 (vi) 28m

(vii) 360° (viii) Centre Angle (ix) 72.7 cm (x) $\frac{1}{2}$ *l*r

Group-B

(1) 44 cm (2) 9.42cm (3) $\frac{77}{8}cm^2$ (4) 38.5cm^2 (5) 123.2 cm^2

(6) 270° and 90° (7) 1056 m^2 (8) 616cm^2 (9) 81.3cm^2 (10) 71.375 cm^2
(11) 38.5cm^2 (12) 264 cm^2

Group-C

(1) 34.84 km/hour (2) 2 hours 20 minuts 48 secs. (3) 3255.21cm^2 (4) 24.6cm^2

(5) 77cm^2 (6) $\frac{14}{5}m$ (7) 8.4 cm. (8) 795.54cm^2 (9) $9\frac{1}{7}$ cm^2 (10) 72cm

(11) 96.25cm^2 (12) 168cm^2 (13) 81.75cm^2, 2744.25cm^2 (14) 6594m^2, ₹13188
(15) 14.875cm^2 (16) 28.5 cm^2 (approx) (17) 42 cm^2 (r) 10cm

CHAPTER-13

SURFACE AREA AND VOLUMES

Key points and formulae

- The surface area of an object formed by combining any two of the basic solids namely, cuboid, cone, cylinder, sphere and hemisphere.
- The volume of an object formed by combining any two of the basic solids namely, cuboid, cone, cylinder, sphere and hemisphere.
- The formulae involving the frustum of a cone are–

i) Volume of the frustum of the cone = $= \frac{1}{3}\pi h\left[r_1^2 + r_2^2 + r_1 r_2\right]$ cubic units

ii) Curved surface area of the frustum of the cone = $\pi(r_1 + r_2)l$, square units

iii) Total surface area of the frustum of the solid cone = $\pi l\ (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$ where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

h = vertical height of the frustum,

l = Slant height of the frustum and r_1 and r_2 are radii of the two bases (ends) of the frustum.

- Solid hemisphere : If r is the radius of a hemisphere, then curved surface area = $2\pi r^2$ total surface area = $3\pi r^2$ and volume = $\frac{2}{3}\pi r^3$

- Volume of a spherical shell = $\frac{4}{3}\pi\left(r_1^3 - r_2^3\right)$, where r_1 and r_2 are respectively its external and internal radii.

Through out this chapter, consider $\pi = \frac{22}{7}$, if not otherwise stated.

Exercise-13

Group-A (1 mark each)

Very short answer type questions :

1. Fill in the blanks :

a) The shape of a Funnel is a combination of ——— of a cone and a cylinder.

b) The volume of a cone is ———, in which radius of the base r and height h.

c) During conversion of a solid from one shape to another, the volume of the new shape will ———.

d) If the radius of a sphere becomes 3 times then its volume will become —— times.

e) The playing top (Lattu) is a solid which is the combination of a cone and the ——.

f) The diameter of a sphere is 14cm. Its volume is ——.

g) g) The radii of the circular ends of a bucket in the form of frustum of a cone of height 30cm are 20cm and 10cm respectively. The capacity of the bucket is —— cm^3. $\left(\pi = \frac{22}{7}\right)$

h) Volume of cylinder of base radius r and height h is given by v = —— cubic units.

i) A hemisphere of radius r cm, has a total surface area ——— cm^2.

j) The curved surface area of a cone of base radius 3cm and height 4cm is —— cm^2.

k) If the radii of the circular ends of the frustum of a cone are R and r respectively and its height is h, then its surface area is —— where $l^2 = h^2 + (R - r)^2$.

l) The radii of the circular ends of a conical bucket of height 15cm are 20cm and 12 cm respectively. The slant height of the bucket is —— cm.

m) The volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is ——.

n) The total surface area of a hemisphere of radius 7cm is ——— cm^2.

2. Multiple Choice Questions :

i) The shape of a glass (tumbler) (Fig 13.1) is usually in the form of

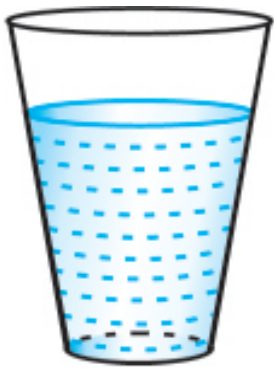


Fig 13.1

(a) a cylinder (b) frustum of a cone (c) a cone (d) a sphere.

ii) A cone is cut by a plane parallal to its base and the upper part is removed. The part that is left over is called–

(a) a cone (b) a sphere (c) a cylinder (d) frustum of a cone.

iii) In a right circular cone, the cross section made by a plane parallal to the base is a

a) sphere b) hemisphere c) Circle d) a semi circle.

iv) A solid sphere of radius 9cm is melted to form a solid cylinder of radius 9cm. The height of the cylinder is

a) 12 cm b) 18 cm c) 36 cm d) 96 cm

v) If the surface area of a sphere is 616 cm^2, its radius (in cm) is –

(a) 3.5 (b) 7 (c) 14 (d) 19

vi) On increasing each of the radius of the base and the height of a cone by 20% its volume will be increased by

(a) 20% (b) 60% (c) 72.8% (d) 40%

vii) The curved surface area of a cylindrical pillar is 264m^2 and its volume is 924m^3. The height of the pillar is –

a) 6m b) 7m c) 8m d) 9m

viii) A solid piece of iron in the form of a cuboid of dimensions (49cm×33cm×24cm)is melted to form a solid sphere. The radius of the sphere is——

a) 17cm b) 19cm c) 21cm d) 23cm

ix) If two solid hemisphers of same base radius *r* are joined together along their bases, then curved surface area of this new solid is–

a) $4\pi r^2$ b) $6\pi r^2$ c) $3\pi r^2$ d) $8\pi r^2$

x) The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is–

a) 9.7 cm^2 b) 77.6 cm^2 c) 58.2 cm^2 d) 19.4 cm^2

3. **Answer the following questions :**

a) The radii of the ends of a bucket, 30cm high, are 21cm and 7cm. Determine, the area of the sheet required to make this bucket?

b) The radius of a cylindrical vessel is 15cm and its height is 25cm. Find its volume.

c) What is the combination of solid shapes, makes a funnel?

d) The surface areas of two spheres are in the ratio 16:9. Find the ratio of their volumes?

e) The areas of three adjacent faces of a cuboid are x, y, z respectively. Find its volume?

f) Five identical cubes, each of edge 5cm, are placed adjacent to each other. Find the volume of the resulting cuboid?

g) The volume of a hemisphere is 19404 cm^3. Find its total surface area.

h) The radii of two cylinders are in the ratio 2:3 and their heights are in the ratio of 5:3. Find the ratio of their volumes?

i) A cylinder with base radius 8cm and height 2cm is melted to form a cone of height 6cm. Calculate the radius of the base of the cone?

j) How many lead shots each 3mm in diameter can be made from a cuboid of dimension 9cm ×11cm ×12cm?

Group-B

Short Answer Type Questions :- (2 marks)

1) The length of a cold storage is double its breadth. Its height is 3 meters. The areas of its four walls (including door) is $108m^2$. Find its volume.
2) A solid sphere of radius 3cm is melted and then recast into small spherical balls each of diameter 0.6cm. Find the number of small balls thus obtained.
3) The radii of the internal and external surfaces of a hollow spherical shell are 3cm and 5cm respectively. If it is melted and recast into a solid cylinder of height $2\frac{2}{3}$ cm, Find the radius of the cylinder.
4) A rectangular water tank of base 11m × 6 m contains water upto a height of 5m. If the water in the tank is transferred into a cylindrical tank of radius 3.5m, Find how high will the water level be in the tank?
5) The volume of a cube is 2744 cm^3. Find its surface area?
6) The diameter of a cylinder is 28cm and its height is 20cm. Find its total surface area?
7) If each edge of a cube increased by 50% than at what percentage of increase may have, its surface area?
8) The radii of the top and bottom of a bucket of slant height 45cm are 28cm and 7cm respectively. Find its curved surface area?
9) The radius of the base and the height of a solid right circular cylinder are in the ratio 2:3 and its volume is 1617 cm^3. Find the total surface area of the cylinder?
10) A hemispherical bowl of internal diameter 30cm is full of a liquid. This liquid is filled into cylindrical shapped bottles each of diameter 5cm and height 6cm. How many bottles are required?
11) The height of a conical tent is 14m and its floor area is $346.5m^2$. How much canvas 1.1m wide, will be required for it.
12) What is the ratio of the total surface area to the lateral surface area of a cylinder with base radius 80cm and height 20cm?
13) How many bricks each measuring (25cm ×11.25cm × 6cm) will be required to construct a wall (8m ×6m × 22.5cm)?

14) How many bags of grain can be stored in a cuboidal granary (8m ×6m × 3m), if each bag occupics a space of $0.64m^3$?

15) If the volumes of two cones are in the ratio of 1:4 and their diameters arc in the ratio of 4:5. Find the ratio of their heights?

16) A copper rod of diameter 2cm and length 10cm is drawn into a wire of uniform thickness and length 10m. Find the thickness of the wire.

17) The raddii of the circular ends of a solid frustum of a cone are 33cm and 27cm and its slant height is 10cm. Find its surface area? (π=3.14)

Group-C

Long Answer Type Questions : (3/4 marks each)

1) A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel?

2) The internal and external diameters of a hollow hemispherical vessel are 24cm and 25cm respectively. If the cost of painting $1cm^2$ of the surface area is Rs. 5.25. Find the total cost of painting the vessel all over?

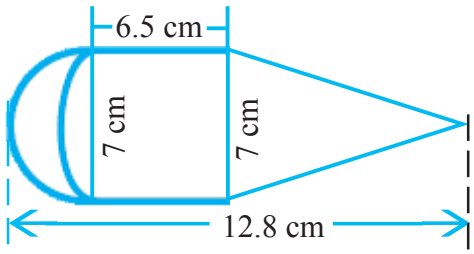
3) How many meters of cloth 5m wide will be required to make a conical tent, the radius of whose base is 7m and height is 24m?

4) A circus tent is cylindrical to a height of 3m and conical above it. If its base radius is 52.5m and slant height of the conical portion is 53m, find the area of the canvas required to make the tent. $(\pi = 22/7)$

5) From a solid right circular cylinder with height 12cm and radius of the base 5cm, a right circular cone of the same height and the same base radius is removed. Find the volume and total surface area of the remaining solid. (use π=3.14)

6) A toy is in the form of a cylinder with hemispherical ends. If the whole length of the toy is 90cm and its diameter is 42cm, find the cost of painting the toy at the rate of 70 paise per sq cm.

7) The given figure represents a solid consisting of a cylinder surmounted by a cone at the one end and a hemisphere at the other. Find the volume of this solid.



8) A solid is in the shape of a cone mounted on a hemisphere of same base, radius. If the curved surface area of the hemispherical part and the conical part are equal then find the ratio of the radius and the height of the conical part.

9) A cylindrical tub of radius 5cm and length 9.8cm is full of water. A solid in the form of a right circular cone mounted on a hemispher is immersed into the tub. If the radius of the hemisphere is 3.5 cm and the total height of he solid is 8.5cm, find the volume of water left in the tub.

10) From a cuboidal solid metallic block of dimensions 15cm×10cm ×5cm, a cylindrical hole of diameter 7cm and height 1.5 cm is drilled out. Find the surfacce area of the remaining block.

11) A toy is in the shape of a cone mounted on a hemisphere of same base radius. If the volume of the toy is 231 cm^3 and its diameter is 7cm. Find the height of the toy?

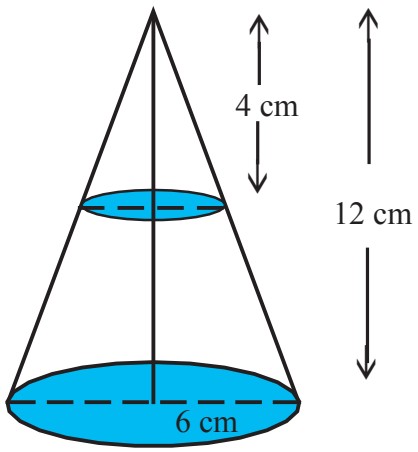
12) A cubical block of side 10cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed at the rate of ₹5 per 100 sq.cm. (use π=3.14)

13) Find the number of coins, 1.5cm in diameter and 2mm thick, to be melted to form a right circular cylinder of height 10cm and diameter 4.5 cm.

14) In a village, a well with 10m inside diameter, is dug 14m deep. Earth taken out of it is spread all around to a width of 5m to form an embankment. Find the height of the embankment.

15) An open metal bucket is in the shape of a frustum of a cone of height 21cm with radii of its lower and upper ends as 10cm and 20cm respectively. Find the cost of milk which can completely fill the bucket at ₹30 per litre.

16) In the figure, from the top of the solid cone of height 12cm and base radius 6cm a cone of height 4cm is removed by a plane parallel to its base. Find the total surface area of the remaining solid. [use $\pi = \frac{22}{7}$ and $\sqrt{5}=2.236$].



17) The radii of the circular ends of a bucket of height 15cm are 14cm and r cm (r<14cm), If the volume of bucket is 5390 cm^3. Find the value of r.

18) The slant height of the frustum of a cone is 4cm and the perimeters of its circular ends are 18cm and 6cm. Find the curved surface area of the frustum.

19) A drinking glass is in the shape of a frustum of a cone of height 14cm. The diameters of its two circular ends are 16cm and 12cm. Find the capacity of the glass.

Answer

Group-A

1. (a) Frustum (b) $\frac{1}{3}\pi r^2 h$ (c) remain unaltered (d) 27 (e) hemisphere

   (f) 1437 $\frac{1}{3}cm^3$ (g) 22000 (h) $\pi r^2 h$ (i) $3\pi r^2$ (j) 15π (k) $\pi\,[R^2+r^2+l\,(R+r)]$

   (l) 17 (m) 16:9 (n) 462 cm^2

2. (i) b (ii) d (iii) c (iv) a (v) b (vi) c (vii) a (viii) c (ix) a

   (x) d

3. (a) 3066.8 cm^2 (b) 5625π cm^3 (c) a cylinder and a frustum of a cone

   (d) 64:27

   (e) $\sqrt{xyz}$ (f) $625cm^3$ (g) 4158 cm^2 (h) 20:27 (i) 8cm (j) 84000

Group-B

(1) $216m^3$ (2) 1000 (3) 7cm (4) 8.6m (appox) (5) $1176cm^2$

(6) $2992cm^2$ (7) 125% (8) $4950cm^2$ (9) $770cm^2$ (10) 60 (11) 525m

(12) 5:1 (13) 6400 (14) 225 (15) 25:64 (16) 2mm (17) 7592.52 cm^2

Group-C

(1) 18cm (2) ₹10110.40 (3) 110 m (4) $9735m^2$ (5) $628cm^3$, $659.4cm^2$

(6) ₹8316 (7) $376.016cm^2$ (8) $1:\sqrt{3}$ (9) $616cm^2$ (10) 553.3 cm^2

(11) 14.5cm

(12) 10cm, ₹53.93 (13) 450 (14) $\frac{14}{3}$m (15) ₹462 (16) $350.592cm^2$

(17) $r=7$

(18) 48 cm^2 (19) $2170.67cm^3$

CHAPTER-14

STATISTICS

Key points and formulae

Measures of Central Tendency

a) *Mean of grouped data*

(i) To find the mean of grouped data, it is assumed that the frequency of each class interval in centred around its mid-point.

(ii) Direct method

Mean $(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$

Where the x_i (class mark) is the mid-point of the ith class interval is the corresponding frequency.

(iii) Assumed Mean method

Mean $(\bar{x}) = a + \frac{\sum f_i d_i}{\sum f_i}$, a is the assumed mean and $d_i = x_i - a$ are the deviations of x_i from a for each i.

(iv) Step - deviation method

Mean $(\bar{x}) = a + h\frac{\sum f_i u_i}{\sum f_i}$

Where a is the assumed mean, h is the class size and $u_i = \frac{x_i - a}{h}$

(v) If the class sizes are unequal, the formula in (iv) can still be applied by taking h to be a suitable divisor of all the d_i's.

b) *Mode of grouped data*

(i) In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class in known as the modal class. The mode of the data is a value inside the modal class.

(ii) Mode of the grouped data can be calculated by using the formula

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where l is the lower limit of the modal class, h is the size of the class, f_1 is frequency of the modal class and f_0 and f_2 are the frequencies of the classes preceding and succeeding the modal class, respectively.

c) *Median of grouped data*

(i) Cumulative frequency table to be made for the less than type and the more than type of the grouped frequency distribution.

(ii) If N is the total number of observations, locate the class whose cumulative frequency is greater than (and nearest to) $\frac{N}{2}$. This class is called the median class.

(iii) Median of the grouped data can be calculated by using the formula :

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f} \right) h$$

Where l is the lower limit of the median class, N is the number of observations. h is the class size, cf is the cumulative frequency of the class preceeding the median class and f is the frequency of the median class.

d) Relation between mean, median and mode : Mode = 3 (Median) –2 (Mean)

e) Graphical representation of Cumulative Frequency distribution is done by two type Ogive- less than type and more than type.

(i) To find median from the graph of cumulative frequency distribution (less than type) of a grouped data.

(ii) To find median from the graphs of cumulative frequency distribution (of less than type and more than type) as the abscissa of the point of intersection of the graphs.

Group-A

Very short answers type questions : **(1 mark each)**

Choose the correct answer

1. The algebraic sum of the deviations of a frequency distribution from its mean is–

a) Always positive b) Always negative

c) 0 d) a non zero number

2. For a frequency distribution, mean median and mode are connected by the relation–

   a) Mode = 3 mean – 2 median

   b) Mode = 2 median – 3 mean

   c) Mode = 3 median – 2 mean

   d) Mode = 3 median + 2 mean

3. The mean of a discrete frequency distribution $x_i \mid f_i$; $i = 1,2, ..., n$ given by

   a) $\frac{\sum f_i x_i}{\sum f_i}$ b) $\frac{1}{n}\sum_{i=1}^{n} f_i x_i$ c) $\frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} x_i}$ d) $\frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} i}$

4. If the mean of observations $x_1, x_1, ... , x_n$ is $\bar{x}$, then the mean of $x_1 + a, x_2 + a, ... x_n + a$ is–

   a) $a\bar{x}$ b) $\bar{x} + a$ c) $\bar{x} - a$ d) $\frac{\bar{x}}{a}$

5. Consider the following distribution.

Class	0-5	5-10	10-15	15-20	20-25
Frequency	13	10	15	8	11

   The upper limit of the median class is–

   a) 5 b) 10 c) 15 d) 20

6. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives us its–

   a) mean b) median c) mode d) none of these.

7. For the following distribution,

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

   The sum of the lower limits of median class and modal class is

   a) 15 b) 25 c) 30 d) 35

8. For the following distribution

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

The Modal class is–

a) 10-20 b) 20-30 c) 30-40 d) 50-60

9. Consider the following distribution

Marks obtained	Number of students
More then or equal to 0	63
More then or equal to 10	58
More then or equal to 20	55
More then or equal to 30	51
More then or equal to 40	48
More then or equal to 50	42

The frequency of the class 30-40 is–

a) 3 b) 4 c) 48 d) 51

10. While computing mean of grouped data, we assume that the frequencies are–

a) Evenly distributed over all the classes.

b) Centred of the class marks of the class.

c) Centred at the limits of the classes.

d) Centred at the upper limits of the classes.

Fill in the blanks :

11. The mode of the data 2, x, 3, 4, 5, 2, 4, 6 (where $x = 4$) is _____.

12. The mean of first 673 natural numbers is _____.

13. If the mean and median of a unimodal data are 34.5 and 32.5 respectively, then mode of the data is _____.

14. Mean of the observations 1, 3, 5, 7....., 99 is _____.

15. If the median of the observations $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$, is m, then the median of the observations x_3, x_4, x_5, x_6 (where $x_1 \angle x_2 \angle x_3 \angle x_4 \angle x_5 \angle x_6 \angle x_7 \angle x_8$) is _____

16. If the mean of 26, 19, 15, 24 and x is x, then the median of the data is ____.

Answer the following questions either in one word or one sentence or as per requirement of the question :

1. Define mean.

2. What is the algebraic sum of deviations of a frequency distribution from its mean?

3. Write the emperical relation between mean, median and mode.

4. Which measure of central tendency is given by the x-coordinate of the point of inersection of the 'more than' ogive and 'less than' ogive.

5. Write the modal class for the following frequency distribution.

Class interval	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	30	35	75	40	30	15

6. Write the median class for the following frequency distribution.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

7. Find the class marks of classes 10-25 and 35-55.

8. Write the median class for the following distribution.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

9. Write the difference of lower limit of median class and Modal class for the following distribution.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

Group-B

Short Answer Type Questions : (2 marks each)

1. Find the mean of the distribution

Class	1-3	3-5	5-7	7-10
Frequency	9	22	27	17

2. Calculate the mean of the following data

Class	3.5-7.5	7.5-11.5	11.5-15.5	15.5-19.5
Frequency	5	4	9	10

3. An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given in the following table

Number of seats	100-104	104-108	108-112	112-116	116-120
Frequency	15	20	32	18	15

Determine the mean number of seats occupied over the flights.

4. The mileage (km per Litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below :

Mileage (KmL^{-1})	10-12	12-14	14-16	16-18
Number of cars	7	12	18	13

Find the mean mileage

5. The following is the distribution of weight (in kg) of 40 persons.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80
Frequency	4	4	13	5	6	5	2	1

Construct of cumulative frequency distribution (of the less than type) table for the data above.

6. The following table shows the cumulative frequency distribution of marks of 800 students in an examination.

Marks	**Number of students**
Below 10	10
Below 20	50
Below 30	130
Below 40	270
Below 50	440
Below 60	570
Below 70	670
Below 80	740
Below 90	780
Below 100	800

Construct a frequency distribution for the above data

7. From the frequency distribution table from the following data.

Marks (out of 90)	**Number of candidates**
More then or equal to 80	4
More then or equal to 70	6
More then or equal to 60	11
More then or equal to 50	17
More then or equal to 40	23
More then or equal to 30	27
More then or equal to 20	30
More then or equal to 10	32
More then or equal to 0	34

8. Given below is a cumulative frequency distribution showing he marks secured by 50 students of a class.

Marks	below 20	below 40	below 60	below 80	below 100
Number of students	17	22	29	37	50

Form the frequency distribution table for the data.

9. Weekly income of 600 families is tabulated below.

Weekly income (in ₹)	Number of families
0 –1000	250
1000–2000	190
2000–3000	100
3000–4000	40
4000 – 5000	15
5000 – 6000	5

Compute the median income

10. The monthly income of 100 families are given as below :

Income (in ₹)	Number of families
0 –5000	8
5000–10,000	26
10,000–15,000	41
15,000–20,000	16
20,000 – 25,000	3
25,000 – 30,000	3
30,000 – 35,000	2
35,000 – 40,000	1

Calculate the modal income

11. The weight of coffee in 70 packets are shown in the following table.

Weight (in gm)	Number of packets
200 – 201	12
201– 202	26
202 – 203	20
203 – 204	9
204 – 205	2
205 – 206	1

Determine the modal weight

Group-C

Long Answer Type Questions : (3/4 marks each)

1. Find the mean age of 100 residents of town from the following data.

Age (in years)	Number of person
Equal and above 0	100
Equal and above 10	90
Equal and above 20	75
Equal and above 30	50
Equal and above 40	25
Equal and above 50	15
Equal and above 60	5
Equal and above 70	0

2. The weights of tea in 70 packets are shown in the following table

Weight (in gm)	Number of packets
200 – 201	13
201– 202	27
202 – 203	18
203 – 204	10
204 – 205	1
205 – 206	1

Find the mean weight of the packets.

3. Following is the cumulative frequency distribution (of less than type) of 1000 persons, each of age 20 years and above. Determine the mean age.

Age	Number of persons
Below 30 years	100
Below 40 years	220
Below 50 years	350
Below 60 years	750
Below 70 years	950
Below 80 years	1000

4. The weights of turmeric in 70 packets are shown in the following table. draw the less than type ogive for this data and use it to find the median weight.

Weight (in gm)	**Number of packets**
200 – 201	13
201– 202	27
202 – 203	18
203 – 204	10
204 – 205	1
205 – 206	1

5. The annual rainfall record of a city for 66 days is given in the following table.

Rainfall (in cm)	0-10	10-20	20-30	30-40	40-50	50-60
Number of days	22	10	8	15	5	6

Calculate the median rainfall using ogives of more than type and less than type.

6. Size of agricultural holdings in a survey of 200 families is given in the following table .

Size of Agricultural holdings (in hec)	**Number of families**
0 –5	10
5–10	15
10–15	30
15–20	80
20 – 25	40
25 – 30	20
30 – 35	5

Compute median and mode size of the holdings.

7. The table below shows the salaries of 280 persons

Salary (in ₹ 1000)	**Number of families**
5–10	49
10–15	133
15–20	63
20 – 25	15
25 – 30	6
30 – 35	7
35 – 40	4
40 – 45	2
45 – 50	1

8. The following is the frequency distribution of duration for 100 calls made on a mobile phone.

Duration (in sec)	**Number of calls**
95 –125	14
125 – 155	22
155 – 185	28
185 – 215	21
215 – 245	15

Calculate the average duration (in sec) of a call and also find the median from a cumulative frequency curve.

9. Find the mean, median and mode of the following data.

Classes	0-50	50-100	100-150	150-200	200-250	250-300	300-350
Frequency	2	3	5	6	5	3	1

10. Find the mean, median and mode of the following data

Classes	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	6	8	10	12	6	5	3

Answers

Very short answers type

Choose the correcct answer

1. (c) 2. (c) 3. (a) 4. (b) 5. (c) 6. (b) 7. (b) 8. (c) 9. (a) 10. (b)

Fill in the blanks :

11. 4 12. 337 13. 28.5 14. 50 15. m 16. 21

Answer the questions :

2. 0 3. Mode = 3 median – 2 Mean 4. Median 5. 20-25 6. 40-50

7. 17.5 and 45 8. 30–40 9. 0

Short Answer Type Questions :

1. 5.5 2. 12.93 3. 109.92 4. 14.98 km/lt.

5.

Weight (in kg)	Comulative frequency
Less than 45	4
Less than 50	8
Less than 55	21
Less than 60	26
Less than 65	32
Less than 70	37
Less than 75	39
Less than 80	40

6.

Class internal	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	10	40	80	140	170	130	100	70	40	20

7.

Class internal	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No of Candidate	2	2	3	4	6	6	5	2	4

8.

Marks	0-20	20-40	40-60	60-80	80-100
Number of Students	17	5	7	8	13

9. ₹ 1263.15 10. ₹ 11,875 11. 201.7 gm.

Long Answer Type Questions :

1. 31 years 2. 201.96 gm 3. 51.3 years 4. 201.8 gm 5. 21.25 cm
6. Median = 17.81, Mode = 17.77 7. Median = 13421, Mode = 12727
8. Mean 170.3 sec, Median 170 sec 9. Median = 170.83, Mean 169, Mode=175
10. Median = 61.66, Mean 62.4, Mode = 65

CHAPTER-15

PROBABILITY

Key points and formulae

For a finite sample space with equally likely outcomes, the probability of an event E, denoted by P(E), is given by

$$P(E) = \frac{\text{Number of outcomes favourable to the event E}}{\text{Total Number of possible outcomes.}}$$

ie, $P(E) = \frac{n(E)}{n(S)}$ where , the set of the possible outcomes is called the sample space, usually denoted by S.

Note:

1. $0 \leq P(E) \leq 1$, for any event E.
2. P (certain event) = 1, P (Sample event space) = 1
3. P (impossible event) = 0
4. $P(\bar{E}) = 1 - P(E)$

Excercise-15

Group-A (1 mark each)

Very short answer type questions :

1. Fill in the blanks

a) ——is a concept which numerically measures the degee of certainly of the occurrence.

b) The Probability of an impossible event is ——.

c) The Probability of a sure event is ——.

d) For any event E, P(E) + P (not E) = ———.

e) The sum of probabilities of all the outcomes of an experiment is —— .

f) A coin is tossed once. The Probability of getting a tail is ——.

g) A dice is thrown. The probability of getting a number more than 6 is ——.

h) In a simultaneous toss of two coins, the probability of getting exactly 2 Heads is ——

2. Choose the correct answer

a) Which of the following cannot be the probability of an event?

i) $\frac{1}{2}$ ii) 0.6 iii) 4% iv) $\frac{14}{13}$

b) If the probability of an event is þ. Then the probability of its complementary event will be

i) þ –1 ii) þ iii) 1– $\frac{1}{-}$ iv) 1–þ

c) The probability expressed as a percentage of a particular occurrence can never be

i) Less then 100 ii) less then 0 iii) greater then 1

iv) anything but a whole number.

d) A bag contains 3 white and 5 red balls. If a ball is drawn at randem, the probability that the drawn ball to be red is.

i) $\frac{3}{8}$ ii) $\frac{5}{8}$ iii) $\frac{3}{15}$ iv) $\frac{5}{15}$

e) The probability of getting an even number when a die is rolled–

i) $\frac{1}{6}$ ii) $\frac{1}{36}$ iii) $\frac{1}{2}$ iv) None of these

f) A card is drawn from a packet of numbered 1 to 100. The probability of drawing a card which is a square number, is–

i) $\frac{1}{10}$ ii) $\frac{9}{100}$ iii) $\frac{1}{100}$ iv) $\frac{2}{100}$

g) If P(A) denotes the probability of an event A, then

i) $P(A) < O$ ii) $P(A) > 1$ iii) $0 \leq P(A) \leq 1$ iv) $-1 \leq P(A) \leq 1$

h) When a die is thrown, the probability of getting an odd number less than 3 in

i) $\frac{1}{6}$ ii) $\frac{1}{3}$ iii) $\frac{1}{2}$ iv) 0

i) The probability of getting a bed egg in a lot of 400 is 0.035. The number of bed eggs in the lot is

i) 21 ii) 14 iii) 7 iv) 28

j) Someone is asked to take a number from 1 to 20. The probability that is a prime number is

i) $\frac{1}{5}$ ii) $\frac{9}{20}$ iii) $\frac{2}{5}$ iv) $\frac{7}{20}$

3. **Answer the following questions :**

a) A coin is tossed two times. list the possible outcomes.

b) In a single throw of two dice, what is the probability of getting a sum of 10?

c) One card is drawn at random from a well-shuffled deck of 52 cards. What is the probability that the card drawn is a face card?

d) Two dice are thrown together. Find the probability of getting a doublet?

e) A bag contain 8 red, 2 black, 5 white balls. One ball is drawn at random. What is the probability that the ball drawn is not black?

f) In a Lottery, there are 8 prizes and 16 blanks. What is the probability of getting a prize?

g) One card is drawn at random from a well-shuffled deck of 52 cards. What is the probability of getting a 6?

Group-B (2 marks each)

Short Type Questions

1) A bag contains 5 red balls, 8 white balls and 7 black balls. If one ball is drawn at random. Find the probability that it is not white?

2) A box contains 20 balls bearing numbers 1,2,3,4, ..., 20. A ball is drawn at random from the box. What is the probability that the number on the balls is an even Number?

3) The king, queen and Jack of clubs are removed from a deck of 52 playing cards and then well shuffled. One card is selected from the remaing cards. Find the probability of getting a king?

4) A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of a red balls, find the number of blue balls in the bag.

5) A letter of English alphabat is chosen at random. Determine the probability that the chosen leter is a consonant?

6) If the probability of winning a game is 0.4 then, what is the probability of losing it?

7) Cards bearing numbers 2,3,4,....., 11 are kept in a bag. A card is drawn at random from the bag. What is the probability of getting a card not a prime number?

8) One card is drawn at random from a well shuffled deck of 52 cards. Find the probability that the card drawn is (i) a red eight (ii) a spade

9) An one rupee coin has tossed in three times. What are the possible outcomes of it?

10) All the black face cards are removed from a pack of 52 playing cards. The remaing cards are well-shuffled and then a card is drawn at random. Find the probability of getting a (i) face card (ii) red card.

11) Two coin are tossed simultaneously. What is the probability of getting at least one head?

12) Determine the probability of a number selected a random from the number 2, 3, 4, ..., 16 is a multiple of 4?

Group-C (3/4 marks each)

Long answer Type Questions

1) Three unbiased coins are tossed simultaneously. Find the probability of getting.

   (i) at least two head (ii) at most two head.

2) Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is (i) 6 (ii) 12

3) Find tthe probability of getting 53 fridays in the year (i) 2019 (ii) 2020

4) There are 30 cards from 1 to 30. One card is drawn at random. Find the probability that the number of the selected card is not divisible by 3.

5) A bag contains 24 marbels of which x are red, $2x$ are white and $3x$ are blue. A marbel is selected at random. What is the probability that it is (i) not red? (ii) White?

6) Box A contains 25 slips of which 19 are marked Rs 1 and others are marked Rs.5 each. Box B contains 50 slips of which 45 are marked Rs 1 each and others are marked Rs. 13 each. Slips of both boxes are poured into a third box and reshuffled. A slip is drawn at random. What is the probability that it is marked other than Re 1?

Answers

Group-A

1. (a) Probability (b) 0 (c) 1 (d) 1 (e) 1 (f) $\frac{1}{2}$ (g) 0 (h) $\frac{1}{4}$

2. (a) iv (b) iv (c) ii (d) ii (e) iii (f) ii (g) iii (h) i (i) ii (j) iii

3. (a) {HH, TT, HT, TH} (b) $\frac{1}{12}$ (c) $\frac{3}{13}$ (d) $\frac{1}{6}$ (e) $\frac{13}{15}$ (f) $\frac{1}{3}$ (g) $\frac{1}{13}$

Group-B

(1) $\frac{3}{5}$ (2) $\frac{1}{2}$ (3) $\frac{3}{49}$ (4) 18 (5) $\frac{21}{26}$ (6) 0.6 (7) $\frac{1}{2}$ (8) i. $\frac{1}{26}$ ii. $\frac{1}{4}$

(9) {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} (10) i. $\frac{3}{23}$ ii. $\frac{13}{23}$

(11) $\frac{3}{4}$ (l2) $\frac{4}{15}$

Group-C

4. (1) i. $\frac{1}{2}$ ii. $\frac{7}{8}$ (2) i. $\frac{1}{9}$ ii. $\frac{1}{6}$ (3) i. $\frac{1}{7}$ ii. $\frac{2}{7}$ (4) $\frac{2}{3}$ (5) i. $\frac{5}{6}$ ii. $\frac{1}{3}$

(6) $\frac{11}{75}$